NUMERICAL METHOD FOR EVALUATING THE ULTIMATE FLEXURAL MOMENT CAPACITY OF CONCRETE SLAB USING YIELD LINE THEORY

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Abstract

Yield line theory offers a simplified nonlinear analytical method that can determine the ultimate flexural moment capacity of flat reinforced concrete plates subjected to distributed , linear or concentrated loads .In this study, a computer program based on a numerical method which depends on the virtual work method is presented .The method consists of computing the ultimate moment capacity based on the geometry of the assumed collapse mechanism defined by means of nodes , planes and lines . One practical limitation of yield line theory is that it is computationally difficult to evaluate some complex mechanisms . This problem is aggravated by complex geometry and reinforcing layouts commonly found in practice , but since the present method is numerical , it allows the yield lines analysis of plates with complex shapes , assumed mechanisms and loadings. Algorithms for calculation of the work done by the external loads on the plate and the internal work dissipated by the yield line in the assumed mechanism are described and a numerical examples of reinforced concrete slab is given.

The computer program developed in this paper allows the use to search on the mechanisms that give the highest ultimate flexural moment capacity using procedure that can change the mechanism and the associated failure geometry of the slab at each mechanism.

Introduction

Yield line theory is a relatively simple analysis method which is accepted by American Concrete Institute (ACI) to calculate the ultimate bending capacity of flat reinforced concrete slabs .The method was developed by Johanson(4) and since that, it has been applied successfully to both concrete and steel slabs. It is based on the observed failure suggested that all of the yielding in a slab can be lumped into a discrete plastic hinges or yield lines. Thus , a slab is idealized as a series of rigid bodies which are connected together by yield lines .At the ultimate load , the total plastic strain energy in the yield lines is equated to the external work done by the external loads to the displaced shape of the assumed yield line mechanism .Yield line theory is an upper bound energy method , and the quality of the solution depends on the assumed yield line mechanism.

A numerical method based on the yield line theory is presented in this paper .The method differs from the conventional yield line method in that it does not use a direct algebraic description of the problem but rather it uses analytical geometry, vector algebra and the specific geometry of the problem on hand to arrive the solution.

The method presented is general and since it is entirely numerical, it can be applied to plates of arbitrary shape which can be assumed to form any arbitrary yield line mechanism .Furthermore, the method has the advantage of requiring no algebraic manipulation and thus it is not limited by complexity of the algebra, as in some time the case with the conventional yield line method.

There are two solution procedures in the yield line theory, the virtual work method and the so-called equilibrium method, both methods lead to identical upper bound solutions. The virtual work method is simpler in principle and it is used for the numerical method presented in this paper.

Since the yield Line method leads to an upper bound solution, different mechanisms as well as different dimensions for each mechanism must be tried in order to find the lowest predicted strength of the plate. In the conventional methods , the optimum solution of simple problems can be found directly by differentiation . For complex problems a trial and error technique is faster and usually satisfactory . For numerical method presented herein , a simple searching procedure is used to find the optimum solution.

Virtual work method

I -Internal Work

The internal work dissipated by the yield lines during a small motion of the assumed collapse mechanism is represented as the following equation

$$D = \sum_{i=1}^{ny} Mp_i \times \theta_i \times L_i \qquad \ldots (1)$$

Where **Mpi** is the plastic moment capacity resistance per unit length

θ_I is the rotation of each yield line .

Li is the length of each yield line.

n_y is the number of yield line in the assumed mechanism.

Consider , as a simple example , an orthotropic rectangular slab with fixed supports subjected to a uniformly distributed load (w) over the area of the plate and assumed to form a yield line mechanism shown in Figure(1) .The yield lines are numbered from (1) to (8).With ends numbered from 1 to 5 , the flat slab segments , or planes , are numbered from 1 to 5 including the plane represented the fixed support plane which is numbered (1) .A right hand rectangular coordinates system is set with the origin located arbitrary , say at the lower left corner with the **z**-axis pointing upward, the (x,y) coordinates of each node are then determined

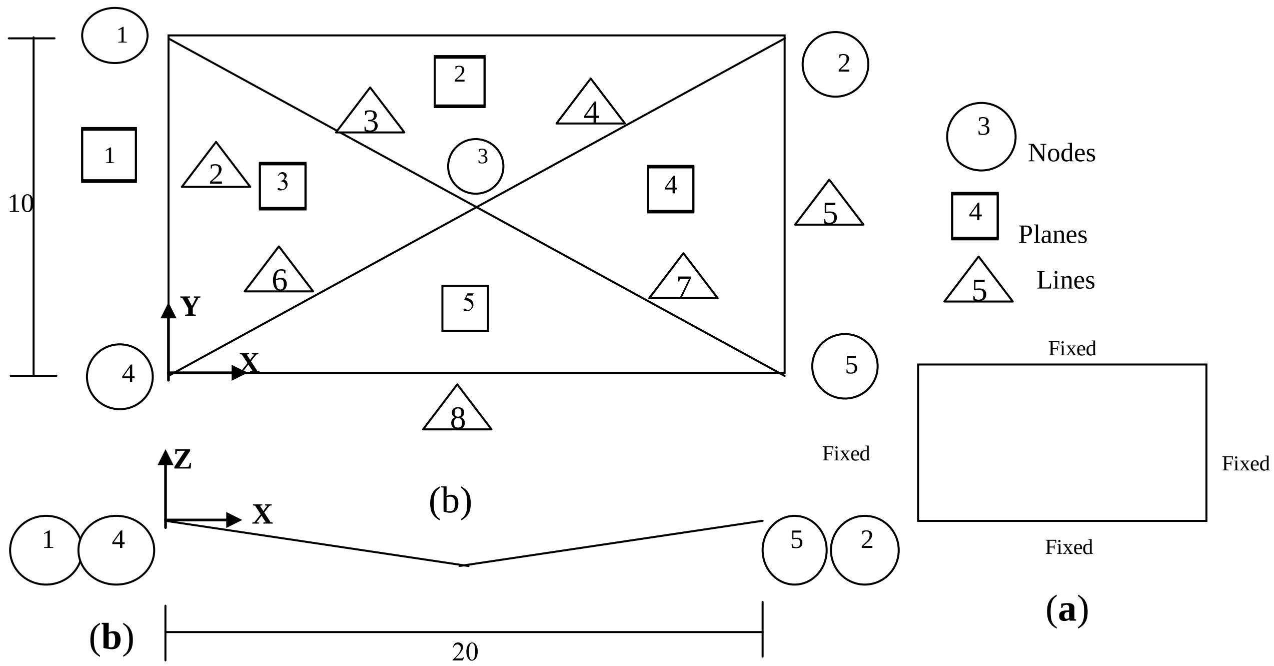


Figure (1) .Fixed edges rectangular slab with uniformly distributed load (a) Plate (b) Model of numerical analysis

The bending resistance per unit length, **Mpi** of a yield line making an angle (α) with the x axis in an orthotropic plate ,(Figure2) if the yield line is sagging :-

$$Mp_i = Mpx\cos^2\alpha + Mpy\sin^2\alpha \qquad \dots (2)$$

and if the yield line is hogging

$$Mp_i = Mp'x\cos^2\alpha + Mp'y\sin^2\alpha \qquad \dots (3)$$

where the function of α are found from

$$\cos^2\alpha = \left[\frac{[y_2 - y_1]}{L}\right]^2 \ , \ \sin^2\alpha = \left[\frac{[x_2 - x_1]}{L}\right]^2 \qquad \dots (4)$$

Mpx and **Mpy** are the sagging resistance in the x and y direction, respectively, and $Mp'x$ and $Mp'y$ are the hogging resistance .x_1, y_1 are the x and y coordinates of the end nodes of the yield line and (l) is the length of the yield line.

In skew concrete slabs the reinforcement may be placed parallel to the edges of the slab, and hence the plate is not orthotropic. Let the reinforcement be placed in the x direction and in the (S) direction, including at an angle β with the x axis ($0 < \beta < 180$) .The bending resistance, **Mpi**, of a yield line is sagging

$$Mp_i = Mpx\cos^2\alpha + Mpy\cos^2(\beta - \alpha) \qquad \dots (5)$$

and if the yield line is hogging]

$$Mp_i = Mp'x\cos^2\alpha + Mp'y\cos^2(\beta - \alpha) \qquad \dots (6)$$

where the functions of (α) are found from :-

$$\cos^2\alpha = \left[\frac{[y_2 - y_1]}{L}\right]^2 \qquad \dots (7)$$

if $(y_2-y_1)(x_2-x_1) > 0$

$$\cos^2(\beta - \alpha) = \left[\frac{\cos\beta(y_2 - y_1)}{l} + \frac{\sin\beta(x_2 - x_1)}{l}\right]^2 \qquad \dots (8)$$

if $(y_2-y_1)(x_2-x_1) \le 0$

$$\cos^2(\beta - \alpha) = \left[\frac{\cos\beta(y_1 - y_2)}{l} + \frac{\sin\beta(|x_2 - x|_1)}{l}\right]^2 \qquad \dots (9)$$

Mpx, $Mp'x$, ***Mps*** and $Mp's$ are the sagging and hogging resistance in the x and s direction, respectively.

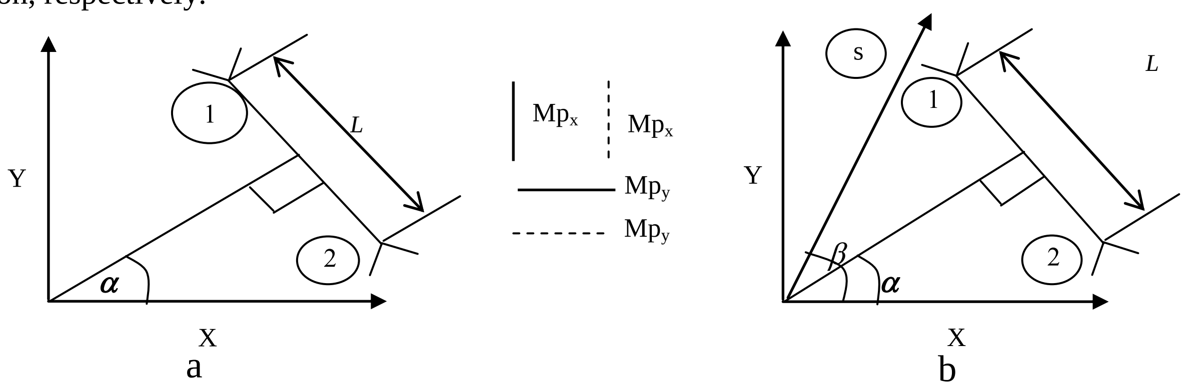


Figure (2) Yield line at general angle (a) In orthotropic plate, (b) In skew concrete slab

Before calculating the rotation of the yield line, planes must be defined, as follows, corresponding to the rigid plate segments of the assumed mechanism. For the plate shown in Figure 1 , plane 2 is defined by nodes 1,2 and 3 , plane 3 is defined the nodes 1,3 and 4,et.Given three points $\mathbf{p_o(x_o,y_o,z_o)}$, $\mathbf{p_1(x_1,y_1,z_1)}$, and $\mathbf{p_2(x_2,y_2,z_2)}$, the algebraic equation of the plane through these points is :-

$$\mathbf{Ax+By+Cz+D=0} \qquad \ldots (10)$$

Where

$$\mathbf{A=(y_1-y_o)(z_2-z_o)-(z_1-z_o)(y_2-y_o)} \qquad \ldots (11)$$

$$\mathbf{B=(z_1-z_o)(x_2-x_o)-(x_1-x_o)(z_2-z_o)} \qquad \ldots (12)$$

$$\mathbf{C=(x_1-x_o)(y_2-y_o)-(y_1-y_o)(x_2-x_o)} \qquad \ldots (13)$$

$$\mathbf{D=-(Ax+By+CZ)} \qquad \ldots (14)$$

In order to define a plane, the three point's p_o, p_1 and p_2 must not to be collinear. This can be checked by comparing the slopes of a line from p_o to p_1 and a line from p_1 to p_2 .For simplicity , the slopes in the x, y plane $(y_2-y_1)/(x_2-x_1)$ are compared .If the slopes are unequal the three points are not collinear and can be used to calculate the algebraic equation of the plane.

Once the equation of a planes has been determined, it can be possible to calculate the deflection of some nodes, which otherwise would have to be calculated by hand using the following equation :-

$$z = -(Ax_c + By_c + D)/C \qquad \ldots (15)$$

The rotation of each yield line is given by the angle θ between the two planes intersecting at that yield line (see Figure 3).Given two planes (**m**) and (**n**) with the following algebraic equations.

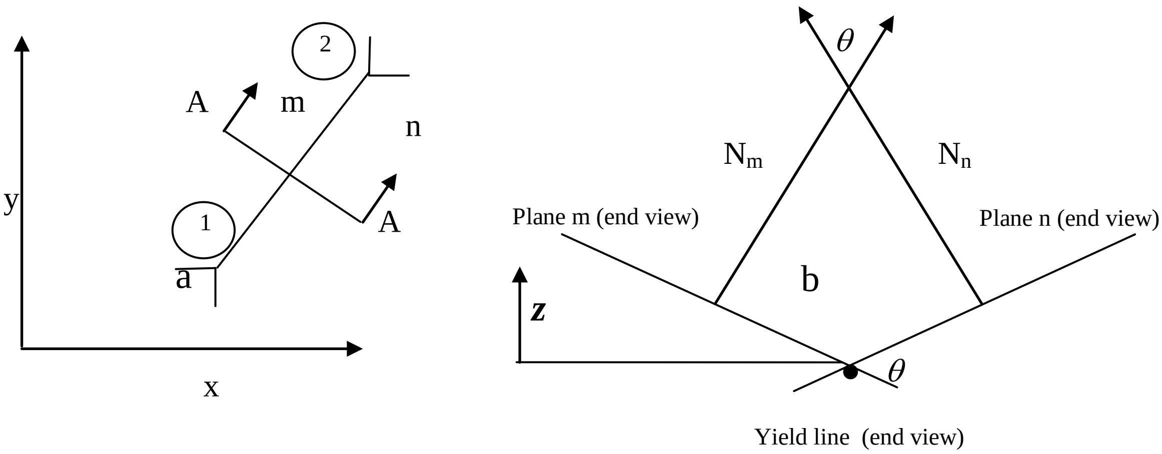


Figure (3).Rotation between the rigid plate segments: (a) Plane view; (b) Section A-A

Plane **m**: $\mathbf{A_mx+B_my+C_mz+D=0}$

Plane **n** : $\mathbf{A_nx+B_ny+C_nz+D=0}$ … (16)

The angle θ between these planes is equal to the acute angle between their normal vectors n_m and n_n and is given by

$$\theta = \tan\theta = \frac{|n_m \times n_n|}{|n_m \cdot n_n|} = \frac{\sqrt{(B_mC_n - C_mB_n)^2 + (C_mA_n - A_mC_n)^2 + (A_mB_n - B_mA_n)^2}}{|A_mA_n + B_mB_n + C_mC_n|}$$

… (17)

Where, since we consider virtual displacement, the angle can be considered small. Such small angles are obtained by assuming small deflections of the yield line mechanism .For example , choosing a maximum value of $1/10^6$ of the plate width , say , for the z coordinate of the nodes in the displaced plate leads to satisfactory results with less errors.
From the numerical description of a yield line mechanism , it is possible to determine the bending sign of the yield lines, i.e. whether they are sagging or hogging .Given a yield line with end nodes 1 and 2 , bounded by planes (**m**) and (**n**) , and using the convention that plane m is on the left hand side of the yield line for an observer standing at node 1 and looking at node 2 , then a point H with coordinates $[x_1+(y_2-y_1), y_1+(x_1-x_2)]$ is always on the right –hand side of the yield line (see Figure 4) .The differences between the z coordinate of point H on plane n and the corresponding coordinate using the equation of plane m indicates whether the yield line is sagging or hogging .When $\mathbf{Z_{Hn}-Z_{Hm}>0}$, the yield line is sagging .When $\mathbf{Z_{Hn}-Z_{Hm}<0}$**,** the yield line is hogging .

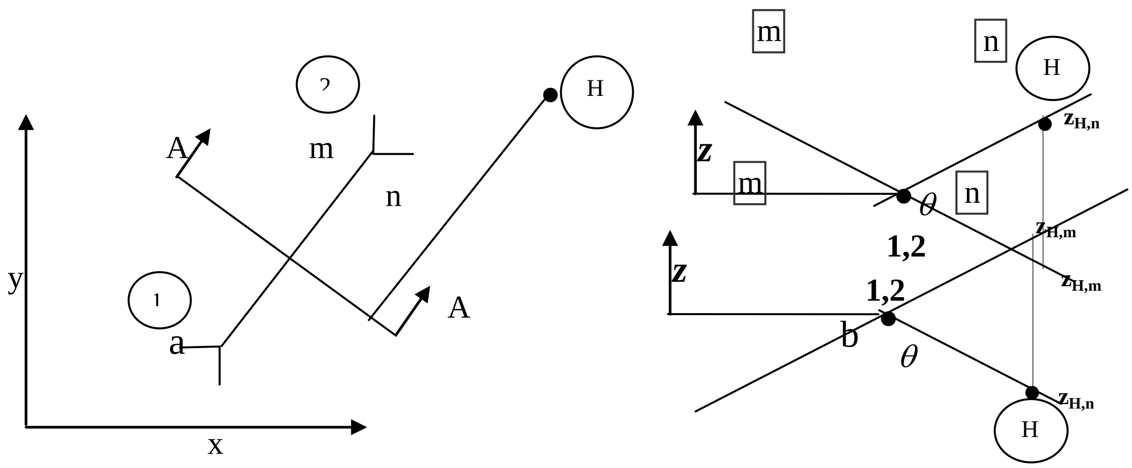


Figure (4) .Bending sign (sagging or hogging) of yield line (a) Plane view (b)Section A-A

The length of each yield line is given by the distance between its end nodes $p_1(x_1,y_1,z_1)$ and $p_2(x_2,y_2,z_2)$, and is equal to

$$\underline{p_1 p_2} = (x_1 - x_2)^2 + (y_1 - y_2)^2 \qquad \dots (18)$$

Where the z coordinates, being very small, are not included.
The plastic moment , the rotation and the length of each yield line having been found using eqns (5-17) , the product of these values is then summed for all yield lines .The sum , $\sum m_p \theta l$, is equal to the total energy , D, dissipated by the yield line .

II-External work

The external work of loads moving the displaced shape of the yield line mechanism is discussed in this section. The external work is the sum of work due to concentrated, line and uniformly distributed loads and is represented in the following equation

$\sum_{i=1}^{NP} P_i \, \delta_i$ Where :- P_i is the loads acing on the slab … (19)

δ is the deflection occurred due to the applied load

The procedure used to calculate the external work for each loading case is as follows:-

1- Concentrated (Point) Load :-

Point load can be defined by a magnitude P_{PL}, a point (node) where the load acts is with coordinates x and y and the plane on which it is acting .The deflection of the load is the $\mathbf{z_{PL}}$ coordinate of the point where the load is acts .If the z coordinate is not specified at the load point, it can be calculated from Eqn. (15) .The work done by the point load (E_{PL})is calculated as:-

$$\mathbf{E_{PL}=P_{PL}.z_{PL}} \qquad \text{… (20)}$$

2- Line Load :-

A uniform or linearly varying line load can be defined by two end nodes with coordinates x_1,y_1 and x_2,y_2 respecively , the magnitude of the line load at each end is P_{LL1} & P_{LL2} and the plane on which the line load is applied is shown on figure(5) .Given this data, the work done by the line load (E_{LL}) is calculated as follows:-

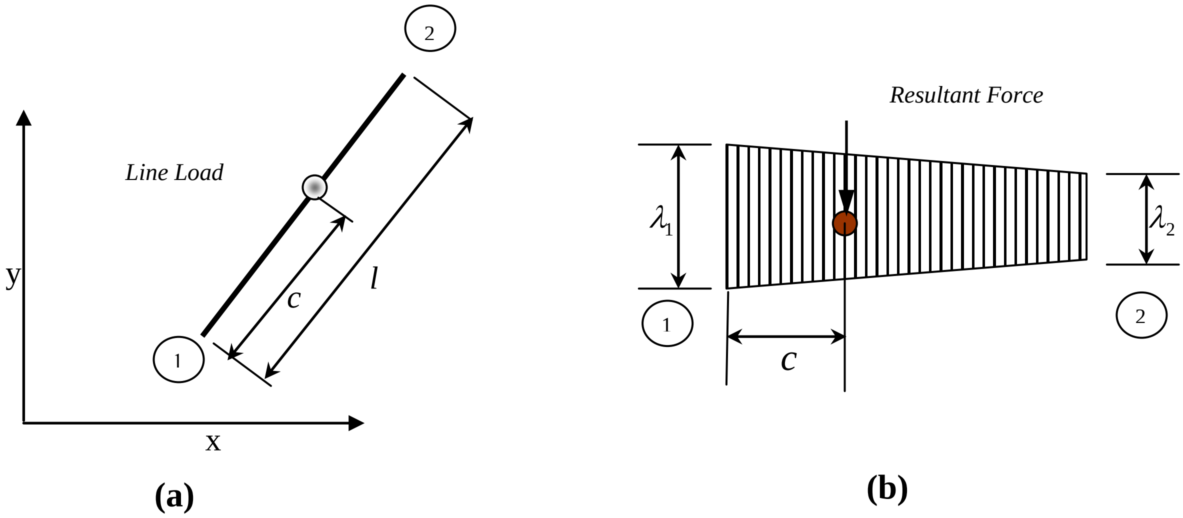


Figure (5) Line load ;(a) Top view ;(b) Front view

The length of the line load is

$$\boldsymbol{l}=\sqrt{(\boldsymbol{x}_2-\boldsymbol{x}_1)^2+(\boldsymbol{y}_2-\boldsymbol{y}_1)^2} \qquad \text{… (21)}$$

The resultant of the line load is

$$\mathbf{P_{LL}=(P_{LL1}+P_{LL2})/2} \qquad \text{… (22)}$$

The location of the resultant is at (x_c,y_c) where

$$\boldsymbol{x_c}=\boldsymbol{x}_1+\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{\boldsymbol{l}}\cdot\boldsymbol{c} \quad , \quad \boldsymbol{y_c}=\boldsymbol{y}_1+\frac{\boldsymbol{y}_2-\boldsymbol{y}_1}{\boldsymbol{l}}\cdot\boldsymbol{c} \qquad \text{…(23)}$$

Where $c=\dfrac{2P_{LL2}+P_{LL1}}{3(P_{LL2}+P_{LL1})}\times l$ … (24)

The deflection at the point through which the resultant load acts is

$$\mathbf{z_{LL}=-(Ax_c+Byc+D)/C} \qquad \text{… (25)}$$

Where A, B, C and D are the coefficients of the algebraic equation of the plane on which the load is acting

Finally, the work done by the line load $\mathbf{E_{LL}}$ is

E$\mathbf{E_{LL}} = P_{LL} \times z_{LL}$ … (26)

3- Uniformly distributed Load (UDL)

A uniformly distributed load can be defined by the N vertices, with coordinates (x_1,y_1), (x_2,y_2),…,(x_n,y_n), of the area covered by the UDL , the magnitude of the UDL , P_{UDL} , and the plane on which it applied. Given these data, the work done by the load is calculated as follows:-

The values and location of the resultant are calculated in a manner similar to that by which the area of a traverse is calculated in surveying (see Figure 6) .The unit resultant load is :-

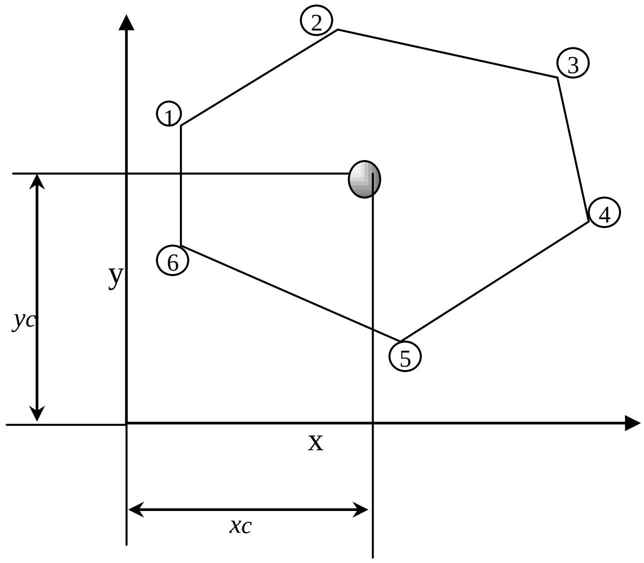


Figure (6); Uniformly Distributed Load

$$Ax = (y_2 - y_1)(x_1 + x_2)/2 + (y_3 - y_2)(x_2 + x_3)/2 + ... + (y_1 - y_n)(x_n + x_1)/2; \quad \text{… (27)}$$

Alternatively,

$$Ay = (x_2 - x_1)(y_1 + y_2)/2 + (x_3 - x_2)(y_2 + y_3)/2 + ... + (x_1 - x_n)(y_n + y_1)/2; \quad \text{… (28)}$$

The location of the resultant is at the centriod of the area covered by the UDL , i.e. at (x_c,y_c) where

$$x_c = \frac{\sum \bar{x}P}{A_y}, y_c = \frac{\sum \bar{y}P}{A_x},$$

where

$$\sum \bar{x}A = \frac{(y_2 - y_1)}{8}\left[(x_1 + x_2)^2 + \frac{(x_2 - x_1)^2}{3}\right] + \frac{(y_3 - y_2)}{8}\left[(x_2 + x_3)^2 + \frac{(x_3 - x_2)^2}{3}\right] \ldots$$

$$+ ... + \frac{(y_1 - y_n)}{8}\left[(x_n + x_1)^2 + \frac{(x_1 - x_n)^2}{3}\right]$$

(29)

and

$$\sum \bar{y}A = \frac{(x_2 - x_1)}{8}\left[(y_1 + y_2)^2 + \frac{(y_2 - y_1)^2}{3}\right] + \frac{(x_3 - x_2)}{8}\left[(y_2 + y_3)^2 + \frac{(y_3 - y_2)^2}{3}\right] + \dots + \frac{(x_1 - x_n)}{8}\left[(y_n + y_1)^2 + \frac{(y_1 - y_n)^2}{3}\right] \quad \dots (30)$$

The deflection at the resultant is:-

$$\mathbf{Z_{UDL}=-(Ax_c+Byc+D)/C} \quad \dots(31)$$

Where A, B, C and D are the coefficient of the algebraic equation of the plane on which the load is acting

Finally the work done by the UDL $\mathbf{E_{UDL}}$ is

$$\mathbf{E_{UDL}=P_{UDL}.z_{UDL}} \quad \dots (32)$$

The work done by the various loading cases having been found using eqns (20) - (32), the values are then summed for all the loads; .This sum $\sum \boldsymbol{P\delta}$ is equal to the work done by the loads (**E**).

Finally, the moment capacity of the slab is found by dividing **E** by the sum of($\boldsymbol{\theta l}$) for all yield lines .

Optimality procedure:-

To find the optimum solution for a yield line mechanism, a series of patterns can be defined and a yield load calculated for each pattern .The pattern that gives the maximum ultimate flexural moment capacity is retained as the solution. Series of patterns can be produced by specifying, for one or more nodes, initial and final positions in the x, y plane, and the number of steps between these positions. These values can be used by an iteration procedures which create the family of patterns .When a yield line mechanism involves several parameters , a corresponding number of iterations procedures can be used to generate all the families of patterns. The iteration procedures should preferably be nested ,so that all possible patterns are created in the same solution .Recursive procedures can be used advantageously for this purpose.

When generating a series of yield line patterns, it is often possible to relate the location of some of the moving nodes to the location of other nodes. This reduces the amount of data required for the optimization, and also it conveniently restricts the movements of the nodes within the limit of validity for the mechanism. One way for establishing the relationship is by locating a node at the intersection of two lines defined by two pairs of nodes .The node at the intersection is called **a slave node**, while the other four nodes guiding the slave node are called **master nodes**. An example to explain this technique will be presented in this paper.

Computer Program

A computer program (**ULTYL)** was developed in this paper to evaluate the ultimate bending moment capacity of a concrete slab with specified dimensions and loading conditions .The program was written using quick basic language (version 4.5) on a PC computer .The flow chart of this program is shown in Figure (7) .This program enable the user to find the maximum bending moment capacity of a slab with an arbitrary shapes and with a various loading conditions (concentrated, line, uniformly distributed)loads and supporting cases by analyzing all the possible mechanism patterns that can be done on the slab and then finding the maximum flexural moment occurred (or. minimum load that causes these patterns) .

Application Examples

Three examples are presented in this paper to demonstrate the capability of the technique and the program developed in this study used to evaluate the ultimate moment capacity of a concrete slabs subjected to a various loading conditions (point, line and uniformly distributed) loads, for each case relationships were found between the ultimate moment capacity and the parameters that relating the failure pattern (mechanism pattern) to show the effect of the mechanism pattern on the total load capacity of the slab , a conclusions was drawn based on these relationships which explain the behavior of the slabs under different loading conditions .

Example (1):-Rectangular slab with simply supported edges

This simple example is one of the common examples used in text books dealing with the yield line theory[2] and it is presented her to compare the results achieved using our program with the results obtained from the classical method of yield line analysis .The dimensions of the slab is shown clearly in figure (8) and the load was assumed to be a uniformly distributed over the area of the slab and its value was assumed to be one unit , the analysis of this slab using the method reported in this study gave a value of (7.07) of a maximum moment capacity which shows reasonable agreement with the value of (7.08) given in Ref.(2)

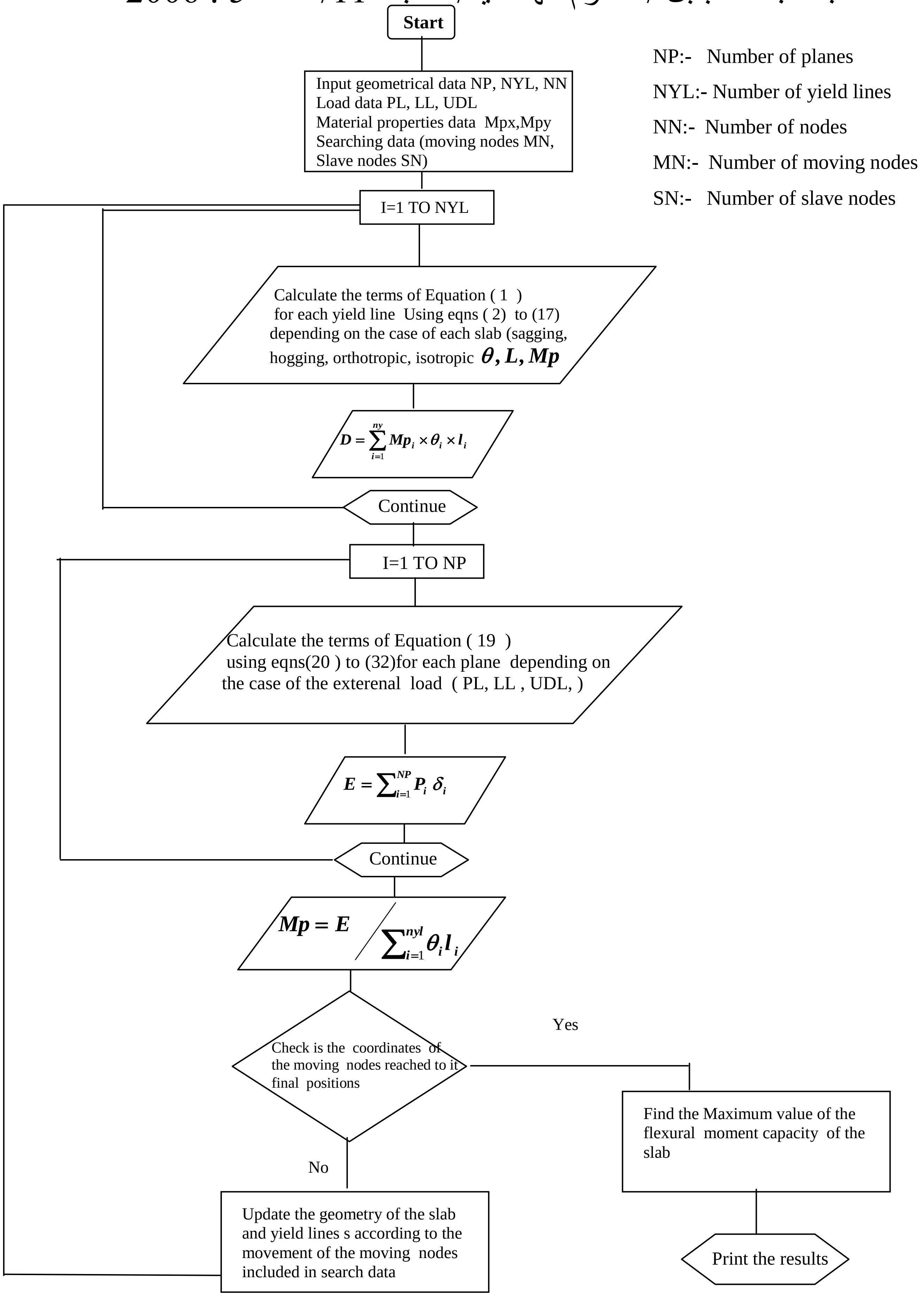


Figure (7) Flow chart of the computer Program (ULTYL)

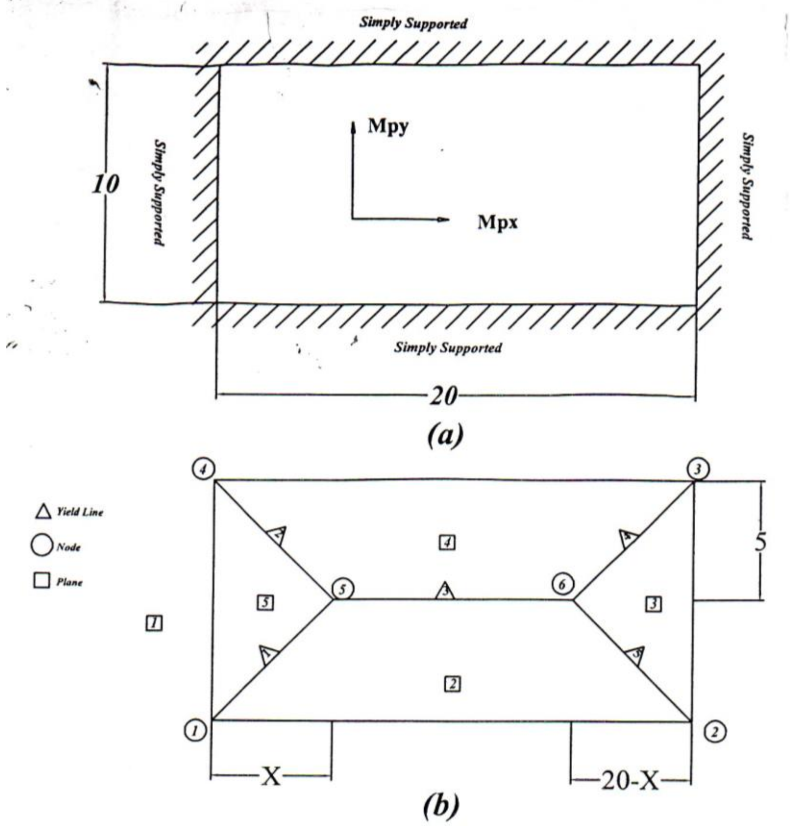


Figure (8): Example (1) Rectangular slab with simply supported edges.

The relationship between the maximum moment capacity of the slab and the location of the intersection of the yield lines from corners of the slab (X – distance in figure 8) is shown in figure (9).From this relation one can observe that when the (X) diverge away from the edge of the slab , a greater capacity of the slab is required to prevent the failure to occur until reach to a specified location (X=6.5 m) over which the relation is conflicted and this point depends on the relative dimensions of the slab and the distribution of the live load on the slab . By this relationship one can re distribute the live loads (line or uniformly distributed) on the slab in a manner that gives a largest load carrying capacity of the structure.

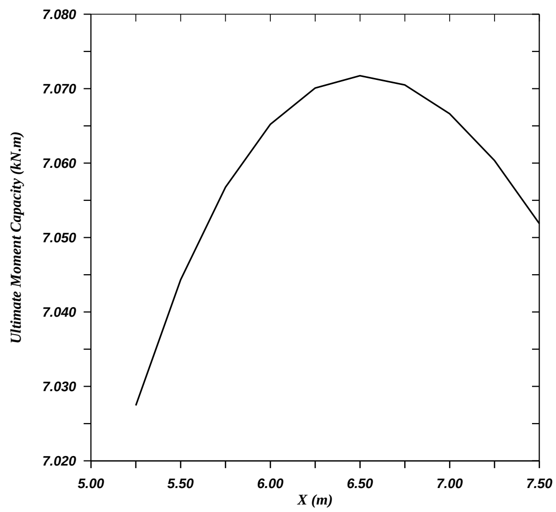


Figure (9) :Relationship between the distance (x) in Figure (8) and the ultimate flexural moment capacity of the slab

Example (2):-Square Concrete Slab with Fixed and free Supporting Edges .

The objective of this example is to demonstrate the efficiency of searching technique used in this paper to trace the ultimate moment capacity of the slab by using the optimality procedure developed in this study and to show the capability of the program used in simulating the line load case . the slab dimensions and supporting case and loading conditions are shown in Figure (10) in this example there is one moving node (7) and two slave nodes (4,6) .The coordinates (x,y) for each slave node was updated by four master nodes (3,72,5) and (7,9,8,5) for nodes 4 and 6 respectively .After the analysis the maximum moment capacity of the slab was found to be (4800) kN.m at (x=10 m) and the curves showing the relationships between the ultimate load capacity an the distances (x) and (y) of the moving node (7) in figure (10) are shown figures (11) and (12).

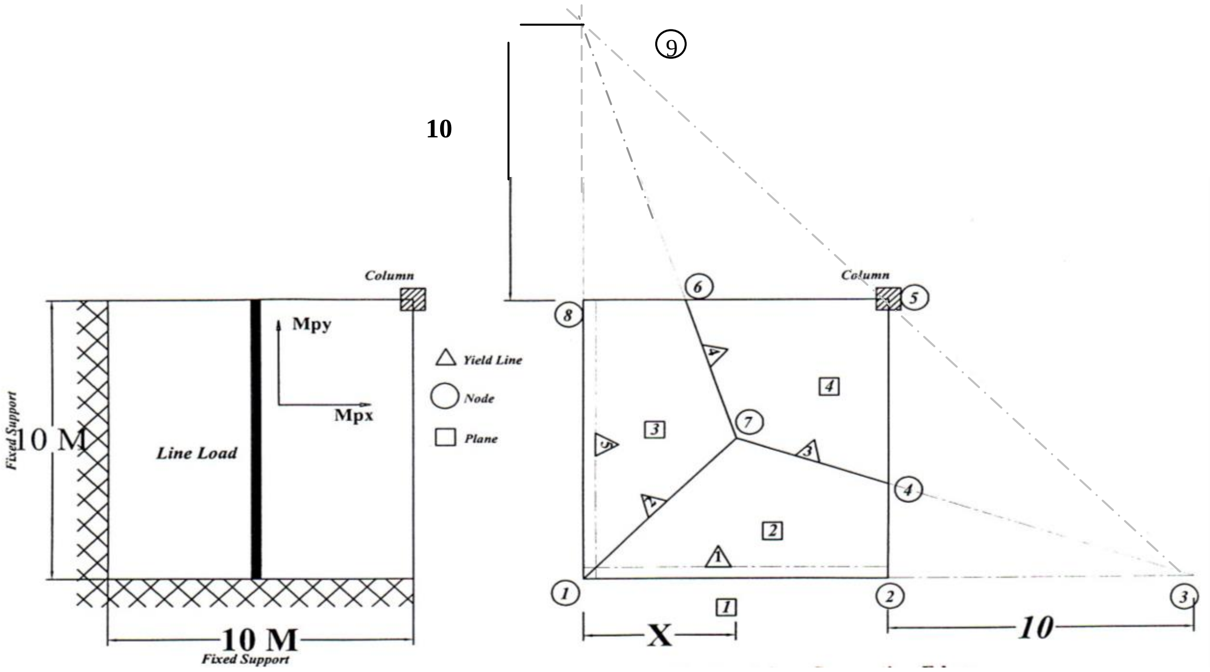
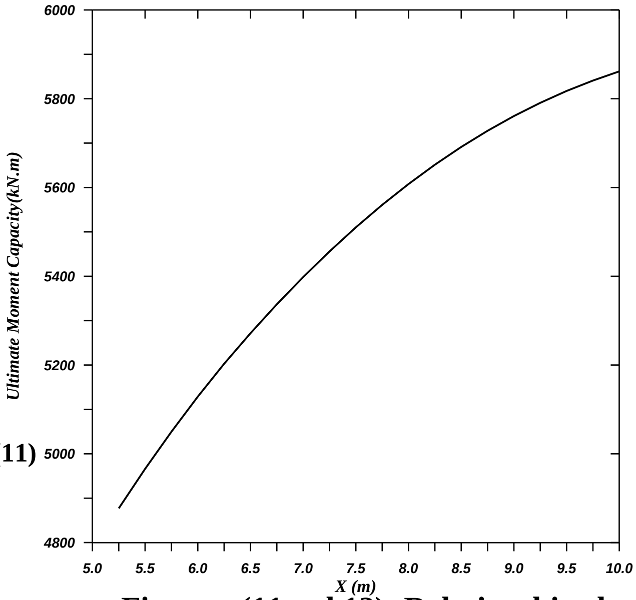


Figure (10): Example (2) Square Concrete Slab with Fixed and free Supporting Edges .

Fig.(11)



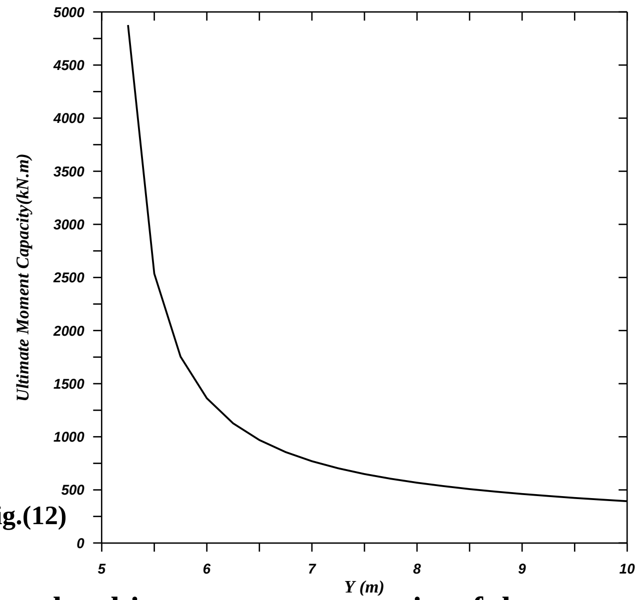


Fig.(12)

Figures (11and 12): Relationships between the ultimate moment capacity of the slab with the distances (x) and(y) in figure (10)

Example (3):-Concrete Slab with different supporting cases

In this example, all the cases of supporting edges and loading conditions are enclosed.This example is taken from Ref.(5) which deals with the problem by the ordinary technique of yield line method .The dimensions and loading conditions and supporting cases are shown clearly in figure (13).This example was analyzed using the developed program. Two moving nodes (4 &9) were considered in searching process .No slaving nodes were needed in this case .The analysis has shown that the ultimate moment capacity of this slab was (3415 Ib.ft)at the positions of the moving node (x_1=9.0 ft ,y=11ft ,x_2=18.5 ft).This indicates good agreement with the (3500 Ib.ft)of Ref.(5).The relationships between the ultimate capacity of the slab with the distances (x_1,x_2,y) of the moving nodes (4&9) in figure (13) are shown in figures (14) (15)and (16) .

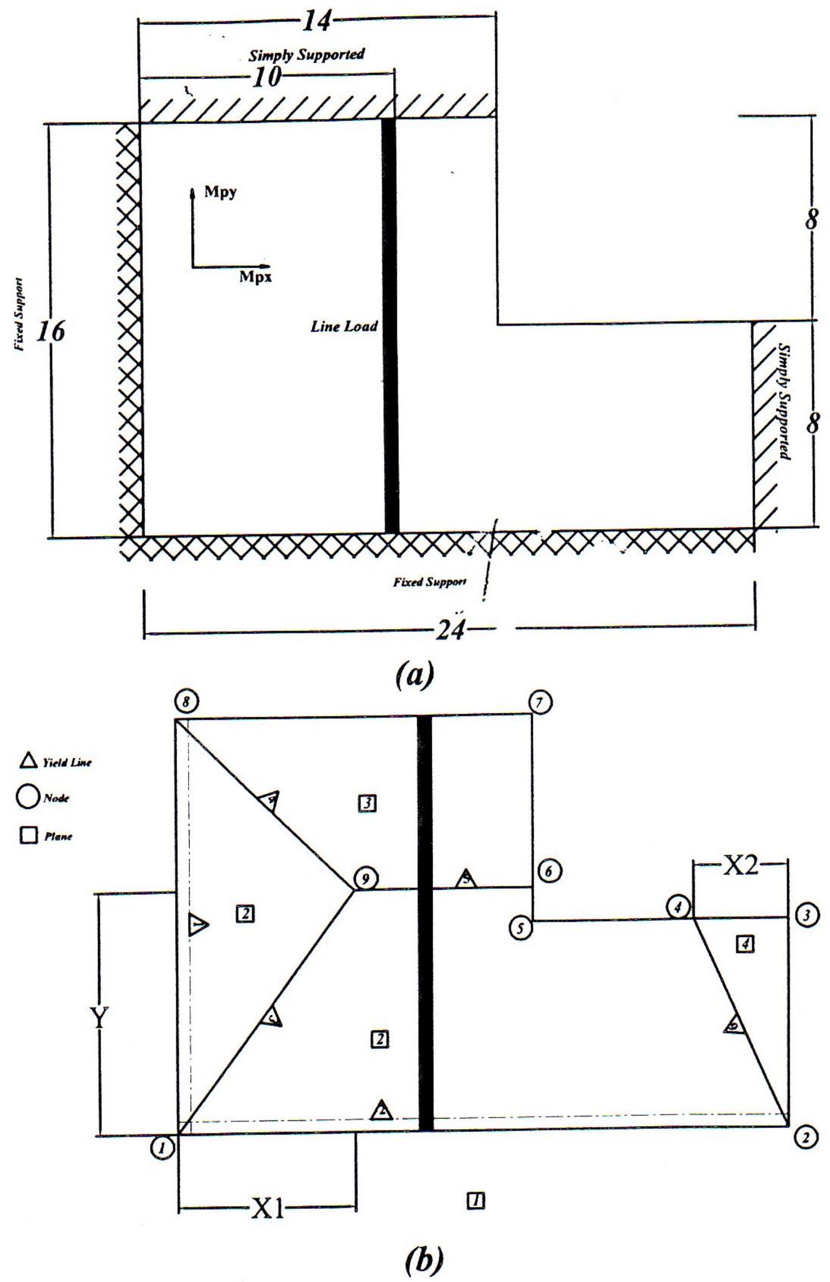


Figure (13):Example (3) Concrete Slab with different supporting cases

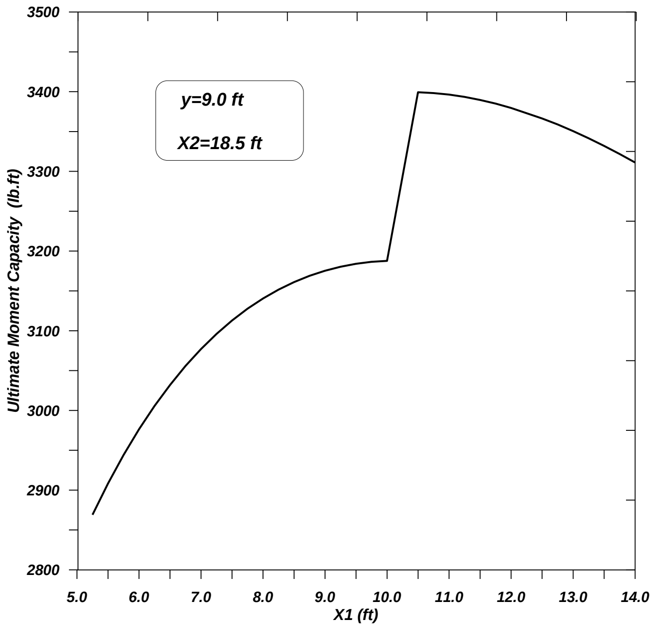


Fig.(14)

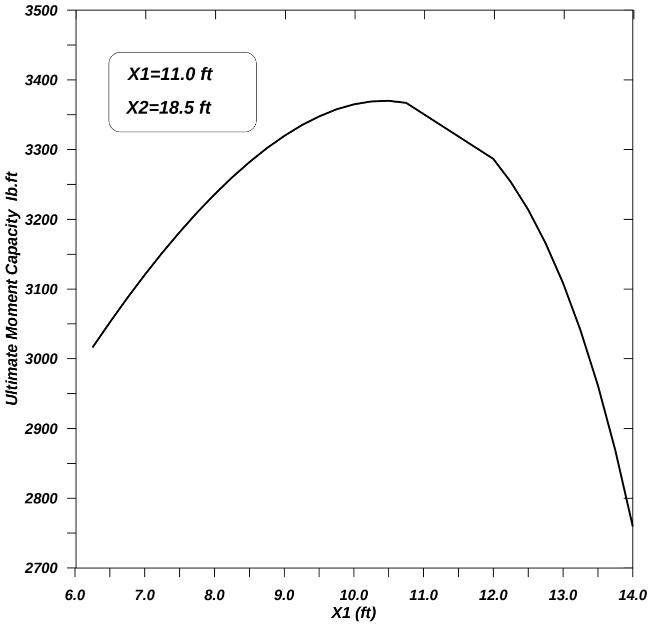


Fig.(15)

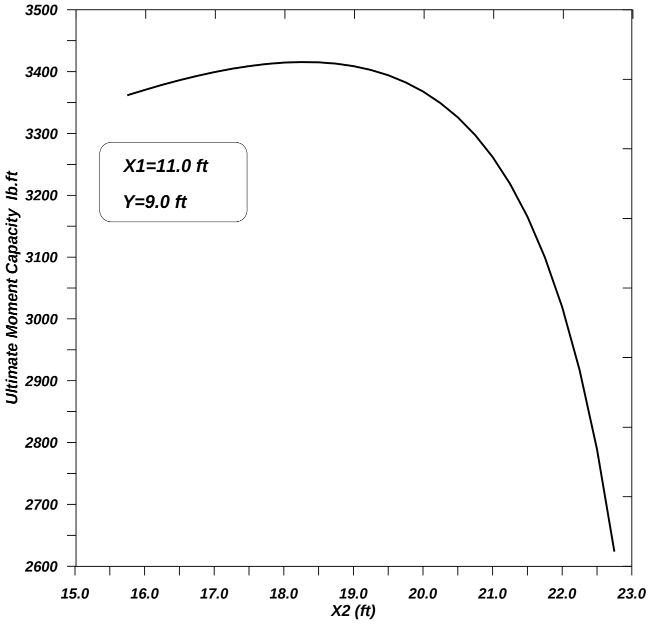


Fig.(16)

Figures (14 , 15 and 16): Relationships between the ultimate moment capacity of the slab with the distances (x_1) , (y)and(x_2) in figure (13)

From figure (14) shown above, one can note that when the distance $\mathbf{X_1}$ exceeds line load location (x=10 ft)the external work done become large suddenly provided that the ultimate moment capacity was become larger too by this conclusion one can practically suppose the weaker point in the slab in order to distribute the live load (line or uniform) in such way that minimize the total external work .Figure(17) shows the relationships between the ultimate moment capacity of the slab with the distances (y) of the moving node (9) in a different locations of distance (X_1) one may conclude that increasing the dimension (X1) leads to increase the total load carrying capacity of the slab when (X_1) exceeds (11 ft) an inversion in the behavior is noticed

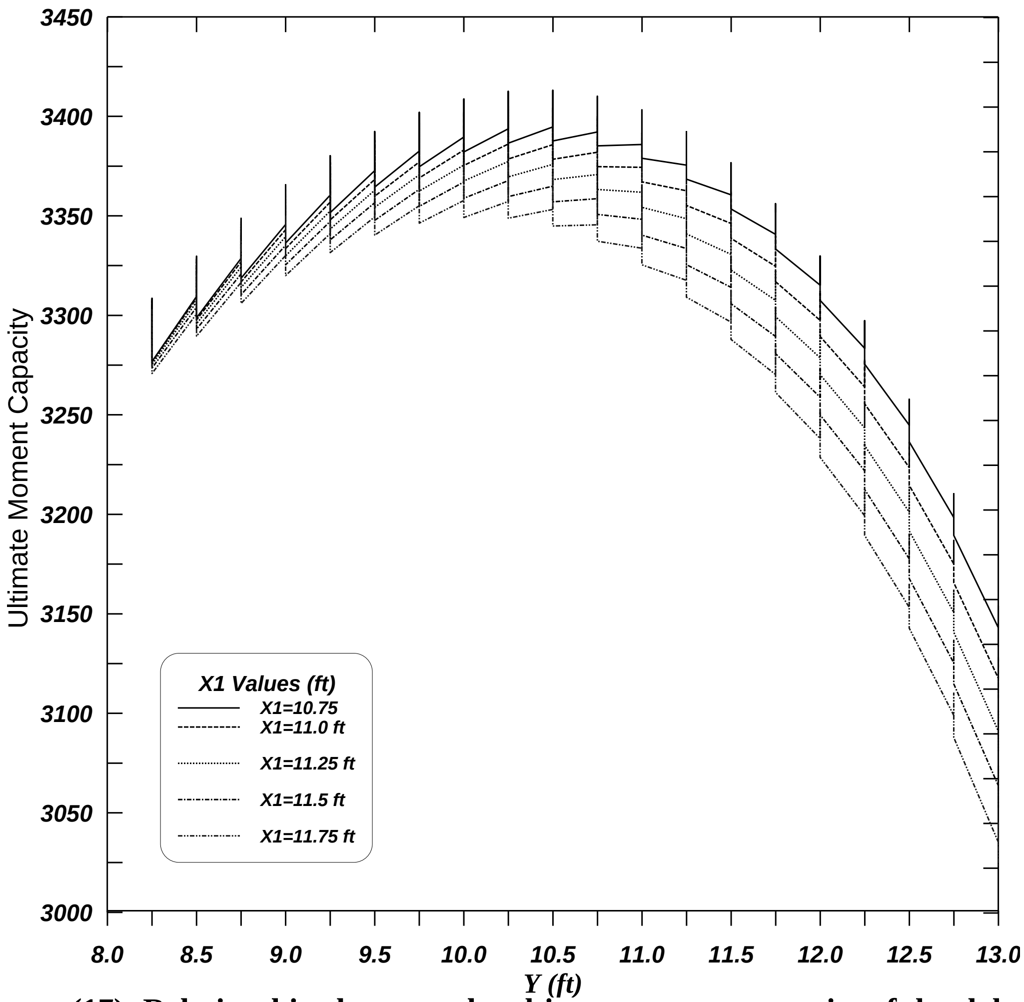


Figure (17): Relationships between the ultimate moment capacity of the slab with the distances (y) at different values of the distance x_1

Conclusions

From the examples presented in this paper one can conclude that the suggested numerical method of analysis and the searching technique adopted to create different yield line mechanisms and then the computer program developed on the bases of these method were very effective in tracing the ultimate moment capacity of the concrete slabs with different loading conditions and supporting cases and with complex geometry shapes .This numerical method and computer program can be considered as a good tool for the civil engineers to tracing the behavior and estimate the maximum moment capacity of the slabs and then design it to carry this moment sufficiently .

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طريقة عددية لإيجاد عزم الانحناء الأعظم للسقوف الكونكريتية باستخدام نظرية خط الخضوع

الخلاصة

تمثل نظرية خط الخضوع **Yield Line Theory** طريقة مبسطة للتحليل اللاخطي للسقوف يتم بواسطتها ايجاد زم الانحناء الأعظم للسقوف الكونكريتية المعرضة ألي حمل موزع أو خطي أو حمل مركز. في هذه الدراسة تم إعداد برنامج حاسوبي بالاعتماد على تمثيل عددي لنظرية خط الخضوع وباستعمال طريقة الشغل الافتراضي **Virtual Work** . تتكون هذه الطريقة من حساب عزم الانحناء الأعظم و بالاعتماد على ميكانيكية الفشل المتوقعة **Collapse Mechanism** والذي يتم تمثيله على شكل مفاصل **Node**s و مستويات **Planes** وخطوط خضوع **Yield lines** .يوجد هناك تحديد في تطبيق نظرية خط الخضوع التقليدية عمليا يتمثل في صعوبة ايجاد وتكوين وحل المعادلات المشتقة للشغل الداخلي و الخارجي للحالات التي تكون فيها أشكال السقوف غير منتظمة أو توزيع التسليح في السقف غير منتظم ولكن الطريقة المطورة في هذه الدراسة تجاوزت هذا التحديد وذلك بسبب اعتمادها على تمثيل عددي **Numerical Idealization** لميكانيكية الحالة موضوعة الدراسة مما يمكنها من حل ميكانيكيات الفشل ذات الأشكال المعقدة وبحالات تحميل مختلفة . تم إعداد مخطط انسيابي لحساب الشغل الخارجي الناتج من الأحمال الخارجية المسلطة على السقف وكذلك الشغل الداخلي المتولد من خطوط الخضوع المتولدة في شكل الفشل المتوقع حيث تم تطبيق هذا المخطط على حالات مختلفة في الشكل و التحميل للتأكد من فعالية الطريقة المقدمة و البرنامج المعد.

يسمح البرنامج المعد في هذه الدراسة للمستخدم بالبحث عن ميكانيكية الفشل التي تعطي اكبر عزم انحناء مسلط على السقف وذلك بواسطة اعتماد أسلوب ارتدادي يمكن من تغيير ميكانيكية الفشل و شكل الفشل المقابل للسقف في كل حالة.