The exact solution of coupled Burgers equation and Boussinesq Regular System via Exp-function Method

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Abstract.

The Exp-function method is one of the powerful methods that appear in recently years for finding exact solutions of different types of nonlinear differential equations. In this work the Exp-function method has been used to find travelling wave solutions of coupled Burgers equation and Boussinesq regular system. The method allows us to found exact solution of coupled Burgers equation and Boussinesq regular system.

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Key Words: Exp-function method, coupled Burgers equation, Boussinesq regular system, exact solution.

1-Introduction

In mathematics many of nonlinear partial differential equations are widely describe an important phenomenon in many branches of sciences such as physical, chemical, economical and biological. More of these equations have been studied to find the exact or approximate solutions by using different methods [1-10]. Recently He and Wu [11] proposed a straightforward and concise method called exp-function method. This method used by many researchers to find the soliton solution and periodic soliton solution of many kind of nonlinear partial differential equations [12-14]. The exp-function method used recently to find the exact solution to the system of nonlinear partial differential equations, for example M. Kazeminia, P. Tolou, J. Mahmoudi, I. Khatami and N. Tolou, used The exp-function method to find the exact solution of Benjamin–Bona–Mahony–Burgers (BBMB) equations [15], J. H. He, and M. A. Abdou, studied systematically New periodic solutions for nonlinear evolution equations using Exp-function method[16], S. Zhang, used the Exp-function method for solving Maccari's system [17]. In this work we introduce two examples to illustrate the Exp-function method, in the first example we investigate the existence of the traveling wave solutions of the homogeneous form of nonlinear coupled Burgers equation of the form [18],

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}+2u\frac{\partial u}{\partial x}+\alpha\frac{\partial}{\partial x}(uv)=0$$
$$\frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial x^2}+2v\frac{\partial v}{\partial x}+\beta\frac{\partial}{\partial x}(uv)=0$$
... (1.1)

where, α , β are parameters.

A. Jawad, M. Petkovic, A. Biswas used Complex tanh method to find the solition solution of the coupled Burgers equation [18]. In the second example we investigate the existence of the traveling wave solutions of the Boussinesq regular system [19], by the form

$$v_t+u_x+(uv)_x-\frac{1}{6}v_{xxt}=0$$
$$u_t+v_x+uu_x-\frac{1}{6}u_{xxt}=0$$
... (1.2)

M. A. Abdul Hussain studied the bifurcation periodic traveling wave solution of the Boussinesq regular system [19].

2. Applications.

In this section, we apply the Exp-function method in the next two subsection to find the exact solution of the coupled Burgers equation and Boussinesq regular system.

2-1. coupled Burgers equation

To apply the Exp-function method to the equations (1.1) we making use the traveling wave transformation
$\eta=kx+qt,\quad u=u(\eta),\quad v=v(\eta)$

where k and q are constants to be determined later. Then equations (1.1) reduces to an ordinary differential equation.

$$qu'-k^2u''+2kuu'+k\alpha(uv)'=0$$
$$qv'-k^2v''+2kvv'+k\beta(uv)'=0$$
... (2.1)

The Exp-function method is based on the assumption that traveling wave solutions can be expressed in the following rational from,

$$u=\sum_{i=0}^{m}\frac{a_i}{(1+\exp(\eta))^i}$$ ... (2.2a)

$$v=\sum_{i=0}^{n}\frac{b_i}{(1+\exp(\eta))^i}$$ ... (2.2b)

Where m and n are positive integer which are unknown to be further determined, a_i and b_i are unknown constants. In order to determine the values of m and n, we balance the linear term u'' with the nonlinear term uu' in the first equation of (2.1), and the linear term v'' with the nonlinear term vv' in second equation of (2.1), by simple calculation, we have

$$u''=\frac{K1}{(1+e^x)^{m+2}}$$ ... (2.3)

$$uu'=\frac{K2}{(1+e^x)^{2m+1}}$$ ... (2.4)

$$v'' = \frac{K3}{\left(1+e^{x}\right)^{n+2}} \quad \text{... (2.5)}$$

$$vv' = \frac{K4}{\left(1+e^{x}\right)^{2n+1}} \quad \text{... (2.6)}$$

where $K1, K2, K3$ and $K4$ are determined coefficients only for simplicity. Balancing highest order of Exp-function in equations (2.3) and (2.4) we have $m=1$. Similarly balancing equations (2.5) and (2.6) we have $n=1$. Equations (2.2a) and (2.2b) becomes,

$$u = a_0 + \frac{a_1}{1+exp(\eta)} \quad \text{... (2.7a)}$$

$$v = b_0 + \frac{b_1}{1+exp(\eta)} \quad \text{... (2.7b)}$$

Substituting equations (2.7a) and (2.7b) into equations (2.1), by the help of software Mathematica 6.0, we have,

$$\frac{1}{A}\left[C_1\, exp(\eta) + C_2\, exp(2\eta)\right] = 0$$

$$\frac{1}{A}\left[D_1 \exp(\eta) + D_2 \exp(2\eta)\right] = 0$$

where $A = \left(1+exp(\eta)\right)^3$

$$C_1 = -(2ka_1^2 + (-k^2 + ((b_0 + 2b_1)\alpha + 2a_0)k + q)a_1 + k\alpha b_1 a_0)$$

$$C_2 = -((k^2 + (2a_0 + \alpha b_0)k + q)a_1 + k\alpha b_1 a_0)$$

$$D_1 = -(2kb_1^2 + (-k^2 + ((2a_1 + a_0)\beta + 2b_0)k + q)b_1 + k\beta a_1 b_0)$$

$$D_2 = -((k^2 + (2b_0 + \alpha a_0)k + q)b_1 + k\beta a_1 b_0)$$

Equating the coefficients of all powers of $exp(n\eta)$ to be zero, we obtain

$$\left[C_2 = 0,\ C_1 = 0,\ D_1 = 0,\ D_2 = 0\right] \quad \text{... (2.8)}$$

solving the system equations (2.8) simultaneously, we get the following solutions

$$a_0 = \frac{a_1 b_0}{b_1},\ a_1 = a_1, b_0 = b_0, b_1 = b_1, \alpha = \alpha, k = a_1 + \alpha b_1,\ \beta = \frac{a_1 + \alpha b_1 - b_1}{a_1}$$
$$q = -\frac{(a_1 + \alpha b_1)(a_1 b_1 + \alpha b_1^2 + 2a_1 b_0 + 2\alpha b_1 b_0)}{b_1} \quad \text{...(2.9a)}$$

$$a_0=a_0, a_1=-b_1, b_0=b_0, b_1=b_1, k=-b_1+\alpha b_1, \alpha=\alpha, \beta=-\alpha+2,$$
$$q=b_1(-1+\alpha)(b_1-\alpha b_1+\alpha a_0-2a_0-\alpha b_0) \quad ...(2.9b)$$

$$a_0=0, a_1=a_1, b_0=0, b_1=b_1, \beta=\beta, k=b_1+\beta a_1,$$
$$q=-(b_1+\beta a_1)^2, \alpha=\frac{b_1+\beta a_1-a_1}{b_1} \quad ...(2.9c)$$

$$a_0=b_0, a_1=-b_1, b_0=b_0, b_1=b_1, \beta=\beta, k=b_1-\beta b_1$$
$$\alpha=2-\beta, \; q=b_1(-1+\beta)(b_1-\beta b_1+2b_0) \quad ...(2.9d)$$

$$a_0=0, b_0=0, q=-k^2, \alpha=\frac{-a_1+k}{b_1}, \beta=\frac{-b_1+k}{a_1} \quad ...(2.9e)$$

$$b_0=0, b_1=-a_1, q=\frac{-a_0a_1k-a_0k^2-a_1k^2}{a_1}, \alpha=\frac{a_1-k}{a_1}, \beta=2-\alpha \quad ...(2.9f)$$

$$a_0=0, b_1=-a_1, q=\frac{-b_0a_1k-a_1k^2+b_0k^2}{a_1}, \alpha=\frac{a_1-k}{a_1}, \beta=2-\alpha, \quad ...(2.9g)$$

$$b_1=-a_1, \beta=\frac{-b_1+k}{a_1}, \alpha=\frac{-a_1+k}{b_1}$$
$$q=\frac{1}{a_1^2}(-a_1^2b_0k+a_0a_1b_1k-2a_0a_1k^2-a_1^2k^2+a_1b_0k^2-a_0b_1k^2) \quad ...(2.9h)$$

$$b_1=\frac{a_1b_0}{a_0}, \alpha=\frac{-a_1+k}{b_1}, \beta=\frac{-b_1+k}{a_1}$$
$$q=\frac{1}{a_1^2}(-a_1^2b_0k+a_0a_1b_1k-2a_0a_1k^2-a_1^2k^2+a_1b_0k^2-a_0b_1k^2) \quad ...(2.9i)$$

we can easily see the equations (2.9a)-(2.9i) reduce to one exact solution. Inserting equations (2.9c) or (2.9e) in to equations (2.7a) and (2.7b) yields the following exact solution

$$u=\frac{a_1}{1+e^{kx-k^2t}},\quad v=\frac{b_1}{1+e^{kx-k^2t}} \qquad \text{...(2.10)}$$

if we chouse $b_0=0, a_0=0$. Then equations (2.9a), (2.9b), (2.9d) and (2.9f)-(2.9h) yields equations (2.10). Equation (2.9i) reduces an exact solution equivalent to the exact solution (2.10) by the form.

$$u=a_0+\frac{a_1}{1+e^{kx+\frac{-2a_0a_1k-a_1^2k^2}{a_1^2}t}},\quad v=b_0+\frac{\frac{a_1b_0}{a_0}}{1+e^{kx+\frac{-2a_0a_1k-a_1^2k^2}{a_1^2}t}} \qquad \text{...(2.11)}$$

The graph of the solution (2.11) is given in figure 1.

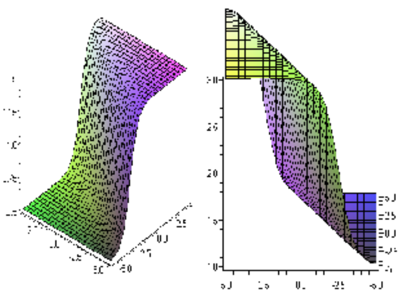

Fig.1 Describe the solution of Eq. (2.11)

2-2. Boussinesq regular system

To apply the Exp-function method to the Boussinesq regular system (1.2) we making use the traveling wave transformation $\eta = k\,x + q\,t,\; u = u(\eta),\; v = v(\eta)$

where k and q are constants to be determined later. Then system (1.2) reduces to an ordinary differential equation.

$$\begin{aligned} &qv' + ku' + k(uv)' - \frac{k^2 q}{6} v''' = 0 \\ &qu' + kv' + kuu' - \frac{k^2 q}{6} u''' = 0 \end{aligned} \qquad \text{... (2.12)}$$

Integrate system (2.12) with respect to η, we have

$$\begin{aligned} &qv + ku + kuv - \frac{k^2 q}{6} v'' = c_1 \\ &qu + kv + k\frac{u^2}{2} - \frac{k^2 q}{6} u'' = c_2 \end{aligned} \qquad \text{... (2.13)}$$

it is easy to see that the system (2.13) can be written in the single equation of the form

$$\begin{aligned} &-\frac{k^3 q^2}{36} u'''' + u''\left(\frac{kq^2}{3} + \frac{k^2 q}{3} u\right) + u\left(-\frac{3q}{2} u - \frac{q^2}{k} + k + \frac{k}{2} u^2 + c2\right) \\ &+ \frac{k^2 q}{6} (u')^2 + \frac{q}{k} c2 - c1 = 0 \end{aligned} \qquad \text{...(2.14)}$$

where

$$v = \frac{kq}{6} u'' - \frac{u^2}{2} - \frac{q}{k} u + \frac{c2}{k} \qquad \text{... (2.15)}$$

Now to apply assumption (2.2a) to equation (2.14), and to determine the value of m, we balance the linear term $u^{////}$ with the nonlinear term u^3 in the equation (2.14), and by simple calculation, we have $m = 2$,and equation (2.2a) becomes

$$u = a_0 + \frac{a_1}{1+exp(\eta)} + \frac{a_2}{(1+exp(\eta))^2} \qquad \text{... (2.16)}$$

Substituting equation (2.16) into equation (2.14), by the help of software Mathematica 6.0, we have,

$$\frac{1}{A}[C_0 + C_1 exp(\eta) + C_2 exp(2\eta) + C_3 exp(3\eta) + C_4 exp(4\eta) + C_5 exp(5\eta) + C_6 exp(6\eta)] = 0$$

where $A = k(1 + exp(\eta))^6$

$$C_0 = \frac{k^2}{2}(a_2 + a_0 + a_1)(a_0^2 + (2a_1 + 2a_2)a_0 + a_2^2 + 2a_2a_1 + a_1^2 + 2) + \frac{k}{36}(-54(a_2 + a_0 + a_1)^2 q - 36c_1 + 36a_2c_2 + 36a_0c_2 + 36a_1c_2) - q((a_2 + a_0 + a_1)q - c_2)$$

$$C_1 = \frac{q^2k^4}{18}(\frac{1}{2}a_1 + a_2) - \frac{2qk^3}{3}(a_1 + a_0 + a_2)(\frac{1}{2}a_1 + a_2) + \frac{k^2}{36}((-12a_1 - 24a_2)q^2 + 108a_0^3 + (270a_1 + 216a_2)a_0^2 + (216 + 108a_2^2 + 216a_1^2 + 324a_1a_2)a_0 + 54a_1^3 + 108a_1^2a_2 + (54a_2^2 + 108)a_1 + 144a_2) + \frac{k}{36}(-108(a_2 + a_0 + a_1)(3a_0 + 2a_1 + a_2)q - 216c_1 + 216a_0c_2 + 180a_1c_2 + 144a_2c_2) - 4q((a2 + \frac{3}{2}a_0 + \frac{5}{4}a_1)q - \frac{3}{2}c_2)$$

$$C_2 = -q^2k^4(\frac{5}{18}a_1 + a_2) + 2qk^3(\frac{2}{3}a_1a_2 + a_2^2 - \frac{1}{12}a_1^2 - \frac{1}{3}a_0a_1) + \frac{k^2}{36}(-24a_1q^2 + 270a_0^3 + (540a_1 + 324a_2)a_0^2 + (324a_1^2 + 540 + 324a_2a_1 + 54a_2^2)a_0 + 54a_1^3 + 216a_2 + 360a_1 + 54a_2a_1^2) + \frac{k}{36}((-810a_0^2 + (-1080a_1 - 648a_2)a_0 - 324a_1^2 - 324a_2a_1 - 54a_2^2)q + 540a_0c_2 - 540c_1 + 360a_1c_2 + 216a_2c_2) - 6q((a_2 + \frac{5}{3}a_1 + \frac{5}{2}a_0)q - \frac{5}{2}c_2)$$

$$C_3 = \frac{11}{6}k^4 q^2 a^2 + 2qk^3 (a_0 a_2 + \frac{7}{6}(\frac{2}{7}a_1 + a_2)a_1) + \frac{k^2}{36}(72a_2 q^2 + 360a_0^3$$
$$+ (216a_2 + 540a_1)a_0^2 + (108a_2 a_1 + 720 + 216a_1^2)a_0 + 360a_1 + 144a_2 + 18a_1^3)$$
$$+ \frac{1}{36}((-1080a_0^2 + (-432a_2 - 1080a_1)a_0 - 108a_1(2a_1 + a_2))q - 720c_1 + 360a_1 c_2$$
$$+ 720a_0 c_2 + 144a_2 c_2) - 4q((a_2 + 5a_0 + \frac{5}{2}a_1)q - 5c_2)$$

$$C_4 = -\frac{4}{9}(a_2 - \frac{5}{8}a1)q^2 k^4 + \frac{4}{3}qk^3((\frac{1}{2}a_1 + a_2)a_0 + \frac{3}{8}a_1^2) + \frac{k^2}{36}((24a_1 + 48a_2)q^2$$
$$+ 270a_0^3 + (270a_1 + 54a_2)a_0^2 + (54a_1^2 + 540)a_0 + 180a_1 + 36a_2)$$
$$+ \frac{k}{36}((-810a_0^2 + (-108a_2 - 540a_1)a_0 - 54a_1^2)q - 540c_1 + 180a_1 c_2$$
$$+ 540a_0 c_2 + 36a_2 c_2) - q((a_2 + 5a_1 + 15a_0)q - 15c_2)$$

$$C_5 = \frac{-k^4 q^2 a_1}{36} + \frac{k^3 qa_1 a_0}{3} + \frac{k^2}{36}(108a_0^3 + 54a_0^2 a_1 + 12a_1 q^2 + 36a_1 + 216a_0)$$
$$+ \frac{k}{36}(-324a_0 q(a_0 + \frac{1}{3}a_1) + 36a_1 c_2 - 216c_1 + 216a_0 c_2) - 6q((a_0 + \frac{1}{6}a_1)q - c_2)$$

$$C_6 = \frac{k^2}{36}((18a_0^3 + 36a_0) + \frac{k}{36}(36a_0 c_2 - 54qa_0^2 - 36c_1) - q(qa_0 - c_2)$$

Equating the coefficients of all powers of $exp(n\eta)$ to be zero, we obtain

$$[C_0=0,\ C_1=0, C_2=0,\ C_3=0, C_4=0,\ C_5=0, C_6=0] \quad \text{... (2.17)}$$

Solving the system, equations (2.17), simultaneously, we get the following solutions

$$a_1 = -a_2,\ a_2 = \frac{4}{3}(-2kq - \sqrt{2}\sqrt{5q^2 + 2k^2 q^2}),\ k = -2\sqrt{\frac{2}{3}},\ a0 = \frac{kq}{24},$$
$$c_1 = \frac{1}{1944}(-1944q + 331q^3), c_2 := \frac{1}{432}(-432k + 85kq^2) \quad \text{... (2.18a)}$$

$$a_0 = \frac{1}{54k^2}\left(38kq - 12k^3 q - \sqrt{31}\sqrt{64k^2 q^2 - 48k^4 q^2 + 9k^6 q^2}\right),\ a_1 = -a_2,$$

$$a_2 = \frac{4}{-8 + 3k^2}\left(9a_0 k^2 - kq\right),\quad c_1 = \frac{1}{24300}\left(-627912a_0^3 k - 24300q + 980748a_0^2 q\right. \quad \text{... (2.18b)}$$

$$\left. - 466080a_0 kq^2 + 151452a_0 k^3 q^2 - 33360q^3 + 49192k^2 q^3 - 12795k^4 q^3\right),$$

$$c_2 = \frac{1}{90}\left(-90k + 351a_0^2 k - 414a_0 q + 186a_0 k^2 q + 66kq^2 - 20k^3 q^2\right)$$

Inserting equation (2.18a) in equations (2.16) yields the following exact solution

$$u = -\frac{\sqrt{6}}{36}q + \frac{-\frac{16}{9}\sqrt{6}q + \frac{4}{9}\sqrt{186}\,q}{1 + e^{-\frac{2\sqrt{6}}{3}x + qt}} + \frac{\frac{16}{9}\sqrt{6}q - \frac{4}{9}\sqrt{186}\,q}{\left(1 + e^{-\frac{2\sqrt{6}}{3}x + qt}\right)^2} \qquad \text{...(2.19)}$$

$$v = \frac{kq}{6}u'' - \frac{u^2}{2} - \frac{q}{k}u + \frac{c2}{k}$$

Inserting equation (2.18b) in equations (2.16) yields the following equivalent exact solution to the solution (2.19) by the form

$$u = \frac{(38kq - 12k^3 q - \sqrt{31}\sqrt{(-8kq + 3k^3 q)^2}\,)}{54k^2} - \frac{4\left(\frac{16}{3}kq - 2k^3 q - \frac{1}{6}\sqrt{31}\sqrt{(-8kq + 3k^3 q)^2}\right)}{(-8 + 3k^2)\left(1 + e^{(kx+qt)}\right)}$$

$$+ \frac{4\left(\frac{16}{3}kq - 2k^3 q - \frac{1}{6}\sqrt{31}\sqrt{(-8kq + 3k^3 q)^2}\right)}{(-8 + 3k^2)\left(1 + e^{(kx+qt)}\right)^2} \qquad \text{...(2.20)}$$

$$v = \frac{kq}{6}u'' - \frac{u^2}{2} - \frac{q}{k}u + \frac{c2}{k}$$

we wrote v by the form of equation (2.15) for simplicity. The graph of the solution (2.20) is given in figure 2.

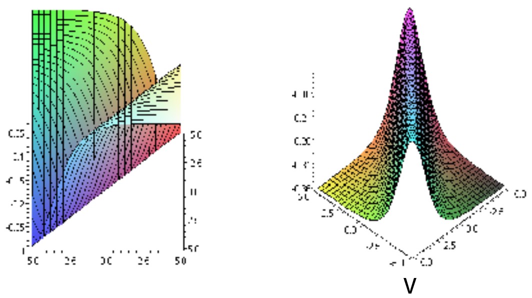

Fig.2 Describe the solution of Eq. (2.20)

3. Conclusion:

In this study we considered the coupled Burgers equation and Boussinesq regular system. We applied the rational form of the exp-function method to find the exact solution of these systems. We think this method can be used to look for exact solutions in a number of differential equations. Of course this method can be implemented in more complicated nonlinear equations by using symbolic computations.

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الحل المضبوط لزوج من معادلة بيركرز وظلم بولسون المنتظم

باستخدام طريقة الدالة الآسية

خالد عبدالاله عبدالزهرة

الملخص

تعتبر طريقة الدالة الآسية واحدة من افضل الطرق التي ظهرت مؤخرا لإيجاد الحل المضبوط لانواع مختلفة من المعادلات التفاضلية الجزئية. في هذا البحث استخدمنا طريقة الدالة الآسية لإيجاد حلول الموجة المتنقلة لزوج من معادلة بيركرز وظلم بولسون المنتظم. إن هذه الطريقة مكنتنا من إيجاد الحل المضبوط لهذه المعادلات.