Counter Example of R.FRAISSE Conjecture

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Abstract:
We present in this paper a Counter Example of R.FRAISSE Conjecture.
((Each Tournament (-1) monomorphic is the dilatation of the points-symmetric Tournament by a chain)). [1]

Introduction and Definitions

- A binary Relation R on a base E is a function R: $E^2 \rightarrow$ {+, -}[1]
- A Tournament T of base E is said to be (-1) - monomorphic if and only if , and $\frac{T}{E} - \{\square\}$) are isomorphic.[3]
- A Chain, or total ordering is a partial ordering whose elements are mutually comparable.[2]
- A relation T of base E is a tournament when it is irrefexive, and that for all $x,y \in E: T(x,y) \neq T(y,x)$ when $x \neq y$. [4]
- A Tournament T of base E is said to be points-symmetric if and only if $\forall x,y \in E, \exists$, an outomorphism $\emptyset$ of T which send x to y. [5]
- A subset I of E base of relation R is an interval of R, if for each $x \in E - I$,and all $y, y' \in I$, we have R (y, x) = R(y', x) and R(x,y) = R(x,y'). [2]
- A binary relation R of base E is said to be decomposable, if we can partition E in intervals, such that at least one of them is of cardinal ≥ 2, if such partition fails to exist, R is, said to be indecomposable. [1]
- The dilatation of a tournament T by a relation R is obtained in replacing each element of T by the relation R, such that R is an interval of T. [5]

- od (x), $x \in$ E, E base of T (respectively ind (x)) ids the number of elements $x \neq y$ such that: x ——> *y* (respectively y ——> x). [4]

A tournament T is said to be regular if and only if $\forall x,y \in T$, od(x) =od(y) [4]

For all $h \in N$, h odd, we define the tournament T_h on the set $I_h=\{0,l,...,2h\}$ by: i——> H if and only if j-I < h+1, (j-i mod 2h+l).[1]

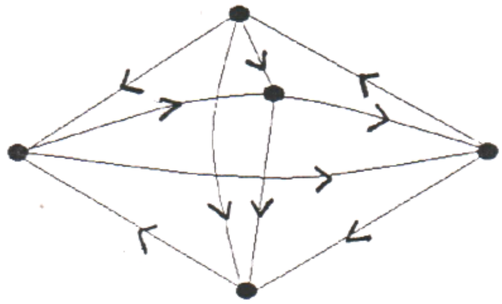
Counter Example

Let CI be the tounament defined on the set I6={ 1,2,3,4,5,6} by the table:

C_I	1	2	3	4	5	6
1	0	0	1	0	0	1
2	1	0	0	1	0	0
3	0	1	0	0	1	0
4	1	0	1	0	1	0
5	1	1	0	0	0	1
6	0	1	1	1	0	0

Proposition:
CI is a tournament (-1)- monomorphic which is not dilatation of the points-symmetric tounament by a chain.

Proof:
CI is a tournament (-1)-monomorphic,since for all elements $x \in I_6$,CI/E-{x}is isomorphic to the structure on five elements as in the following fig.



On the other side to prove that, this tournament not dilatation of the points-symmetric tournament by a chain, we note that this tournament is indecompasable, then, suppose that it satisfies such structure, then the dilatation must be trivial, so it is points-symmetric, but it is not regular (which is easy to prove).

Definition:
$\forall$ $h \in N$ (h odd), we denote by Cl_h the dilatation of T_h by the tournament **CI.**[2]

Theorem:
For all $h \in N$ (h odd), Cl_h is the tournament (-l)-monomorphic which is not dilatation of the points-symmetric tournament by a chain.

<u>Proof.</u>
Clh is the tournament (-1)- monomorphic (obviously) because the dilatation of points-symmetric, on the other hand if Cl_h satisfies such structure, the chain must be of cardinal two, according to the decomposition of Cl_h. By stages of regular tournaments, but there is no chain of cardinal two which is not interval in Clh. In fact, for each element x , y of Cl_h if x, y lies in the same interval, x and y don't form an interval, because the tournament CI is indecompasable, suppose that x , y lies in two different intervals, so they don't form an interval because the tournament T_h is indecompasable, i.e we can always find an element z in the interval different from x and y, such that the segment [x , x] is different from the segment [y , z]. The same argument applies for tournament of large arbitrary cardinals.

References:

[1] Fraisse, R, Theory of relations, North Holland publ., camp Amsterdam, 1987.

[2] Fraisse, R., Abritement Entre Relations et specialement Entre chaines, Symoria Math. Inst Nas, Alte Math. 5, 1970.

[3]Moon, J., structure interieure of of tournaments (-1)-monomorphics J. Grapg Theory, 1988.

[4] Moon, J., Topics on tournaments, New York, Holt. Rinehart and Winston, 1968.

[5] Stockmyer, P., (-1)- Reconstruction of tournaments (-1)- monomorphics, Notices Amer, Math. Soc. 1976.

الخلاصة

لقد وجدت مثال مضاد الى حدسية فرايسي القائلة بأن ((لكل علاقة دورية (1−) مونومورفي هي تمديد لعلاقة دورية اخرى متناظرة الرؤوس بواسطة سلسلة .