ONE–DIMENSIONAL FINITE ELEMENT MODEL FOR ANALYSIS OF BONDED REINFORCING BARS

Ammar Yaser Ali **Jabbar Hmood** **Salih Essa**
College of Engineering , University of Babylon

Abstract

A one-dimensional finite element model for predicting the behavior of deformed bars embedded in concrete is developed. In this model, the bond between steel and concrete is simulated with discrete springs that connect the bar to concrete along the anchorage length, whereas the reinforcing bar is subdivided into axial elements. This technique involves the construction of the total tangent stiffness matrix of the anchorage for use in an incremental solution algorithm of the nonlinear problem.

The proposed model is used for simulating the behavior of bonded bar with common types, such as the anchored bars in beam-column connections and spliced bars of the reinforcement lap splices. The analytical results are compared with available results of pull- out and flexural tests, and suitable agreement is found.

Notations

d_b	:	**Bar diameter**	δ_i	:	**Displacement (or slip) of node i**
E_s	:	**Steel modulus of elasticity**	c	:	**Minimum thickness of concrete surrounding the bar**
fc'	:	**Cylinder conc. comp. Strength**	σ_{si}	:	**Steel stress of element i**
f_{cu}	:	**Cube conc. comp. strength**	σ_{bi}	:	**Bond stress of node i**
K_{bi}	:	**Stiffness of spring i**	fy	:	**Yield strength of steel**
K_{si}	:	**Stiffness of bar element i**	Ls	:	**Splice length**
K_t	:	**Stiffness of spring per unit area**	σ_{so}	:	**Steel stress at splice end**
K_T	:	**Total stiffness matrix**	f_t'	:	**Tensile strength of concrete**
l_i	:	**Length of bar segment i**			
P_i	:	**Applied load at node i**			

1. Introduction

The interaction of deformed bars with concrete is a complex phenomenon that has important effects on the response characteristics of reinforced concrete elements and structures under static and dynamic loads. For example, the beam fixed-end rotation that contributes significantly to the overall beam deflections is caused by pullout of beam longitudinal bars anchored in beam-column connections(1,2), as shown in Fig.(1). Also, in case of discontinuity of reinforcing bars which is often encountered in concrete structures, lap splices are preferred means for providing continuity because of their practical and economical characteristics(3), as shown in Fig.(2). The presence of lap splices are generally recognized to represent potential weakness in components of concrete structures(4).

A common way to describe the bond between a steel bar and concrete is through the relationship between the local bond stress and the relative slip of the bar. The bond stress versus slip relationship represents the overall behavior at the interface between a steel bar and concrete. The bond stress distribution along the bar is of major importance and attempts to predict the bond stress distributions were found in many works. Martin[5] derived a solution utilizing the ascending part of the bond-slip relationship which was found too complicated for calculations[6]. The complexity of the bond-slip law led some of the works to numerical solution of the equilibrium differential equation by stepwise integration, using small increments of length The bond stress distribution may be predicted using the finite elements method, where the bar-concrete interface is modeled either by springs or by special interface elements[10,11].

Tepfers[12] utilized modulus of displacement theory to determine the distribution of bond and steel stresses along bars of the lap splice with and without interaction of surrounding concrete.

2. Object1ve And Importance Of The Present Study

In this paper a one-dimensional finite element model for predicting the behavior of deformed bars anchored in concrete is developed. In this model, the steel bar and concrete are subdivided into finite elements and the bond between steel and concrete is simulated with discrete springs that connect the bar to concrete along the anchored length. This technique involves the construction of the tangent stiffness matrix of the anchorage zone for use in an incremental solution algorithm.

The proposed model can be used for simulating the behavior of bonded bars with common types, as in single anchored bars and lap spliced bars, which can mostly encountered in construction of reinforced concrete structures. The model can be used for efficient idealization of anchorages in analytical studies of structural subassemblies and complete structural systems.

3. Bonded Bar Model

In two and three dimensional finite element modeling of reinforced concrete, the bond between steel and concrete is usually idealized by discrete springs connecting the bar at different points along its length to the concrete[13,14,15,16]. In this study, the idea of idealizing a bond with discrete springs is employed, whereas the reinforcing bar is subdivided into finite axial elements (i.e. truss element). The proposed model is shown in Fig.(3). Each spring in this model represents the bond resistance along its tributary length of the bar. The concrete strains are assumed to have negligible effects on an anchored bar behavior, and thus the springs are assumed to be rigidly fixed at the ends connected to concrete.

4. Formulation Of Total Stiffness Matrix

Construction of the tangent stiffness matrix of this bonded model at any stage during the loading history requires knowledge of the steel and bond constitutive laws. With these values available, the bond tangent stiffness per unit interfacial area k and the steel tangent stiffness k can be evaluated. Then the stiffness for the steel segment K_{si} and for bond spring K_{bi}, can be computed respectively as follows:

$$\mathbf{K_{si}} = \begin{bmatrix} \mathbf{k_{si}} & -\mathbf{k_{si}} \\ -\mathbf{k_{si}} & \mathbf{k_{si}} \end{bmatrix} \qquad \text{and} \quad \mathbf{k_{si}} = \frac{\frac{\pi}{4}\mathbf{d_b^2 E_s}}{\ell_i} \quad \text{..................................} (1)$$

$$\mathbf{K_{bi}} = (\pi\ \mathbf{d_b})\,\ell_i\,\mathbf{k_{ti}} \quad \text{...} (2)$$

where:

d_b = bar diameter

l_i = length of the ith steel segment

E_s = steel modulus of elasticity.
k_t = the bond tangent stiffness per unit area, which represents the slope of ascending branch of the available bond-slip constitutive relationship.

The total tangent stiffness matrix KT of the idealized system shown in Fig.(3), with N degree of freedom (N is number of discrete points along the bar length), can then be constructed as follows:

$$\mathbf{K}_T = \begin{bmatrix} k_{s1}+K_{b1} & -k_{s1} & 0 & 0 & 0 & 0 \\ -k_{s1} & k_{s1}+k_{s2}+K_{b2} & -k_{s2} & 0 & 0 & 0 \\ 0 & -k_{s2} & k_{s2}+k_{s3}+K_{b3} & -k_{s3} & 0 & 0 \\ 0 & 0 & -k_{s3} & k_{s3}+k_{s4}+K_{b4} & & \\ & & & & k_{s(N-2)}+k_{s(N-1)}+K_{b(N-1)} & -K_{s(N-1)} \\ & & & & -K_{s(N-1)} & K_{bN} \end{bmatrix} \quad \text{.......(3)}$$

This tangent stiffness matrix of Equation (3) defines the relationship between the incremental nodal forces (ΔP_1, ΔP_2, ΔP_N) and the incremental nodal displacements (or slips) values along the bar length ($\Delta\delta_1$, $\Delta\delta_2$,…, $\Delta\delta_N$), as follows:

$$\begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_N \end{Bmatrix} = \mathbf{K}_T \begin{Bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \vdots \\ \Delta\delta_N \end{Bmatrix} \quad \text{.......(4)}$$

or assuming that inversion of stiffness matrix,

$$F_T = K_T^{-1} \quad \text{.......(5)}$$

Then

$$\begin{Bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \vdots \\ \Delta\delta_N \end{Bmatrix} = F_T \begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_N \end{Bmatrix} \quad \text{.......(6)}$$

From Equation (6) and taking advantages of the fact that in anchored bars the loads are applied only at one end, as in pullout bars and bars of lap splices, then it can be concluded that

$$\begin{Bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \vdots \\ \Delta\delta_{N-1} \\ \Delta\delta_N \end{Bmatrix} = \mathbf{F_T} \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{\Delta P_N} \end{Bmatrix} \quad \text{.........(7)}$$

or

$$\begin{Bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3 \\ \vdots \\ \Delta\delta_{N-1} \\ \Delta\delta_N \end{Bmatrix} = \Delta \mathbf{P_N} \begin{Bmatrix} \mathbf{f_{1,N}} \\ \mathbf{f_{2,N}} \\ \mathbf{f_{3,N}} \\ \vdots \\ \mathbf{f_{N-1,N}} \\ \mathbf{f_{N,N}} \end{Bmatrix} \quad \text{.........(8)}$$

Equation (8) gives the incremental nodal displacement (or slip) values ($\Delta\delta_i$)in terms of the incremental end force (ΔP_N), where $f_{i,N}$ is the element in the ith row and N the column of matrix F.

5. Bond And Steel Constitutive Laws

The local bond stress-slip relationship adopted in the present study consists of two branches. There are elastic branch and flat plastic branch with yielding point represents the bond stress necessary for split cracking τ_{cr}, as shown in Fig.(4).
The slope of ascending branch k is evaluated as follows [17]

$\mathbf{k_t = 2.4 f_{cu}}$ (N/mm^3) for steel of Grade 40 .... (9a)

$\mathbf{k_1 = 3.4 f_{cu}}$ (N/mm^3) for steel of Grade 60............................ (9b)

where f_{cu} he cubic compressive strength of concrete so the bond stress for split cracking (τ_{cr}) is calculated as follows [12]

$$\tau_{cr} = \frac{f'_t}{d_b}(1.30\,c + 0.15\,d_b) \qquad \text{for single bar ....(10a)}$$

$$\tau_{cr} = \frac{f'_t}{2\,d_b}(1.30\,c + 0.15\,d_b) \qquad \text{for spliced bar......................(10b)}$$

For steel, a simple bilinear constitutive law, as shown in Fig.(5) is used. Both the steel and bond constitutive laws are incorporated into Equation (3) for deriving and updating the total stiffness matrix of bonded bar.

6. Solution Algorithm Of The Nonlinear Problem

The procedure adopted in the nonlinear solution of the present work involves the following steps:

a. For the nth loading increment, add the current increment of load A to the previous total load p^{n-1},

$$\mathbf{P^n = P^{n-1} + \Delta P^n} \quad \text{.........(11)}$$

b. Evaluate the stiffness of each element (steel bar and bond element) in accordance to the stress level reached, as described in item (4) above. These stiffnesses are assumed to get the total tangential stiffness matrix according to Equation (3).

c. The system of equations is solved for incremental displacement (or slip) A& according to Equation (8).

d. Add incremental displacements to the total displacements (or slips) of the previous load step, to get:

$$\{\delta^{n}\} = \{\delta^{(n-1)}\} + \{\Delta\delta^{n}\} \quad \text{.......} (12)$$

e. 1. For each discrete spring i, calculate the bond stress σ_{si},

$$\sigma_{bi}^{n} = Kt\ \delta_{i}^{n} \quad \text{.......} (13)$$

2. For each steel bar element i, calculate the steel stress o

$$\sigma_{si}^{n} = \frac{\delta_{i+1}^{n} - \delta_{i}^{n}}{\ell_{i}} E_{s} \quad \text{.......} (14)$$

f. According to the current stresses, the stiffness of each element (steel bar element, bond element) is evaluated and stored for the next increment of load (n+1).

g. At this point, the load increment for the next load step Δp^{n+1} is added and go back to step (a).

A computer program NABB 1 (Nonlinear Analysis of Bonded Bar in One Dimension) is written for the solution algorithm illustrated above. The flow chart of this program is shown in Fig.(6).

7. Numerical Examples And Results

Two examples of typical comparisons between the analytical and experimental bonded bar are made in this section.

A. In the first(1), a deformed bar of diameter (25-mm) and yield strength of (450-MPa) were anchored in concrete specimen with a compressive strength of (30-MPa), as shown in Fig.(7a). The anchored bar in the test specimen is modeled as shown in Fig.(3), with (10) springs simulating the bond between steel and concrete. In this model, thickness of concrete surrounding the bar was four times the bar diameter on each side.

Fig.(7-b) compares the experimental and theoretical end force end slip relationship for the above specimen tested under monotonic pull only, and good agreement can be observed.

B. Specimen of reinforced concrete beam containing tensile reinforcement lap splices and subjected to a constant moment have been studied (12). The geometric and material properties of the tested specimen are shown in Fig.(8.a). The measurements of the steel strains were recorded at selected point of lap region for two steel stress levels at spliced end, 133 MPa and 309 MPa. The spliced bar is modeled as shown in Fig.(3), with 20 springs simulating the bond between steel and concrete.

Comparison of measured steel strains with those calculated from the proposed model are described in Fig.(8.b), for reinforcing bar coming from right side of the lap region. Again, suitable consistent between the experimental and analytical results can be concluded.

8. Summary and Conclusions

A new modeling technique based on one-dimensional finite element method of analysis, is developed for predicting the behavior of deformed bar embedded in concrete. In this model, the bond between steel and concrete is simulated with discrete springs connecting the bar to concrete along the anchorage length, whereas the reinforcing bar is subdivided into several axial elements. The global tangent stiffness matrix is built up for use in an incremental solution algorithm of the nonlinear problem.

Two types of bonded bars are considered: anchored bar under effect of pull-out and spliced bars of tensile reinforcement lap splices. In spite of its simplicity, the proposed model to be efficient and it is capable of estimating test results with suitable accuracy.

References

1. Viwathanatepa, S., Popov, E.P., and Bertero, V.V., "Effects of Generalized Loadings on Bond of Reinforcing Bar Embedded in Confined Concrete Blocks", Report No. UCB/EERC-79/22, Earthquake Engineering Research Center, University of California, Berkely, Calfi., (1979), p.
2. Filippou , F.C., and Popov, E. P., and Bertero V. V. "Effect of Bond Deterioration on the Hysteric Behavior of R/C Joints", Report No. UCB/EEkC- 83/19, Earthquake Engineering Research Center, University of California, Berkely, Calfi., (1983), p.
3. Au, A.Y., "Behavior of Lap Splices in Reinforced Concrete Beams Under Inelastic Cyclic Loads", M.Sc. Thesis, Department of Building and Construction, University of Technology, Baghdad, December (1990).
4. ACI Committee 318, "Building Code Requirements for Reinforced Concrete and Commentary-ACI 31 8R-95", American Concrete Institute, Detroite (1995), 369 pp.
5. Martin, H., "Zusammenhsng zwischen oberflachenflachebeschaffenheit, Verbund and Sprengwirkung von Bewehrungsstahlen unter Kurzzeibtbelastung, Deutscher Ausschuss fur stahibeton, Heft 228, 1973.
6. Yankelevsky, D.Z., "New Finite Element for Bond Slip Analysis", Journal of Structural Engineering, ASCE, Vol.111, No.7, July, 1985, pp.1533-1542.
7. Ciampi, V. *et al*., "Analytical Model for Deformed Bar Bound Under Generalized Exitations", Proceedings, IABSE Colloquium on Advanced Mechanics in Reinforced Concrete, Deift, June, 1981.
8. Morita, Kaku, "Local Bond Stress-Slip Relationship Under Repeated Loading", Symposium on resistance and Ultimate Deformability of Structures acted on by Well Define Repeated Loads, JAB SE, Lisbon, 1973.
9. Tassios, T.P., and Yannopoulos, P.J., "Analytical Studies on Reinforced Concrete Under Cyclic Loading Based on Bond Slip Relationships", ACI-Journals, May-June, 1981.
10. Scha H., "A Contribution to the Solution of Contact Problems with the Aids of Bond Elements", Computer Methods in Applied Mechanics and Engineering, Heft6, 1975.
11. Bresler, B., and Bertero, V.V., "Behavior of Reinforced Concrete Under Repeated Loads", Journal of the Structural Engineering Division, ASCE, Vol.94, No. ST-6, June, 1968.
12. Tepfers, R., "A Theory of Bond -Applied to Over Lapped Tensile Reinforcement Splices for Deformed Bars", Application No.73:2, Division of Concrete Structures, Chalmers University of Technology, Goteborg, Sweden (1973), 328 pp.
13. Naji, J.H., "Nonlinear Finite Element Analysis of Reinforced Concrete Panels and Infilled Frames Under Monotonic and Cyclic Loading", Ph.D. Thesis, University of Bradford, (1989).

14. Al- Sha'arbaf, I.A.S., "Three-Dimensional Non-Linear Finite Element Analysis of Steel Fiber Reinforced Concrete Beams Subjected to Combined Bending and Torsion", Ph.D. Thesis, University of Bradford, (1990).
15. Al-Mahaidi, R.S.H., "Nonlinear Finite Element Analysis of Reinforced Concrete Deep Members", Ph.D. Thesis, Cornell University, Ithaca (1978).
16. Ahmad, M. and Bangash, Y., "A Three Dimensional Bond Analysis Using Finite Element", Computers and structures, Vol.25, No.2 (1987), pp. -296.
17. Berggren, L., "Utdragsprov med Kort Vidhaftningslangd", Chalmers Tekniska Hogskola, Instituionen for Betongbyggned, Goteborg, Nr686 (1965).

موديل طريقة العناصر المحددة ببعد واحد لتحليل قضبان التسليح المترابطة

الخلاصة

نموذج رياضي بطريقة العناصر المحددة ببعد واحد تم اشتقاقه في هذا البحث لدراسة واحتساب عناصر السلوك لقضبان حديد التسليح المطمورة في الخرسانة. في هذا الموديل تم تمثيل الترابط بين الحديد والخرسانة على شكل نوابض منفصلة تربط قضيب التسليح بالخرسانة المحيطة في نقاط محددة وعلى طوله. في حين ان حديد التسليح تم تقسيمه الى عدد من العناصر المحورية. وقد تضمنت الطريقة كيفية انشاء مصفوفة الجساءة الكلية واستخدامها لاحقاً في الحل الرياضي للمسألة غير الخطية.

هذا الموديل الرياضي المقترح يمكن ان يطبق في حالات عديدة شائعة لتمثيل تصرف الترابط لضبان حديد التسليح كما في القضبان المثبتة في مفصل الاعمدة–الجسور وكذلك القضبان الموصلة في وصل تعاقب التسليح.

النتائج التحليلية تم مقارنتها في هذه الدراسة مع تلك المتوفرة من نتائج مختبرية لفحص السحب وفحص الانحناء واعطت توافقاً مقبولاً وملائما.

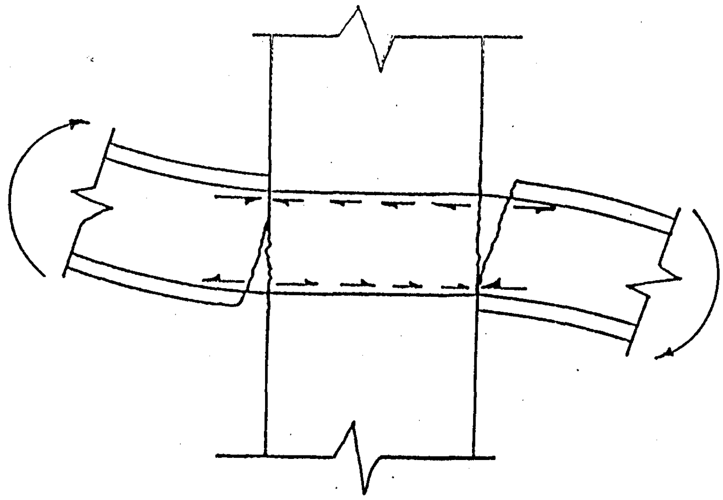


Fig.(1) Beam fixed end rotation resulting from anchored bar pullout

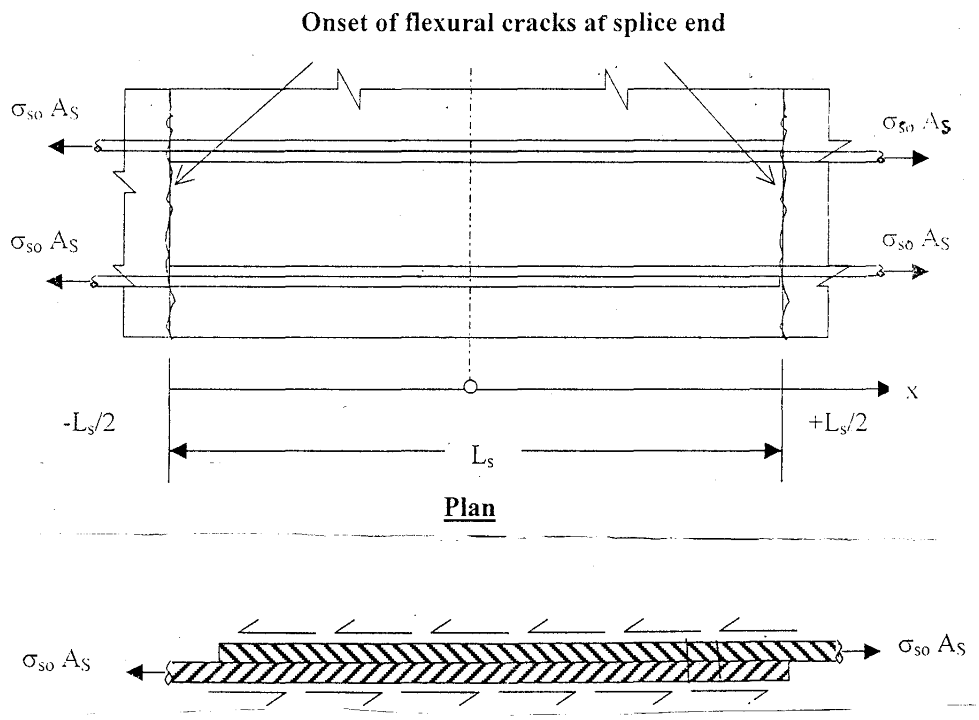


Fig.(2) Beam with tensile reinforcement lap splices

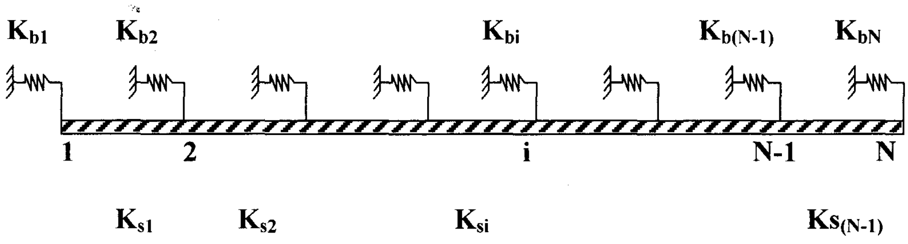


Fig.(3) Proposed model of anchored bar with discrete springs representing bond

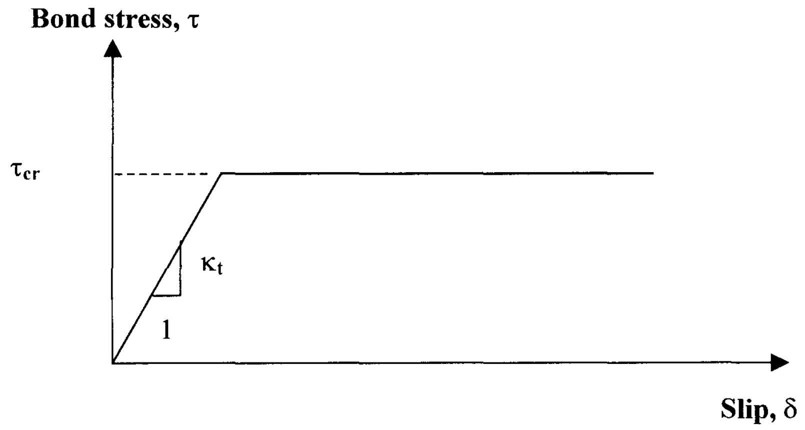


Fig.(4) Typical bond stress – slip relationship adopted in the present study

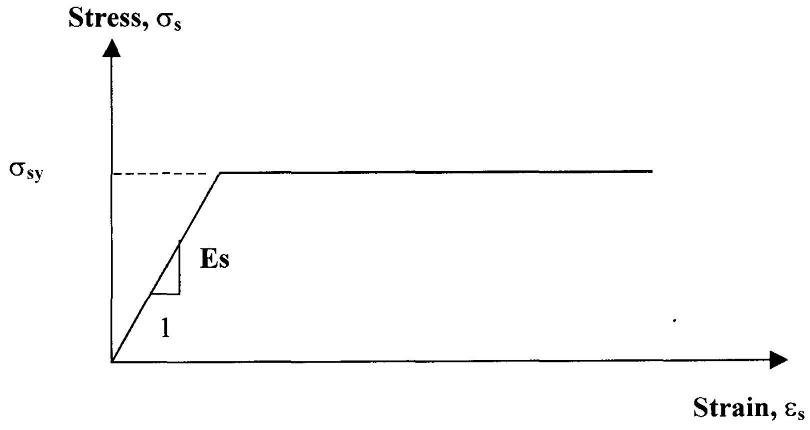


Fig.(5) Typical stress- strain relationship adopted in the present study for steel bar

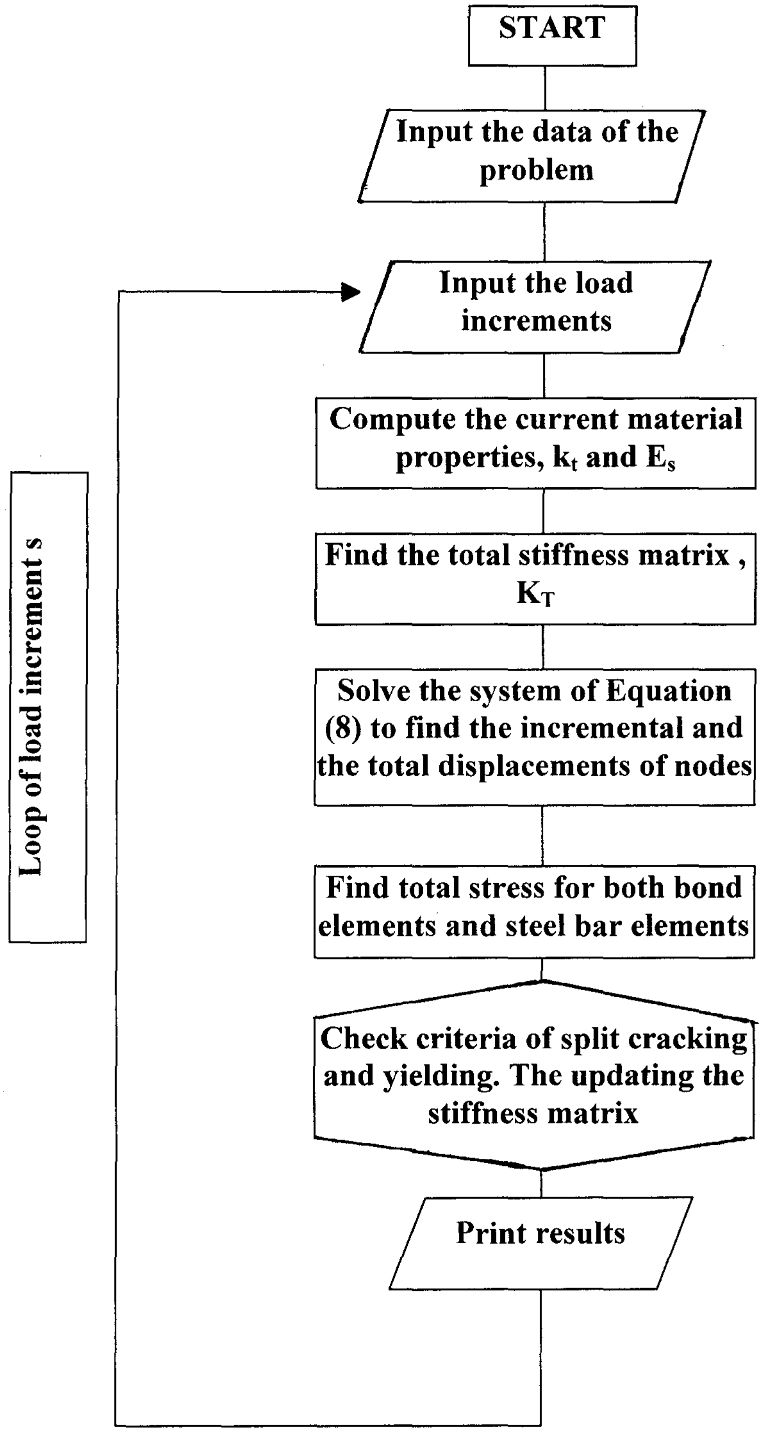
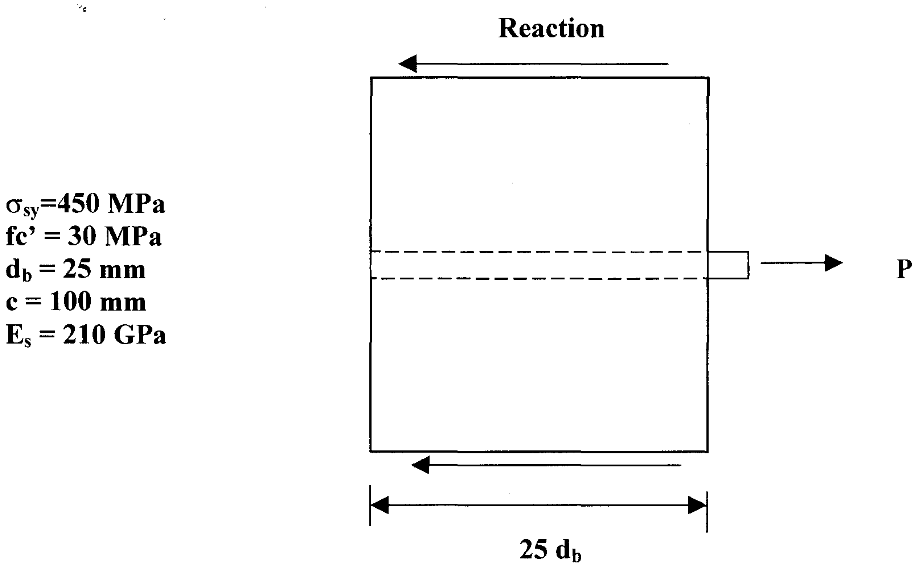
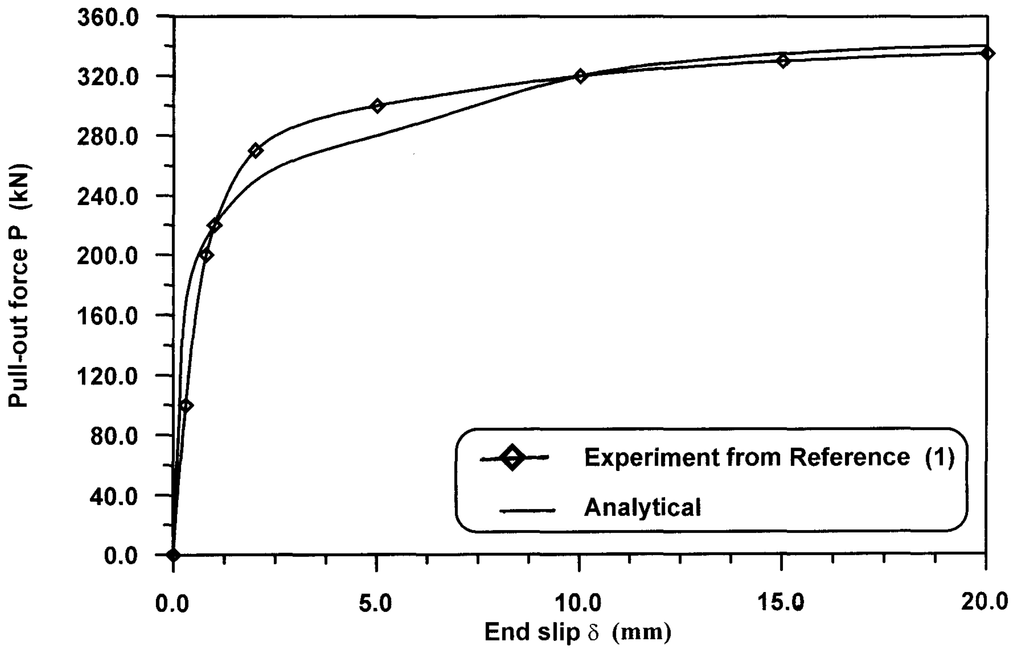


Fig.(6) Flowchart of computer program NABB1.
(Nonlinear Analysis of Bonded Bar in One Dimension)



(a)



(b)

Fig.(7) Comparison of experimental and theoretical results for specimen subjected to monotonic pull-out load
(a)Test specimen, (b) Results of end force - end slip relationship

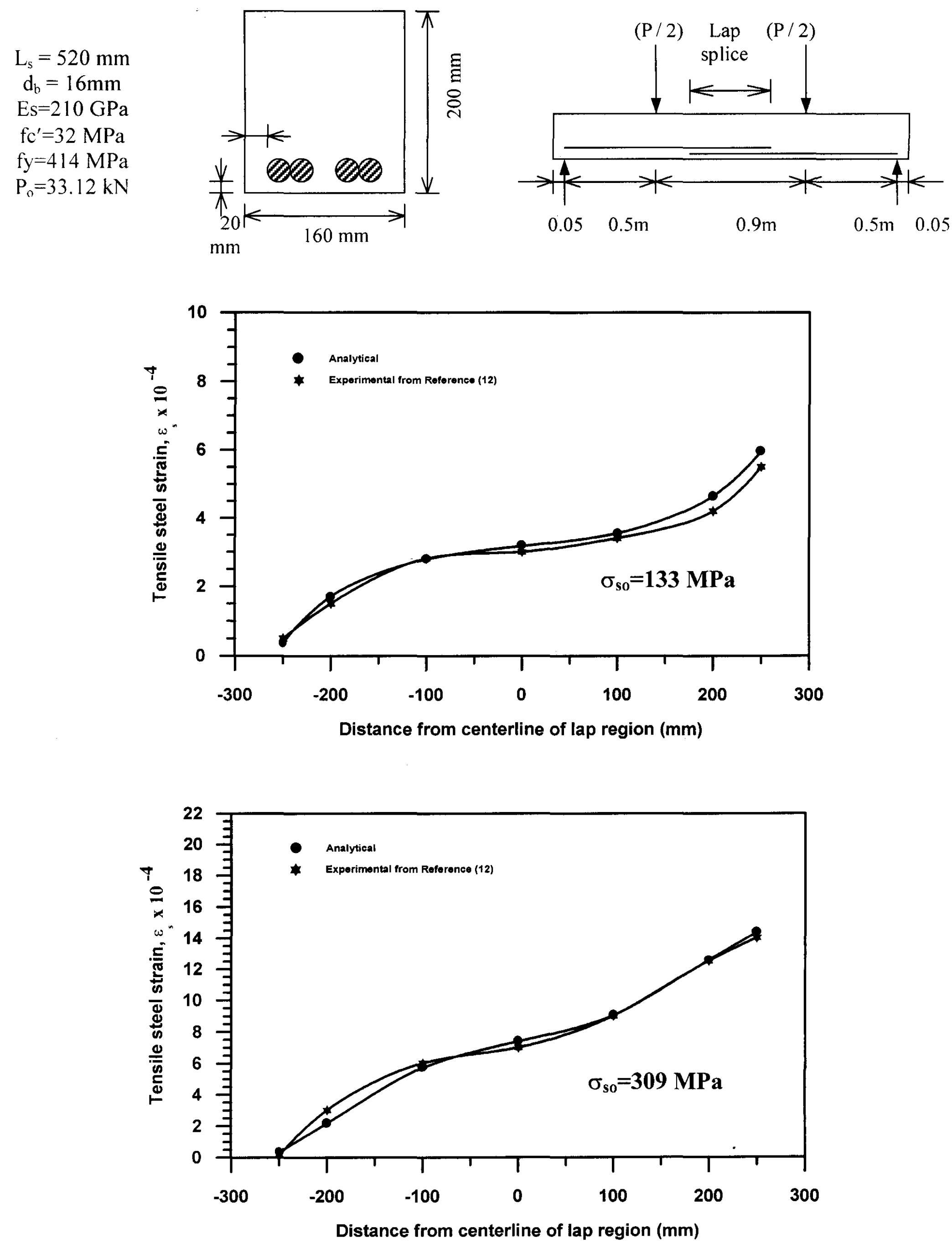


Fig.(8) Comparison of experimental and theoretical results for reinforced concrete beam containing tensile reinforcement lap splices a) Test specimen , b) Result of steel strains along spliced bar coming from right side