Determination the optimal portfolio by using the cut- off rate

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Abstract:

The determination of the optimal portfolio has a lot of concern from 1950s until now, therefore the researches introduce many techniques for delineating the optimal portfolio.

In this paper we try to use one of these techniques to determine an optimal portfolio using a real world data from al-Saudi exchange, which is the cut-off rate.

The issue of this study is how we can select the security to be incorporated in the portfolio, and how many securities should be in it and then what are the fractions or the weights of any securities in the optimal portfolio.

1- The Optimal Portfolio;

Here we are not trying to study the principles of the optimal portfolio or the evidence of existed a one optimal portfolio for the investor, but we will give a definition of it in order to use it in our study, we can state that the optimal portfolio is;

a- Have a higher return at one level of risk, or

b- Have a minimum degree of risk at one level of return.

(Beterson, 2000, 33)

These features demonstrate that the optimal portfolio should be one and only one portfolio for any investor.

Now we can move to another step in our study, we know that there are many different elements have an effect on optimal portfolio, from the very important element the investor behavior or preference, the beta coefficient, rate of return on every security and the utility function of the investors (the income, consuming, and his investing) . But to facilitate the procedure of select optimal portfolio we will use a single-index model depended on the average rate of return of the security, and then we can compute the unsystematic risk of security by the variance of return and the systematic risk by beta coefficient.

The determining of optimal portfolio would be greatly facilitated, and the ability of practicing security analysts and portfolio managers to relate to the construction of optimal portfolio greatly enhanced, if there were a

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single number that measure the desirability of including a security in the optimal portfolio, if one is willing to accept the standard form of single-index model as describing the co-movement between securities, such number will exist. In this case the desirability of any security is directly related to its excess return to beta ratio, the excess return is the difference between the expected return on the security and riskless rate of interest like rate on Treasury bill, this excess return measure the additional return on a security (beyond that offered by riskless assets) per unit of non-diversification risk. (Edwin .et, al, 2003, 183).

Here we can put the excess return as a formula;

Excess return= Ri –Rf /Bi …………... 1

Where; Ri = the expected return on security I.

Rf = the expected return on riskless asset.

Bi = the expected change in the rate of return on security I associated with a 1% change in market return (Beta).

2- The cut-off rate;

Now after determining the concept of optimal portfolio and the formula of excess return, we can answer the most important question in this paper "what is the security which selected to be in the optimal portfolio? ", to answer this question we need to determine the excess rate of return of all securities under the interest and then chose those securities that have an excess return higher than the cut-off rate. (Toyaco, 2003, 44).

In other words if securities are ranked by excess return to beta (from highest to lowest), the ranking represents the desirability of any securities inclusion in the portfolio, then if the security with a particular ratio of excess return is included in an optimal portfolio, all security with a higher ratio will be included also, and if the security with a particular ratio of excess return is excluded in an optimal portfolio, all security with a lower ratio will be excluded also, when the single-index model is assumed to represent the covariance structure of security return, then a security is included or excluded, depending on the size of its excess return to beta ratio . How many securities are selected depending on a unique cut-off rate such that all securities with higher ratio of excess return will be included and all securities with lower ratios excluded, we call this ratio the cut-off rate C* .(Edwin .et, al, 2003, 183).

3- The cut-off rate calculation

what is the cut-off rate and how can we determine it, to clarify that we should return to the first point discuss earlier, that is the definition of optimal portfolio, if we want to maximize the return of portfolio at specified level of risk, then we should choose those securities that added more return than risk to portfolio (jullany, 2004, 534). In other word, when we add a security to a portfolio we will get a better trade-off between return and risk, and this will be achieved when the security have an excess return more than the average excess return of all the securities included in the portfolio .

Then we can follow these two steps to determine which securities are included in the optimal portfolio:

1- Find the excess return to Beta ratio for each stock under Consideration, and rank from highest to lowest.

2- The optimal portfolio consist of investing in all stocks for which have an excess return to Beta ratio greater than cut-off ratio C*. Shortly, we will define C* and interpret its economic significance.

Once C* has been determined, the securities to be included can be selected by inspection.

The C* can be determined as follow; (Edwin .et, al, 2003, 184-186).

$$C = \frac{Qm^2 \sum \frac{(Rj - Rf)\, Bj}{Qj^2}}{1 + Qm^2 \sum \left[\frac{Bj^2}{Qj^2}\right]} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Where;

Qm^2 = the variance of market index

Qj^2 = the variance of assets I (unsystematic risk).

Now to determine C* it is necessary to calculate its value as if there were different numbers of securities in the optimal portfolio, designated Ci as a candidate for C*. The value of Ci is calculated when (i) securities are assumed to belong to the optimal portfolio. Since securities are ranked from (highest excess return to Beta) to lowest, we know that if particular security belongs to optimal portfolio, all the higher ranked securities also belong to optimal portfolio.

To do that we will calculate values of a variable Ci as if the first ranked securities was in the optimal portfolio (i=1), then the first and second ranked securities are in the optimal portfolio (i=2),then the first , second and third ranked securities are in the optimal portfolio (i=3) and so forth . these Ci are candidate for C*, we know we found the optimum Ci (that is C*) when all securities used in the calculation of Ci have excess return to Beta above Ci and all securities not used to calculate Ci have excess return to Beta below Ci .

That is the securities are added to the portfolio as long as;

Ri – Rf / Bi > Ci

But what about the percent invested in each securities , the percentage invested in each securities is;

$$X_i = Z_i / \sum Z_j \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

Wher

$$Z_i = \frac{B_i}{Q_i}\left[\frac{R_i - R_f}{B_i} - C^*\right] \quad \ldots\ldots\ldots\ldots\ldots 4$$ **(Edwin .et, al, 2003, 189).**

4- The practical framework

Here we will use the data from Saudi Arabia exchange for (7) indexes there are, agriculture , telecommunication , insurance , banks, services, and power for the last (10) weeks in 2005.

The first step for calculation is to determine the excess return to Beta ratio from highest to lowest , to do that we should compute the return of every index by the following formula:

Ri= P2 – P1/P1, here we ignore the dividing profits because we use the index not the real securities , the first week will be th basis period and we need to determine the risk free rate so we use the interest rate on saving deposits as risk free rate it was (2.5%). The rate of (10) weeks was (0.48) .Here we need also to compute the variance and covariance of every index with market index.

These computations shown in table (1), this table shown that the services index have the highest excess return to Beta ratio (4.44), then agriculture (0.85), banks (0.84) , industry (0.75) , insurance (0.18) , telecommunications (0.17) and power (-0.07).

The second step is computing the Ci for every index as candidate of cut-off-rate C* according to the formula 2. Table (2) show these computations, from this table the cut-off-ratio will be (0.83) ,then the only indexes will constitute the portfolio are the service index with excess rate- of- return to beta ratio (4.44), agriculture (0.85) , banks (0.84). They are only the

indexes have excess rate of return to Beta ratio more than the cut-off-rate ratio (0.83), then will not inter in the portfolio.

The third step is determining the percent of every index in portfolio by using formula 3 and 4.

As shown in table (3), the Zi of services index was (0.108), agriculture (0.028), banks (0.12)

Our portfolio will constitute of three indexes only with the percentage above, the return of this portfolio will be (4.35) and we get that from the formula :

Rp= ∑ Xi Ri , where Xi = the percentage of every securities

Ri= the average return of every securities.

And the risk of it will be only (2.33), we get it from the formula:

$Qp^2 = \sum Xi^2\ Qi^{\ 2} + \sum i \sum j\ X\ ij\ Q\ ij$

Where : Qi = the variance of return

Qij= the covariance between return.

This portfolio has a best tradeoff between the risk and return than any one security (index) under concern , by using the differentiation coefficient (Qi^2/ Ri) we can see that the degree of D.C. for the portfolio is (0.54) whereas the minimum D.C for any index was the service index (3.33), and this tradeoff will be best from any combination any indexes theoretically.

Table (1)
The excess rate of return to Beta ratio

index	Ri	Ri-RF	Bi	Q2	Ri-Rf/Bi
SER.	2.7	2.22	0.5	7.4	4.44
AGRI.	5.6	5.12	6.1	25.9	0.84
COMU.	0.2	0.28	0.4	4.3	0.07
INSU.	0.8	0.32	1.75	6.3	0.18
BANKS.	2	1.52	1.9	24.8	0.8
INDUS.	1.2	0.72	0.96	8.8	0.75
POWER.	0.6	0.12	0.7	6.8	0.17

Table (2)
The cut-off-rate for the portfolio

index	Ri-Rf/Bi	(Ri-RF) Bi /Q2	Bi^2/Q^2	∑(Ri-RF) Bi /Q2	∑ Bi^2/Q^2	C*
SER.	4.44	0.15	0.03	0.03	0.03	0.63
AGRI.	0.84	1.2	1.4	1.35	1.43	0.73
COMU.	0.07	0.1	0.1	1.45	1.53	0.83
INSU.	0.18	0.1	0.1	1.55	1.63	0.82
BANKS.	0.8	0.1	0.49	1.56	2.12	0.72
INDUS.	0.75	-0.1	0.04	1.55	2.16	0.65
POWER.	0.17	0.08	0.07	1.63	2.23	0.65

Table(3)
The percentage of indexes

index	Zi	Xi%
SER.	0.108	70
AGRI.	0.028	18
COMU.	0.02	12
∑ Zi	0.156	100

5- Conclusions and Recommendations

a- Conclusions

1- The number of securities and the percrntage of every on of them in the portfolio can be determined by using the cut-off-ratio.

2- Only securities that there excess rate of return more than the cut-off-ratio will be consisted in optimal portfolio.

3- Using this technique will lead to portfolio has best tradeoff between risk and return from any other portfolio under concern.

b- Recommendation:

1- Using this technique in Iraqi security market by investor when they developing there portfolios.

2- This technique should studied from other viewpoint and compare it with other techniques by using a longer time series to obtain a better understanding of the optimal portfolio.

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