Post Buckling Behavior of Prismatic Structural Steel Members with Semi Rigid Connections

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ABSTRACT
In most designs of steel frames,beam-column connections are assumed either perfectly pinned or fully rigid. This assumption results in an incorrect estimation of the structural behavior of these frames. Practically, beam-column connections are between these two extreme assumptions and possess some rotational stiffness; therefore, it is called semi rigid connection. This research, presents a numerical analysis using finite element method to study the effect of semi-rigid connection on post buckling behavior of prismatic structural steel frames. The beam-column connection is modeled by linear elastic rotational spring element with specified rotational stiffness. The obtained results reveal that changing beam-column connection from rigid to semi rigid with different rotational joint stiffness values make the load-displacement curve of post buckling behavior less stiffer, the ultimate load decrease and ultimate vertical displacements increase with percentages depend on joint stiffness value and type of frame ;thus, semi-rigid connection should be considered in analysis and design of steel frames which exhibits post buckling behavior to obtain more realistic results.
Keyword: Post Buckling Behavior, Structural Steel Members and Semi-rigid Connections.

INTRODUCTION
Two types of beam-column connections are generally considered in the design of steel structures in practice. These are classified as completely rigid (moment) and simple (shear) connections. In theory, completely rigid connections cannot undergo rotation and simple connections cannot transfer moment. However, in reality rigid connections have a relative flexibility which makes them to rotate and simple connections have some reserve capacity to transfer moments [1]. In many modern design specifications, this fact is realized and another type which is called partially restrained or semi-rigid connection is introduced. These types of connections have got the transfer of some beam moment to column together with shear.
The early attempts for analyzing and studying semi rigid connections were undertaken in 1942 by Johnston and Mount [2], in their study they presented methods applicable to the analysis of building frames with semi-rigid riveted or welded connections. In 1987, Chasten et al [3] presented a testing program to study the behavior of four common connections types with semi rigid connections. In 2002, Citipitioglu et al [4], presented models include the effects of slip by utilizing a general contact scheme to analysis partially-restrained bolted steel beam-column connections. Also , In 2002 , Lima et al [5] , presented experimental tests and numerical analysis to predict moment resistance and rotation capacity of minor axis beam-to-column semi-rigid connections. In 2004 ,Degertekin and Hayalioglu [6], presented nonlinear analysis included P-Δ effects and design method for steel frames with semi-rigid connections and semi rigid column bases. In 2005, Öztürk and Seçer [7]

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presented an investigation for dynamic response of semi rigid frames by different connection models. In 2008, Xinwu [8] , presented nonlinear finite element analysis using Fortran program on two dimensional steel frame with semi rigid connections. In 2009 Uslu [9] in his thesis presented three dimensional analyses of semi rigid steel connections. Dave and Savaliya [10], in 2010 Frye and Morris used polynomial model [11] to analysis and design of semi-rigid steel frames ,values of secant stiffness were incorporated in analysis for all alternatives using STAAD Pro. In 2013 Bogatinoski [12] presented theoretical and experimental researches of the steel beam-column connections made of rigid and semi-rigid connections in which four connections with IPE-profile and four connections with tube's section for the beam were used.

On the other hand , in structures which has slender elements such as slender frames and slender trusses ,after a load reaches its buckling value, the load value may remain unchanged or it may decrease, while the deformation continues to increase, for some problems, after a certain amount of deformation, the structure may start to take more loading to keep deformation increasing, and a second buckling can occur, the cycle may repeat several times, this phenomena is called post buckling behavior [13]. Estimating post buckling behavior for structures having slender members is very important since post buckling means loss the stability of structure associated with large displacement and it may be lead to destruction the structure .

All the studies of semi-rigid connections did not investigate effect of semi-rigid connections on post buckling behavior of steel frames.

Aim of the research

This research aim at presenting a numerical analysis to investigate the effect of semi-rigid connections on the post buckling behavior of plane structural steel members in elastic materials stage.

Modeling semi rigid connection in finite element method (FEM)

Practically steel connection possesses some rotational stiffness; therefore, in this study semi rigid beam-column connection was represented in FEM byrotational spring as proposed by Chen et al **[14]**. The rotational spring is modeled in ANSYS using Combin14 element. This element has longitudinal or rotational capability in 1-D, 2-D, or 3-D applications. The rotational spring-damper option is a purely rotational element with three degrees of freedom at each node: rotations about the nodal x, y, and z axes. Fig.(1) shows COMBIN14 element **[15]**. COMBIN14 element is modeled with two coincident nodes. The element is introduced into the frame with one rotational degree of freedom in the z axis. Since each two nodes of this element shall have translation in the x and y directions, each pair of the nodes that belong to each of the COMBIN14 should be coupled in the x and y directions. This will maintain analysis stability.

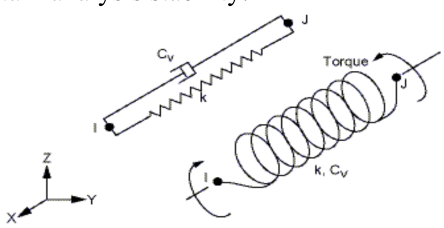

Figure. (1) COMBIN14 element [15]

The parts of frames in this study were modeled in ANSYS program using BEAM3 element which is a uniaxial element with tension, compression, and bending capabilities. The element has three degrees of freedom at each node: translations in the nodal x and y directions and rotation about the nodal z-axis**[15]**. Fig.(2) shows BEAM3 element. All the frames were analyzed by discretizing the each member into different number of elements and the mesh that gave a satisfactory solution is implemented.

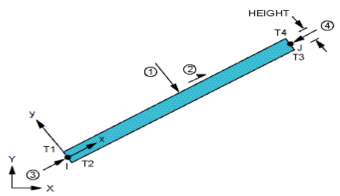

Figure.(2) BEAM3 element [15]

Post buckling analysis was achieved in ANSYS by activate large displacement option and using arc-length method. The maximum and minimum multiplier of arc-length method are set to be 10 and 0.0001 respectively.Table (1) lists the solutions controls and the parameters used in this analysis.

Table(1) The solution controls and its parameters

Solution Controls	Parameters
Analysis Options	Large Displacement Static
Number of Substep	100
Equilibrium Iteration	40
Arc-Length Options	Activate Arc-Length Method(Max Multiplayer=10, Min Multiplayer=0.0001)

In order to assess finite element models proposed to predict post buckling behavior, two frames were analyzed with the proposed models, and comparisons were made with previously published results.

Framesanalysis

Lee′sframe

This frame was analyzed using variable displacement control (modified displacement control) by Fujii et al [16], using beam-column theory by Al-Mahdawi[17]and using finite element method by Ismael and Salman [18], Ismael and Salman used BEAM188 element to modeling the frame in ANSYS to predict fully load-displacement relationship in post buckling behavior. In this study BEAM3 element was used to modeling the frame and to predict load-displacement relationship until initial peak load in post buckling behavior since BEAM3 is more appropriate with COMBIN14 element to modeling semi rigid connection. The geometry, loading and section properties of Lee′s frame are shown in the Fig.(3).

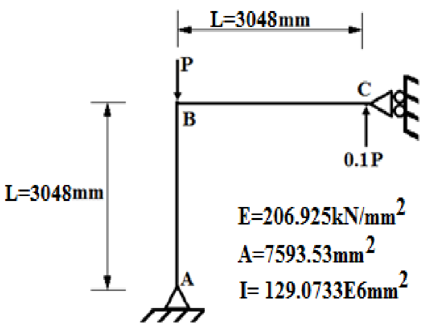

Figure (3) Geometry , loading and section properties of Lee′s frame

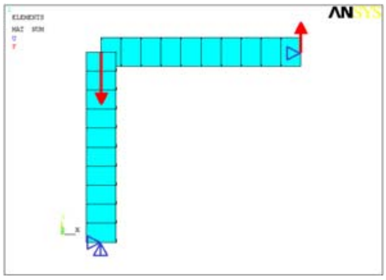

Figure.(4) Modeling the lee′s frame in ANSYS

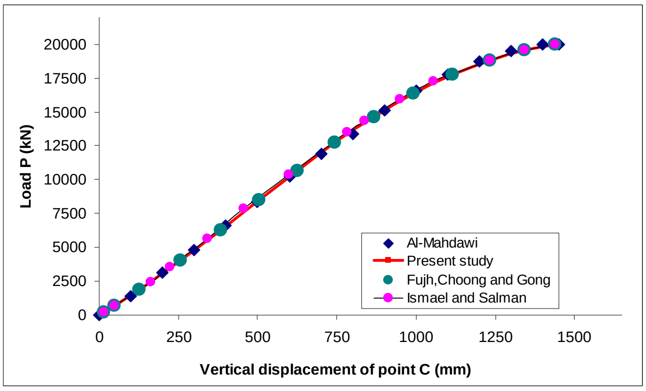

Figure.(5) Load-vertical displacement of point C for lee′s frame

Fig.(4) shows modeling of the frame in ANSYS and Fig.(5) shows load-vertical displacement of joint C curves until initial peak load in post buckling analysis. It can be seen that the results of this study is with good agreement with the results of previous studies. Fig.(6) shows the original shape of the frame, the frame at the initial peak load, and variation of vertical displacement along the frame in post buckling stage.

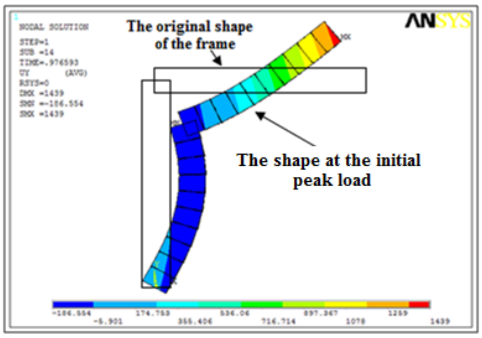

Figure.(6) Variations in vertical displacement for lee′s frame at ultimate load using ANSYS

Effect of semi rigid connection on post buckling behavior of Lee′sframe

Four values of joint stiffness (rotational spring stiffness) (4EI/L , 6EI/L , 8EI/L and 10EI/L N.mm) were used (where E is modulus of elasticity , I is moment of inertia and L is length of the member) as shown in the Fig.(1) , as well as rigid connection case to study effect ofsemi rigid connection on post buckling behavior of Lee′s frame. It is important to mention that the minimum value of rotational spring stiffness used in this study is the minimum value that does not cause rigid body motion in the frame.The rotational spring was placed in the joint of beam-column as shown in the

Fig.(7). The results of finite element analysis are listed in the Table(2) and load-vertical displacement of point C is shown in the Fig.(8). It can be noted from these results that, as the connection change from rigid case to semi rigid with rotational stiffness 10EI/L , 8EI/L , 6EI/L and 4EI/L N.mm. The initial peak load decrease with percentages 0.8%, 3.2%,7.6 and 20.2% , the ultimate displacement increase with percentages 3.4%, 4.4%,8.2% and 16.1% respectively and the curve become less stiffer. These results reveal that semi rigid connection with joint stiffness 10EI/L and 8EI/L N.mm has little effect on post buckling behavior of Lee′s frame but the effect become significant as joint stiffness decrease to 6EI/L and 4EI/L. This can be attributed to the fact that changing the joint connection to semi rigid connection with less rotational stiffness decrease the stability of the frame and increase the flexibility, thus the frame exhibit less carrying capacity against the applied loads and increase the displacement. Fig.(9) shows post buckling behavior of lee′s frame with semi rigid connection in different load steps.

Table(2) Effect of joint stiffness on initial peak load andultimate vertical displacement

Stiffness	Rigid	10EI/L	8EI/L	6EI/L	4EI/L
Initial peak load (kN)	18990	18840	18380	17550	15150
Percentage decrease in initial peak load (%)	-	0.8	3.2	7.6	20.2
Ultimate vertical displacement (mm)	1506	1557	1573	1629	1749
Percentage increase in ultimate vertical displacement (%)		3.4	4.4	8.2	16.1

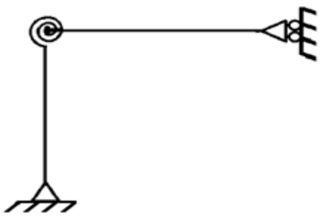

Figure.(7) Schematic representation of semi rigid connection of Lee′s frame

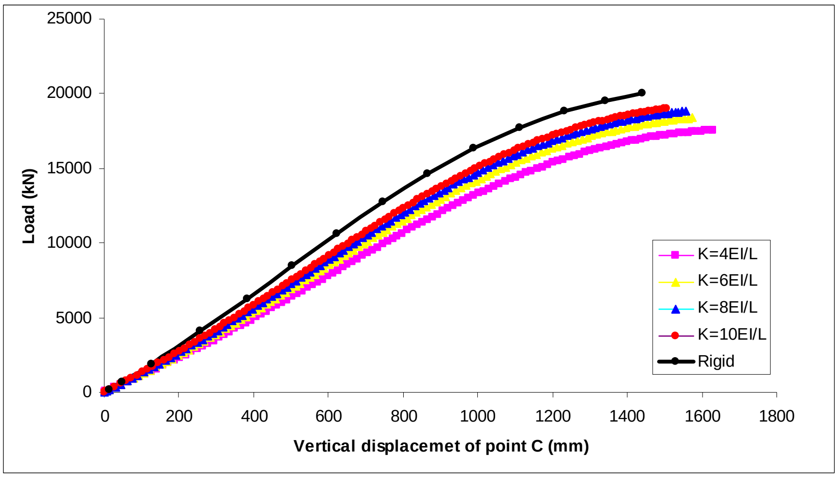

Figure.(8) Effect of spring stiffness value (semi rigid connection) on load-vertical displacement of point C for Lee′s frame

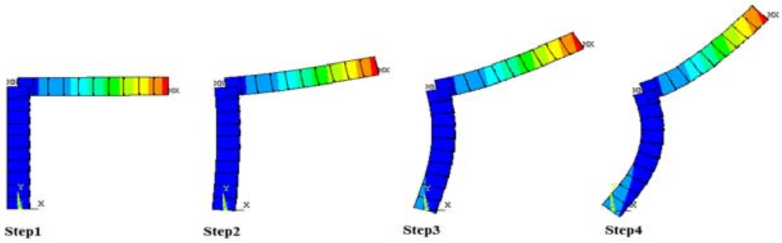

Figure. (9) post buckling behavior of Lee′s frame semi rigid connection in different load steps

One-bay one-story frame

The second example which used to verify the validity of the FEM models used in this study is one-bay one-story frame. This frame was analyzed by Al-Mahdawi**[17]** using beam-column theory. The structure load system of this frame is shown in Fig.(10) and modeling it in ANSYS is shown in Fig.(11). The post-buckling behavior in term of load-vertical displacement curve of joint C, obtained from finite element analysis of this study along with the curve obtained from beam-column theory by Al-Mahdawi**[17]** study are presented and compared in Fig.(12).

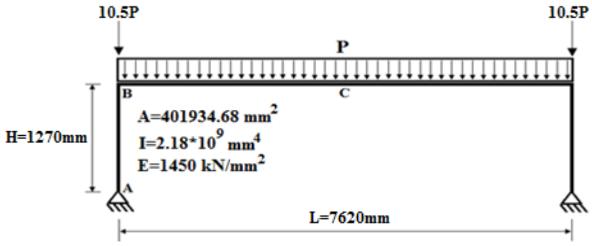

Figure.(10) Geometry and loading of the frame

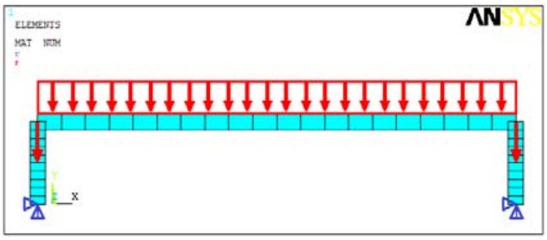

Figure.(11) Modeling one-bay one-story frame in ANSYS

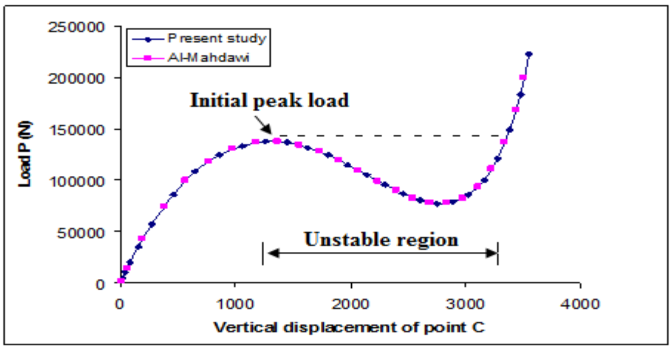

Figure.(12) Load- vertical displacement of point C

It can be noted that the results of this study show good agreement with the results of Al-Mahdawi**[17]**.Fig.(13) shows the original shape of the frame, the frame at the ultimate load and variation of vertical displacement along the frame. Also ,It can be noted that the frame exhibits snap through buckling ,in which the vertical displacement continue to increase after initial peak load although the load began decrease and later the load began to increase again until the load value equal to initial peak load , in this region of load-displacement curve the frame become unstable. After this region the frame return to be stable again.

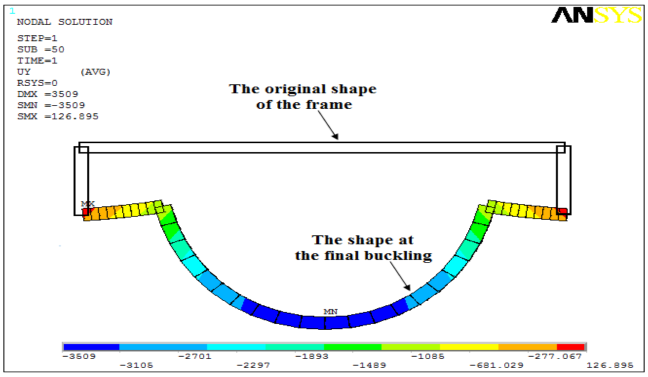

Figure.(13) Variations in vertical displacement for one- bay one-story frame at ultimate load using ANSYS

Effect of semi rigid connection on post buckling behavior of one-bay one-story steel frame

In this frame the rotational spring elements were placed as shown in the Fig. (14). Effect of different joint stiffness (6EI/L to 50EI/L N.mm) as well as rigid connection on post buckling behaviorof one-bayone-story steel frame in term of load-vertical displacement of point C and load-horizontal displacement of point B are shown in the Fig.(15) and Fig.(16) respectively.

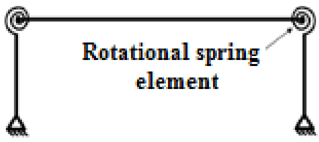

Figure.(14) Schematic of semi rigid connection ofone-bay one-story steel frame

It can be noted from these figures that under the same applied load (200 kN) , as the joint stiffness increase the curves become more stiffer until first point in which all the curves are met ,beyond this point , increase the joint stiffness make the curves less stiffer until second point in which all the curves are met again, beyond the last meeting point the curves return to be more stiffer with increase the stiffness. This phenomenon can be attributed to that decrease the joint stiffness make the frame more flexible and undergoes large displacement; therefore, snap through buckling will extend to larger displacement points. Also, it can be noted that the ultimate vertical displacement increase with decrease the joint stiffness as listed in the Table (3) ,whilethe final displacement of load-horizontal displacement is approximately corresponding, this behavior can be attributed to that ultimate horizontal displacement depend on length of the column which at the final of buckling change its position from vertical to horizontal, while ultimate vertical displacement depend on flexibility of beam-column connection and curvature of beam as shown in the Fig.(17) which shows post buckling behavior of the frame in different load steps.

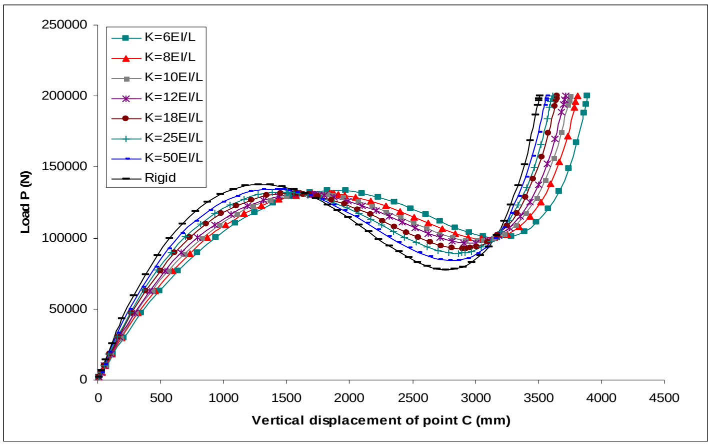

Figure. (15) Effect of joint stiffness on post buckling behavior in terms of load-vertical displacement of point C of one- bay one-story steel frame

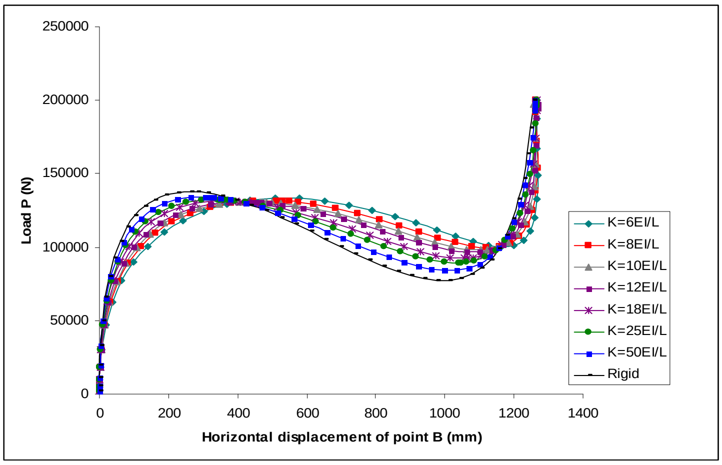

Figure. (16) Effect of joint stiffness on post buckling behavior in terms of load-horizontal displacement of point B of one- bay one-story steel frame

Table (3) Effect of joint stiffness on ultimate vertical displacement

Stiffness	Rigid	50EI/L	25EI/L	18EI/L	12EI/L	10EI/L	8EI/L	6EI/L
Ultimate vertical displacement (mm)	3508	3559	3610	3648	3715	3761	3807	3890
increase in ultimate vertical displacement (%)	-	1.5	2.9	4.0	5.9	7.2	8.5	10.9

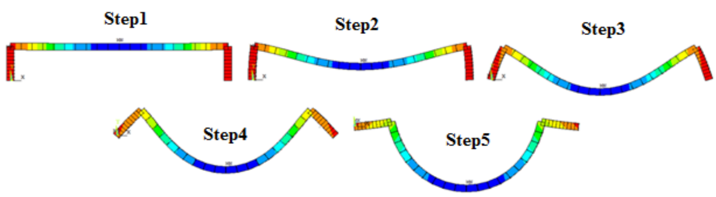

Figure.(17) Post buckling behavior of one-bay one-story frame in different load steps

Effect of semi rigid connection on post buckling behavior ofone-baytwo-story frame

This frame was analyzed by Yan**[19]**in his studying behavior of one-bay steel frames with semi-rigid connections.The dimensions, loading, section properties and positions of spring element are shown in the Fig.(18).Modeling of the frame in ANSYS is shown in the Fig.(19).

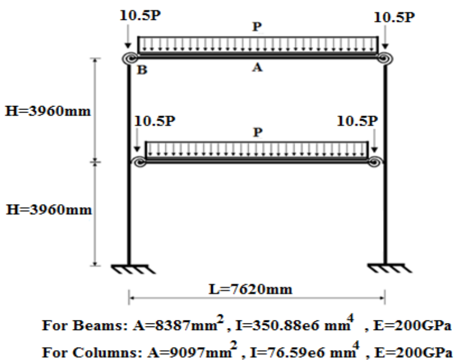

Figure.(18) Schematic of semi rigid connection of one-bay two-story frame

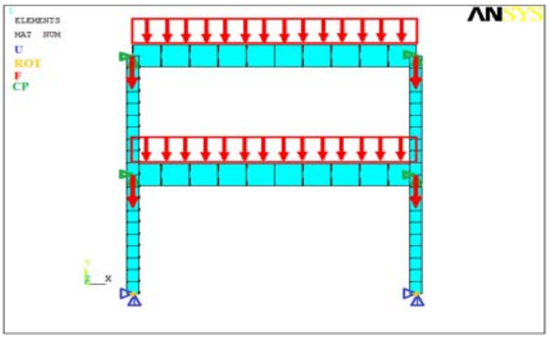

Figure.(19) Modeling one-bay two-story frame in ANSYS

Effect of different joint stiffness as well as rigid connection case on post buckling behavior in terms of load-vertical displacement of point Ais shown in the Fig.(20) and Table(4) showseffect of joint stiffness on ultimate load , ultimate vertical displacement . It can be noted that the load-vertical displacement of point A is bilinear curve with large transition zone, changing beam-column connection from rigid to semi rigid with joint stiffness (10EI/L ,8EI/L , 6EI/L and 4EI/L) results in decrease the ultimate load with percentages (4.1% , 5.7%, 7.8% and 15.3%) and increase the ultimate vertical displacement with percentages (3.4%, 7.1%, 8.6%and10.9%) respectively. It can be noted also that the curves become less stiffer with decreasing joint stiffness and the second part of the curve is more affected by changing joint stiffness value than the first part, this can be attributed to that the vertical displacement at early stages of loading as shown in the Fig.(22) depend largely on the deflection of the beam and partially depend on the rotation of the beam-column joints and at the next stages effect of beam-column rotation become significant.

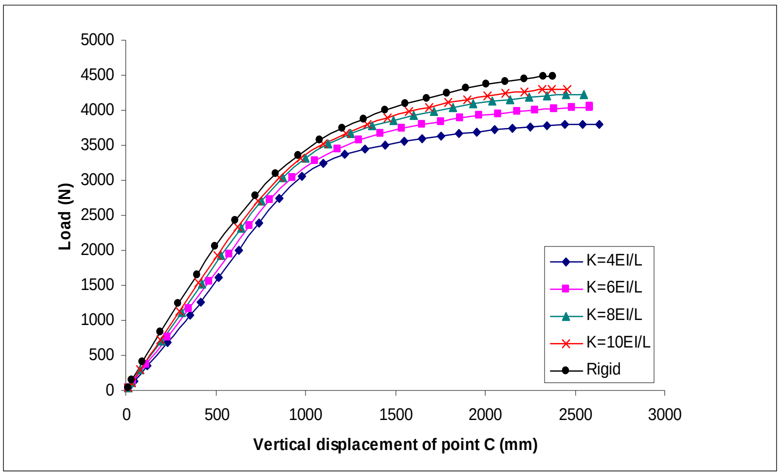

Figure. (20) Effect of joint stiffness on post buckling behavior in terms of load-vertical displacement of point A of one- bay two-story steel frame

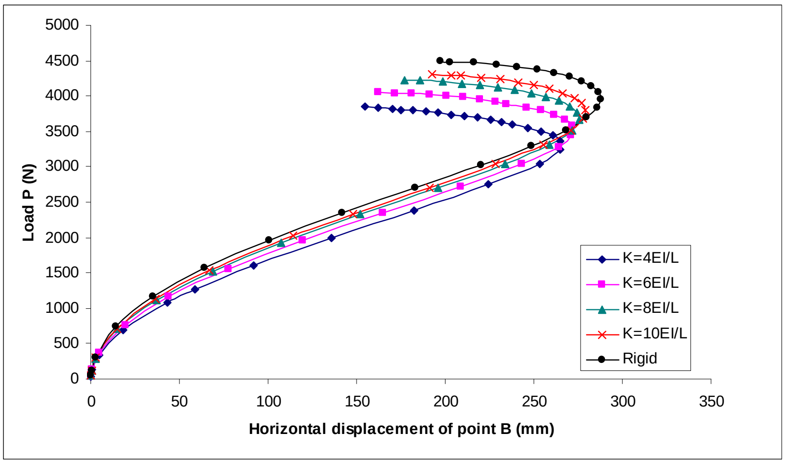

Figure. (21) Effect of joint stiffness on post buckling behavior in terms of load-horizontal displacement of point B of one-bay two-story steel frame

Table (4) Effect of joint stiffness on ultimate load ,ultimate vertical and horizontal displacement of one-baytwo-story frame

Stiffness	Rigid	10EI/L	8EI/L	6EI/L	4EI/L
Ultimate load (kN)	4490	4305	4235	4140	3805
Percentage decrease in ultimate load (%)	-	4.1	5.7	7.8	15.3
Ultimate vertical displacement (mm)	2378	2459	2547	2582	2637
Percentage increase in ultimate vertical displacement (%)	-	3.4	7.1	8.6	10.9
Ultimate horizontal displacement (mm)	197	193	177	162	155
Percentage decrease in ultimate horizontal displacement (%)	-	2.0	10.2	17.8	21.3
Final horizontal displacement (mm)	288	279	276	272	266
Percentage decrease in final horizontal displacement (%)	-	3.1	4.2	5.6	7.6

On the other hand, effect of joint stiffness on load-horizontal displacement curves of point B is shown in the Fig.(21). It can be seen that these curves consists of two parts, in the first part the displacements increase with increase the applied loads and in the second part the displacements decrease with increase the applied loads, this behavior can be attributed to that the value of horizontal displacement of point B is the algebraic product of columns displacementsvalues in upper bay and columns displacements values in lower bay as shown in Fig. (22); thus, at the first stages of loading, the buckling of lower columns is larger than that of upper columns which buckles in opposite sense to buckling of lower columns resulting in increase displacement with increase the load and later at the next stages of loading, the buckling of upper columns become larger than that of lower columns; therefore the horizontal displacement began to decrease.

Fig.(21) also reveals that decrease joint stiffness make load-horizontal curve less stiffer but the final and ultimate horizontal displacement increased; however, according to the results listed in the Table(4) changing beam-column connection from rigid to semi rigid with joint stiffness (10EI/L ,8EI/L , 6EI/L and 4EI/L) results in decrease ultimate horizontal displacement with percentages (2.0 %, 10.2%, 17.8% and 21.3%) and decrease the final displacement to (3.1%, 4.2 % , 5.6% and 7.6%). This behavior can be explained by Fig.(23) which shows the original shape of the frame and the shape at the final buckling for the different joint stiffness, it can be seen that, as the beam-column joint stiffnesschange from rigid to semi rigid with joint stiffness (10EI/L ,8EI/L , 6EI/L and 4EI/L) the beam column angle at point B turn from right angle to acute angle; thus, the range of horizontal displacement decrease with decrease joint stiffness resulting in increase horizontal displacement. These results reveal that the joint stiffness values (6EI/L and 4EI/L) have significant effect on the ultimate load and ultimate displacement.

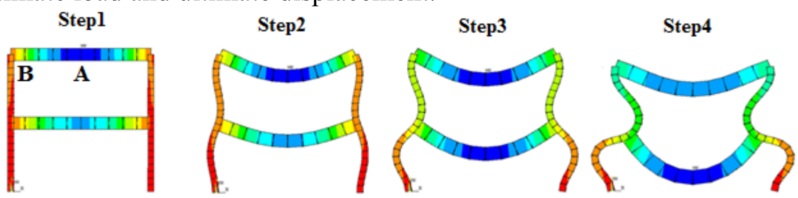

Figure.(22) Post buckling behavior of one-bay two-story frame

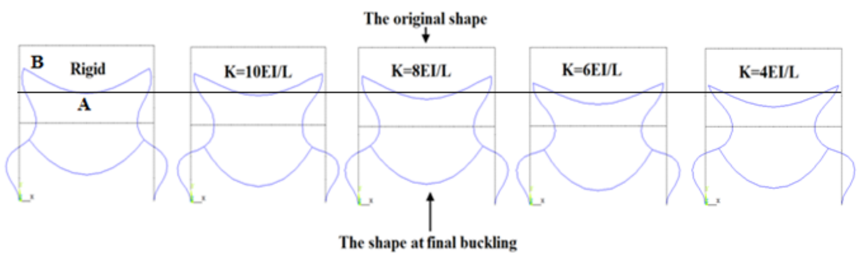

Figure.(23) One-baytwo-story frame at the original shape and at the final buckling with different joint stiffness (rotational spring stiffness)

Effect of semi rigid connection on post buckling behavior ofthree-bay two-story

To investigate the influence of semi rigid connection on post buckling behavior of three-bay two story, four values of rotational spring stiffness (2EI/L, 3EI/L, 4EI/L and 5EI/L N.mm) were used, in additional to rigid connection case. The dimensions, loading, section properties and positions of spring element are shown in the Fig.(24) and modeling it in ANSYS is shown in Fig.(25).

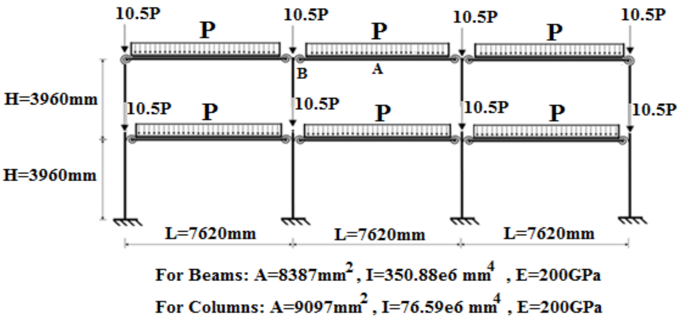

Figure.(24) Schematic representation of semi rigid connection ofthree-baytwo-story frame

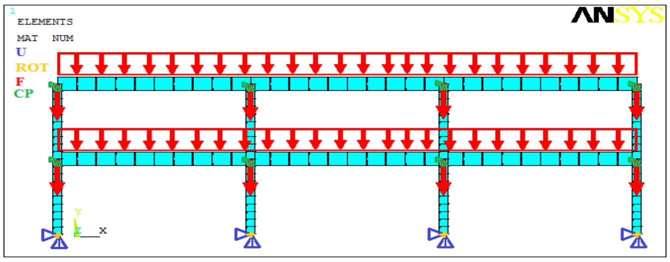

Figure.(25) Modeling three-baytwo-story frame in ANSYS

Effect of joint stiffness on post buckling behavior in terms of load-vertical displacement of point A is shown in the Fig. (26) .It can be noted that the curves consist from two linear parts with small transition zone , increasing joint stiffness value has very little effect on the first part and has significant effect on the second part, the reason of this behavior is similar to that of one-bay two-story but in the present case the vertical displacement at early stages of loading depend so far on the deflection of the beam due to restrict beam-column joint rotation by other members in the frame but in the next stages of loading the beam-column joint become less constrained due to buckling as shown in the Fig.(28).

Table (5) shows effect of joint stiffness on ultimate load, ultimate vertical displacement; however, changing beam-column connection from rigid to semi rigid with joint stiffness (5EI/L ,4EI/L , 3EI/L and 2EI/L) results in decrease the ultimate load with percentages (3.4% ,5.8% , 8.6% and 13.1%) and increase the ultimate displacement with percentages (2.5% , 5.4% ,7.2% and 9.4%) respectively. On the other hand, effect of joint stiffness on load-horizontal displacement curves of point B is shown in the Fig.(27). It can be seen that these curves are similar to load-horizontal displacement curves of one-bay two-story but the transition zone here is sharper. Table (5) shows also results of effect joint stiffness on horizontal displacement ; however, changing beam-column connection from rigid to semi rigid with joint stiffness (5EI/L ,4EI/L , 3EI/L and 2EI/L) results in decrease ultimate horizontal displacement with percentages (2.6% , 4.0% , 5.2% and 5.3%) and decrease the final displacement to (2.8%,3.9 %, 4.8% and 5.9%) respectively.

A comparison between the results of one-bay two-story frame and three-bay two-story frame reveals that, effect of joint stiffness (semi rigid connection) on post buckling behavior is more significant for one-bay two- story than three-baytwo-story , this can be attributed to that the joints in multi bay multi story is more braced by other members which restrict the rotation and movement of joint.

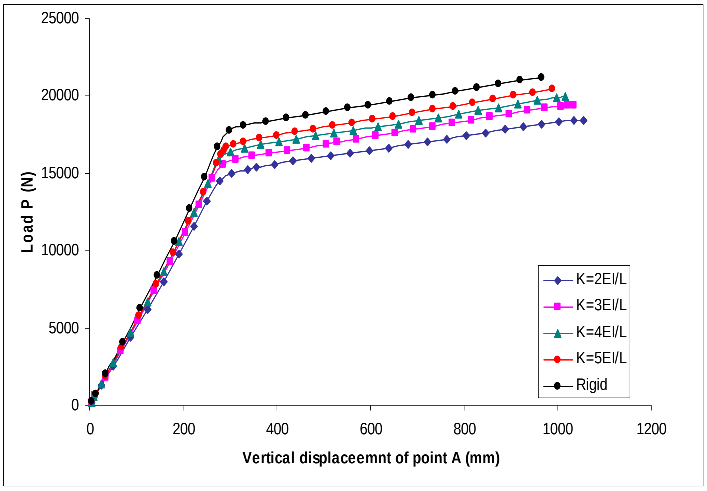

Figure. (26) Effect of joint stiffness on post buckling behavior in terms of load-vertical displacement of point A of three-baytwo-story frame

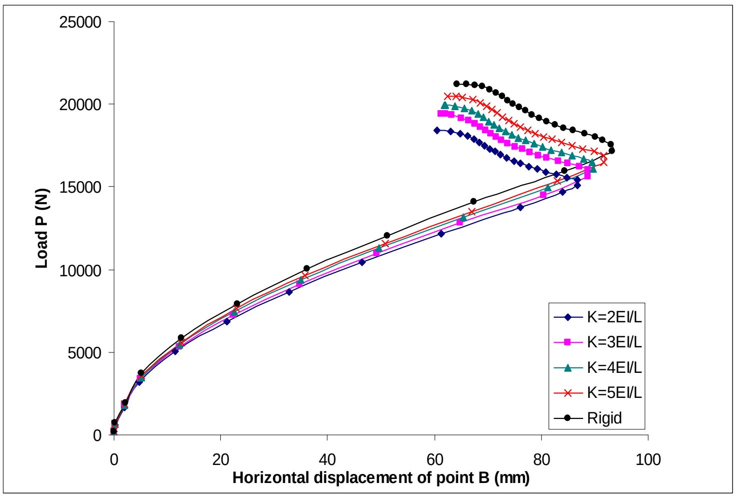

Figure. (27) Effect of joint stiffness on post buckling behavior in terms of load-horizontal displacement of point B of three-bay two-story frame

Table (5) Effect of joint stiffness on ultimate load, ultimate vertical and horizontal displacement of three-bay two-story frame

Stiffness	Rigid	5EI/L	4EI/L	3EI/L	2EI/L
Ultimate load (kN)	21200	20475	19970	19385	18415
Percentage decrease in ultimate load (%)	-	3.4	5.8	8.6	13.1
Ultimate vertical displacement (mm)	964	988	1016	1033	1054
Percentage increase in ultimate vertical displacement (%)	-	2.5	5.4	7.2	9.4
Ultimate horizontal displacement (mm)	104.2	101.5	100	98.8	98.7
Percentage decrease in ultimate horizontal displacement (%)	-	2.6	4.0	5.2	5.3
Final horizontal displacement (mm)	64.3	62.5	61.8	61.2	60.5
Percentage decrease in final horizontal displacement (%)		2.8	3.9	4.8	5.9

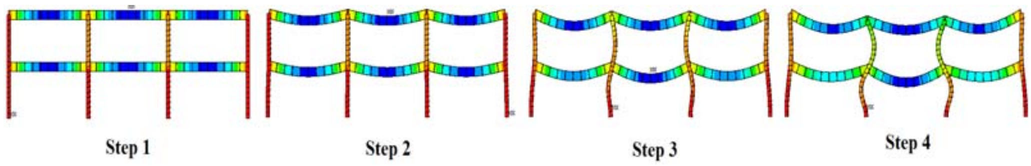

Figure.(28) Post buckling behavior of three-bay two-story frame

CONCLUSIONS

Post-buckling behavior of semi rigid connection steel frames using finite element method is studied in this research. Rotational spring element was used to modeling semi rigid connection in finite element method. The material of members is assumed to be elastic. The arc length method was adopted for predicting large deformations.

Based on the results obtained from this study it can be conclude:

١.For lee′s frame, one-bay two-story and three-bay two-story, changing beam-column connection from rigid to semi rigid make load-displacement curve of post buckling less stiffer, the ultimate load decrease and ultimate vertical displacement increase. This can be attributed to that semi-rigid connection makes the joint more flexible and undergoes large buckling under smaller loads.

2.For one-bay one-story frame, which exhibits snap through buckling, increasing joint stiffness make load-vertical displacement curve become more stiffer in stable regions and become less stiffer in the unstable region. Also decrease joint stiffness result in increase ultimate vertical displacement.

3.Effect of changing beam-column connection from rigid to semi rigid on load-displacement curve of post buckling behavior is more significant for Lee′s frame and less significant for one-bay two-story and three-bay two-story, this can be attributed to that the joints in multi-bay multi-story is more braced by other members which restrict the rotation and movement of joints. However for lee′s frame changing the connection from rigid to semi rigid with joint stiffness value of 4EI/L results in decrease the ultimate load with percentage 20.2% and increase ultimate vertical displacement with percentage 16.1%, for one-bay two-story frame changing to semi rigid with joint stiffness value of 4EI/L results in decrease the ultimate load with percentage 15.3% and increase ultimate vertical displacement with percentage10.9%, while for three-bay two-story frame changing to semi rigid with joint stiffness value of 2EI/L results in decrease the ultimate load with percentage 13.1% and increase ultimate vertical displacement with percentage 9.4%.

4. Semi-rigid connection should be considered in analysis and design of steel frames which suffer from post buckling behavior to obtain more realistic results.

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