

## **A Cantor Fractal Based Printed Monopole Antenna for Dual-band Wireless Applications**

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### **ABSTRACT**

Cantor fractal geometry and its variants are found attractive for microwave antenna designers seeking for compact size multiband antennas. This paper presents the design of a new microstrip fed printed monopole antenna for use in dual-band wireless applications. The radiating element of the monopole antenna is in the form of Cantor fractal geometry of the second iteration as applied on rectangular patch. The monopole radiating element has been etched on a substrate with relative permittivity of 4.4 and 1.6 mm thickness and is fed with a 50 ohm microstrip line. A reduced ground plane has been etched on the reverse side of the substrate. Modeling and performance evaluation of the proposed antenna have been carried out using the commercially available EM simulator, IE3D. Simulation results reveal that the proposed antenna offers dual-band resonant behavior with  $-10$  dB impedance bandwidths and radiation characteristics suitable for almost most of the recently available services in the 1-6 GHz range. A parametric study has been carried out to explore the effect the aspect ratio of the proposed antenna radiating element on its performance. The study reveals that the radiating element aspect ratio has a considerable effect on the coupling of the two resonant bands. Besides the simple design, the antenna offers reasonable radiation characteristics. Simulated  $-10$  dB impedance bandwidths for the lower and the upper resonant bands are (2.30–2.84 GHz) and (5.56–6.01 GHz) respectively. This makes the proposed antenna suitable to cover numbers of operating bands of the wireless communication systems (2.4 GHz-Bluetooth, 2.4 GHz ISM, 2.5/5.8 GHz-WLAN, 5.8 GHz-ITS).

**Keywords:** Cantor Fractal Antenna, Dual-band Printed Antenna

### **INTRODUCTION**

Various fractal geometries are characterized by two unique properties; space-filling and self-similarity. These properties have opened new and essential approaches for antennas and electronic solutions in the course of the most recent 25 years. This preliminary stage gives a prologue to the benefits given by fractal geometry in antennas, resonators, and related

structures. Such profits incorporate, among numerous, wider bandwidths, small sizes, no electronic parts, and better performance. Additionally, fractals give another era of optimized design tools, initially utilized effectively in antennas but applicable in a general manner [1]. Conventionally, the design of dual-band printed dipole antenna design can be performed by using multi-elements in the radiating structure [2-4] or by making slots in it [5-6]; each element or slot resonates at certain frequency. However, the design of dual-band planar antennas for various communications has been still attractive for antenna designers.

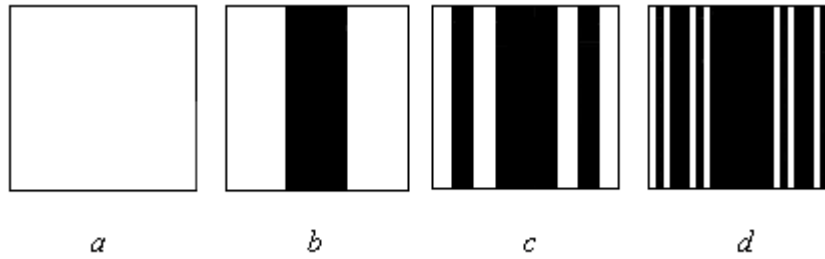
In this context, various fractal geometries have been successfully incorporated in the design of compact and multiband antennas benefiting from their unique properties; space filling and self-similarity respectively [1,7]. The application of various fractal geometries in the design of planar antennas for various communication applications is either in microstrip patch or in printed slot structures [8-17]. To provide bandwidth enhancement of the resonant bands, fractal based slot structures are widely used in the design of multiband printed antennas. Conventional fractal geometries such as Koch, Hilbert, Sierpinski, Minkowski and other fractal curves have been successfully used to produce dual-band and multiband printed slot antennas for various wireless applications [8-14]. However, printed and microstrip patch antenna structures have been also reported in the literature to design compact dual-band and multiband for multifunctional communication services [15- 17].

Furthermore, antenna structure based on Cantor fractal geometry and its variants have been attractive to antenna designers to produce compact and ultrawideband, multiband and dual-band antennas for a wide variety of communication applications [18-27]. It is worth to note here that most of the applications of Cantor fractal geometries are in the design of antennas for wideband and ultrawideband applications [18-21]. Printed slot antennas based on square Cantor fractal geometries have been reported in [22-23] to design a dual-band antenna for 2.5/ 5.8 GHz WLAN applications. In both fractal based structures, the resulting printed slot antennas have offered a dual-band resonant response with enhanced bandwidths of both resonant bands besides reasonable radiation characteristics meeting the requirements of the recently available communication applications. Tri-band antennas have been designed using fractal shaped structures [23-24]. However, many Cantor based antenna structures have been successfully designed to produce multiband radiation characteristics [23-27]. The designed antennas cover most of the currently available communication services operating below 6 GHz.

In this paper, a fractal based printed monopole antenna has been presented as a candidate for use in dual-band wireless applications operating within the frequency range of 1–6 GHz. The radiating structure of the proposed printed monopole antenna is in the form of Cantor fractal geometry of the second iteration as applied to a rectangular structure. The antenna has been directly fed with a 50  $\Omega$  microstrip line etched on the substrate while a shorted ground plane etched on the reverse side.

### **The Antenna Structure**

The radiating element of the monopole printed antenna, presented in this paper, is in the form of Cantor fractal geometry of the second iteration as applied on rectangular patch. The Cantor fractal is made of lines as shown in Figure 1. Here, the generation process of this fractal involves starting with a rectangle, dividing it into three equal rectangles, and cutting out the rectangle at the middle; the process is repeated for the consequent steps. Figure 1 demonstrates the generation process of the Cantor square fractal structure up to the 3rd iteration.



**Figure 1. The steps of growth of the proposed Cantor square fractal up to the third iteration.**

The fractal structure can be constructed using string rewriting beginning with a cell and iterating the rules [28]

$$\left\{ 0 \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, 1 \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right\} \quad \dots (1)$$

The width of the unit element after the  $n$ th iteration is:

$$L_n = \left(\frac{1}{3}\right)^n \quad \dots (2)$$

and the number of elements (white cells) is:

$$N_n = 2N_{n-1} \quad \dots (3)$$

Consider a unity side length square, the width of each white cell,  $W_n$ , in the consequent iterations is given by:

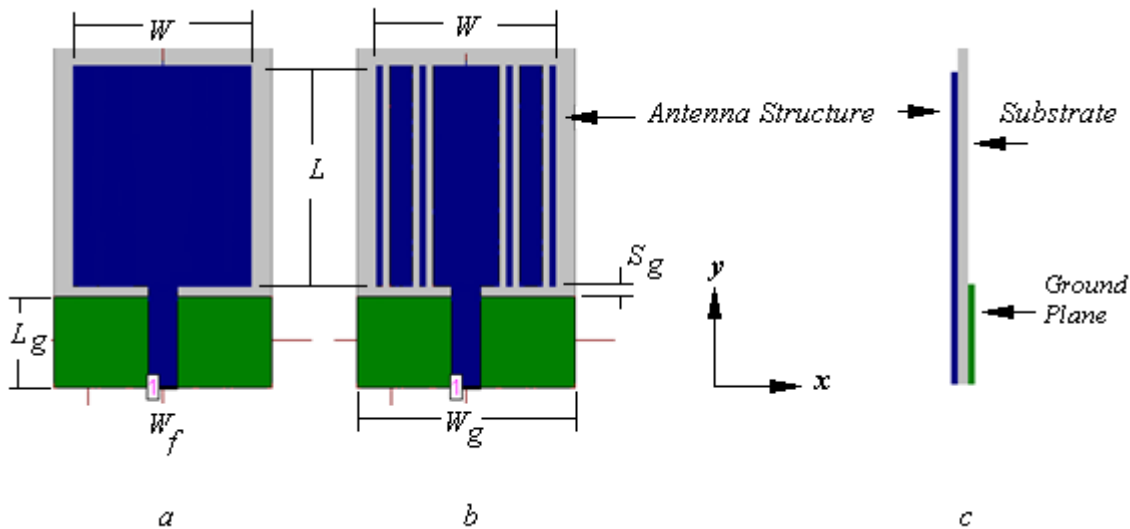
$$W_n = \frac{1}{(3)^n} \quad \dots (4)$$

This means that as  $n$  increases, the width of the white cell exponentially decrease and approach zero as  $n \rightarrow \infty$ , in which case the resulting structure will no longer be a fractal. The fractal dimension,  $D$ , is therefore given by:

$$D = - \lim_{n \rightarrow \infty} \frac{\ln N_n}{\ln L_n} = 2 \quad \dots (5)$$

### The Antenna Design

The application of fractal geometries on the conventional Euclidean antenna structures will result in different current distributions which in turn lead to different resonant behaviors [29]. To confirm this fact, the modeling and performance evaluation of the conventional printed monopole antenna with rectangular radiating element, shown in Figure 2(a), has been started with. The modeled antenna is supposed to be etched on an FR4 substrate with relative dielectric constant of  $\epsilon_r = 4.4$  and  $h = 1.6$  mm. The performance responses of the modeled antenna have been evaluated in the swept frequency range of 1–6 GHz. The width of the  $50 \Omega$  microstrip feed line,  $W_f$ , is found to be about 3.0 mm. Numerical analysis of the antenna performance is carried out using the commercially available method of moment based EM simulator, IE3D[30].



**Figure 2. (a). The layout of the conventional printed monopole antenna with reduced ground plane with respect to coordinate system, (b). The proposed Cantor based monopole printed antenna and (c) The side view of both antenna structures.**

the influence of the various parameters on the antenna performance has been examined. It has been found that the dominant factor in the antenna is the radiating element side length  $L$  in terms of the guided wavelength  $\lambda_g$ :

$$\lambda_g = \frac{\lambda_o}{\sqrt{\epsilon_{eff}}} \quad \dots(6)$$

Where  $\epsilon_{eff}$  is the effective dielectric constant. In terms of the slot side length  $L$  and the guided wavelength  $\lambda_g$ , the lower resonant frequency,  $f_1$ , is given by:

$$f_1 \approx \frac{c}{2L\sqrt{\epsilon_{eff}}} \quad \dots(7)$$

Where  $c$  is the speed of light in free space. The guided wavelength  $\lambda_g$  has been calculated in terms of the lower resonant frequency. The modeled antenna has been designed such that the lowest resonant frequency is to be allocated around 2.5 GHz.

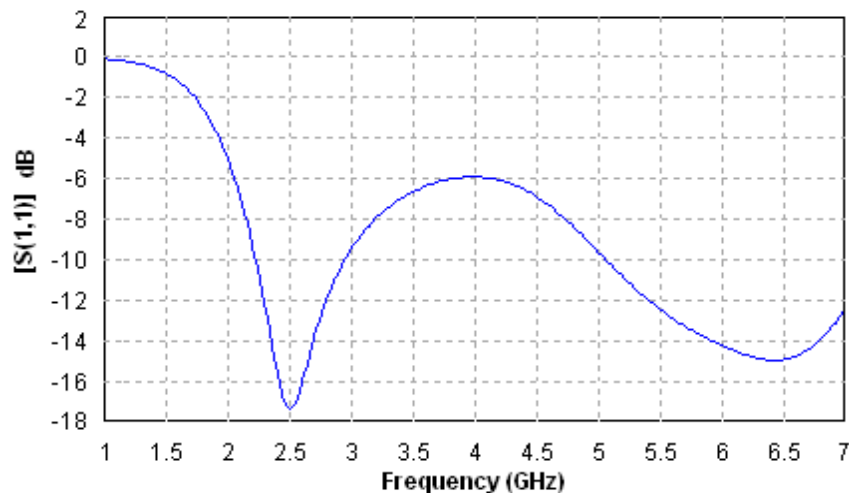
Afterwards, the Cantor fractal geometry shown in Figure 1(b) is applied to the antenna structure depicted in Figure 2(a). The resulting fractal based printed monopole antenna structure is shown in Figure 2(b). This antenna has been modeled using the same substrate and its performance has to be evaluated as will be shown in the following section.

### Numerical Results

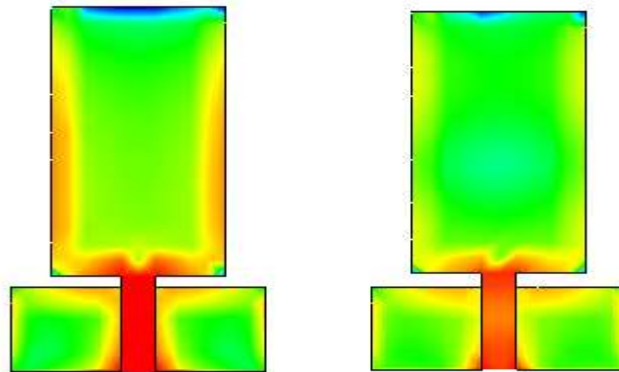
The conventional printed dipole antenna with a rectangular radiating element and reduced ground plane, depicted in Figure 2(a), has been started with as a reference antenna. By appropriate dimension scaling together with antenna parameters optimization, the lowest resonant band has to be allocated at the design frequency, 2.5 GHz. The antenna has been modeled using the said substrate and its performance, in terms of the input reflection coefficient  $S_{11}$  response, has been evaluated within the sweep frequency range of 1–6 GHz using the commercially available method of moment based EM simulator, IE3D [30]. The resulting antenna dimensions are summarized in Table 1.

**Table 1. Summary of the modeled antenna depicted in Figure 2(a)**

Parameter	$L$	$W$	$L_g$	$W_g$	$S_g$	$W_f$
Dimensions (mm)	26	15.1	9.18	21.98	1.1	3.01



**Figure 3. Simulated input reflection coefficient response of the antenna depicted in Figure 2(a).**

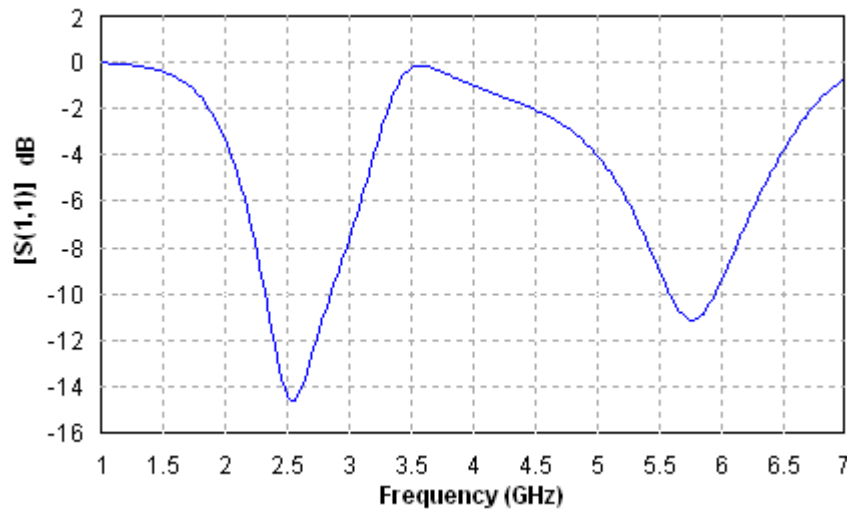


**Figure 4. Simulated current distributions on the surface of the modeled antenna depicted in Figure 2(a) at 2.5 and 5.8 GHz.**

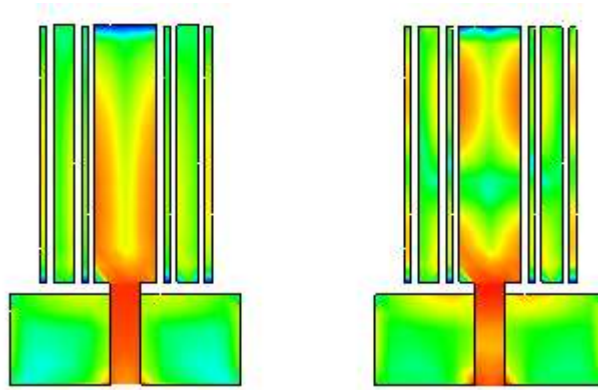
Figure 3 shows the input reflection coefficient response of this antenna in the prescribed frequency range. The results imply that the lowest resonant frequency has been located at about 2.5 GHz. With an antenna radiator side length  $L$  of 26 mm, the lower resonant frequency is clearly what is predicted by Equation (7). Furthermore, the response depicted in Figure (3) implies that the antenna tends to produce an ultra-wideband behavior which is attributed to the reduced ground plane. This behavior confirms the findings reported in [29, 31-32]. However, as stated before, the current distribution on the surface of the antenna radiator at the design frequency provides more physical insight. Figure (4) shows the surface current distribution at 2.5 and 5.8 GHz.

The same process is repeated to model the proposed antenna depicted in Figure 2(b). The side length  $L$  and the width  $W$  of the monopole radiator have been kept unchanged. The dimensions of the modeled antenna are the same as those summarized in Table 1. The dimensions of the embedded slots in the radiator structure can be determined by Equation 4 with  $n = 2$ . The performance of the antenna has been evaluated using the same substrate and throughout the same sweep frequency range.

Figure 5 shows the input reflection coefficient of the modeled antenna. It is clearly shown that the antenna offers dual-band resonant behavior throughout the swept frequency. The lower resonant band is centered at 2.5 GHz. Again, this confirms that the lower resonant band of both antennas is primarily determined by the radiator side length  $L$  as predicted by Equation 7. The upper resonant band is centered at 5.8 GHz. This resonant band is attributed to the slots embedded in the rectangular radiating structure depicted in Figure 2(b). The current distribution, on the surface of the radiating element shown in Figure 6, confirms this fact. The existence of the slots provides a different current path as compared with that shown in Figure 4. This disturbance in the current path contributes in the creation of the upper resonant band.

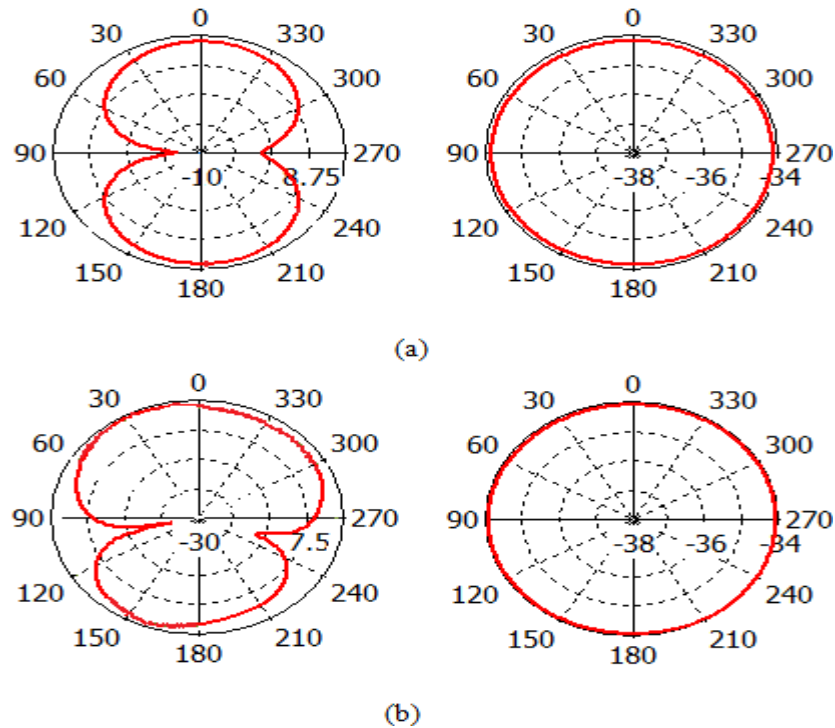


**Figure 5. Simulated input reflection coefficient response of the antenna depicted in Figure 2(b).**



**Figure 6. Simulated current distributions on the surface of the fractal based antenna depicted in Figure 2(b) at 2.5 and 5.8 GHz.**

The simulated  $-10$  dB return loss response bandwidths for the lower and the upper resonant bands depicted in Figure 5, are of about (2.30–2.84 GHz) and (5.56–6.01 GHz) respectively. This makes the proposed antenna suitable to cover many operating bands of the wireless communication systems (2.4 GHz-Bluetooth, 2.4 GHz ISM, 2.4/5.8 GHz-WLAN, 5.8 GHz-ITS).



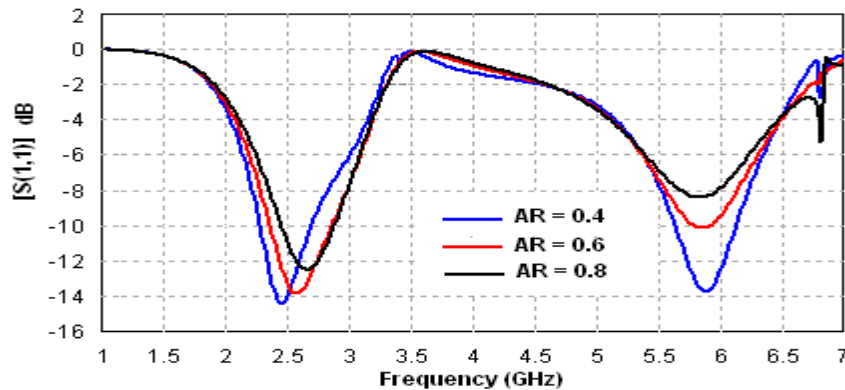
**Figure 7. Simulated far field radiation patterns of the proposed antenna at (a). 2.5GHz and (b). 5.8 GHz.**

The far field radiation pattern characteristics of proposed fractal monopole antenna are shown in Figure 7. In this case, the proposed antenna resonates at 2.50 GHz and 5.80 GHz in broad side direction at  $\varphi=0^\circ$  and  $\varphi=90^\circ$ . The results show almost monopole like radiation patterns with almost omnidirectional radiation.

### Parametric Study

The proposed printed fractal based printed monopole antenna depicted in Figure 2 has been modeled and its performance has been evaluated within the swept frequency range of 1–6 GHz using the prescribed substrate. Simulation results show that the antenna offers a dual-band response within the prescribed sweep frequency. The proposed antenna has many degrees of freedom that are attractive to antenna designers. The effect of varying the gap width between the antenna radiator and the ground plane and the width of the reduced ground plane have been investigated by many authors [31-32] and no need to repeat it again. In this section a parametric study has been carried out to explore the effect of the radiator aspect ratio on the antenna performance in terms of the input reflection coefficient response. The aspect ratio of the antenna radiator structure is defined as the ratio of slot width  $W$  to its length  $L$ .





**Figure 8. Simulated input reflection coefficient responses of the proposed antenna with the monopole radiator aspect ratios  $W/L$  as a parameter.**

In this context, when varying the values of the radiator aspect ratio, it is kept in mind that the antenna lowest resonant bandwidth has to be maintained constant and centered at 2.50 GHz. It has been shown earlier the antenna lower resonant frequency has been determined by the radiator length  $L$  as stated by Equation (7). Thus the values of the radiator aspect ratio are obtained by only varying the antenna radiator width  $W$  and keeping its length  $L$  constant. Figure 8 demonstrates the simulated return loss responses of the proposed antenna for selected values of the aspect ratio in the range of 0.40–0.8, in steps of 0.20. As it is implied from Figure 8, varying the radiating element aspect ratio results in very small changes of the upper resonant frequency as compared with those of the lower resonant frequency.

**Table 2. The coupling of the two resonant bands with different values of the aspect ratio**

Aspect Ratio	Lower Band $S_{11}$ (dB)	Upper Band $S_{11}$ (dB)
0.25	-16.98	-21.73
0.30	-13.59	-16.08
0.40	-14.34	-13.72
0.50	-14.21	-11.72
0.60	-13.82	-10.12
0.75	-12.71	-8.67
0.80	-12.48	-8.40

However, the changes in the lower frequency corresponding to the different values of aspect ratio are still small. An interesting result of varying the radiator aspect ratio is that the coupling of the upper resonant band is highly affected as compared with that of the lower resonant band; resulting in enhancement of the upper resonant bandwidth as the aspect ratio becomes lower. To demonstrate this effect in more details, additional values of the aspect ratio and the corresponding values of the

coupling levels have been listed in Table 2. The results demonstrated in Table 2 reveals that increasing the aspect ratio will worsen the coupling of the two resonant bands and the deterioration of the upper band is more as compared with that of the lower one. However, the coupling of the two bands becomes approximately equal for an aspect ratio equal to 0.4.

### Practical Measurements

The proposed antenna is fabricated using an FR4 substrate with relative dielectric constant of  $\epsilon_r=4.4$  and  $h=1.6$  mm. Figure 9 shows a photo of the fabricated prototype. The measured and simulated performance responses of the fabricated antenna have been evaluated in the swept frequency range of 1–6 GHz as shown in Figure 10.

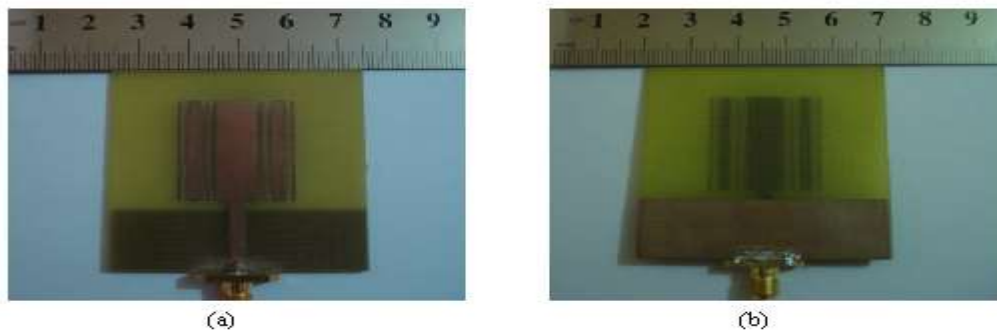


Figure 9. A photo of the fabricated prototype; (a) top view, and (b) the bottom view.

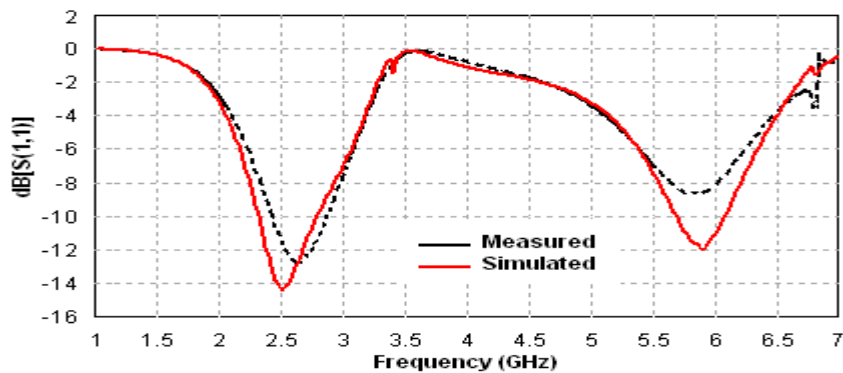


Figure 10. Measured and simulated input reflection coefficient responses of the fabricated antenna.

Measured and simulated input reflection coefficient responses of the fabricated antenna, depicted in Figure 10, with a reasonable agreement. The measured antenna response shows a little difference in the position of the lower resonant band, while the upper band position is approximately the same as that predicted by simulation. However, there is a small difference in the coupling of the first resonant band between measured and simulated results and the coupling becomes poorer in the upper band as compared with the simulated one. This can be attributed in

part to the slight mismatch in the fabricated prototype and also to the tolerances in the fabricated antenna. Because of the unavailability of the required measuring instruments and the echoic chamber, antenna radiation patterns have not been presented in this paper.

## CONCLUSIONS

A Cantor fractal based printed monopole antenna has been introduced in this paper, as a candidate for use in dual-band wireless applications. Simulation results of the modeled antenna have shown that the proposed antenna structure offers interesting features in that it can be a suitable candidate covering most of the recently available communication services operating below 6 GHz. The antenna has offered dual-band behavior with resonant bandwidths and accepted radiation characteristics. Simulation results of  $-10$  dB return loss response bandwidths for the lower and the upper resonant bands are centered at 2.53 GHz and 6.01 GHz and extending from about (2.30–2.84 GHz) and (5.56–6.01 GHz) respectively. This makes the proposed antenna suitable to cover many operating bands of the wireless communication systems (2.4 GHz-Bluetooth, 2.4 GHz ISM, 2.4/5.8 GHz-WLAN, 5.8 GHz-ITS). Because of the high degree of freedom offered by the proposed antenna design, it is hopefully considered as an attractive choice of the antenna designer.

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