The Most Effective Parameters of Nanofluids Flow over Flat Stretching Surface in the Presence of Variable Stream Conditions

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Abstract

Heat transfer and mass concentration in the boundary layer of steady nanofluids flow (Nanofluid Defined as a Fluid Containing Nanometer-sized Particles) which occurs because the stretching of a flat surface was studied numerically. The mathematical system contains ten parameters which are effect on this type of flow such as Brownian motion (Brownian motion defined as is the random movement of particles suspended in a base fluid), variable thickness magnetic effect, non-linear velocity, thermophoresis, chemical reaction, Reynolds number, Prandtl number and heat source. The Partial differential equations PDE (Which Represents the Boundary Layer Equations BLE for the Steady Two-dimensional Flow) are turned into ordinary deferential equations ODE using similarity transformation and then analyze the issue numerically by the method “shooting technique”. The profiles of velocity, temperature and concentration were analyzed in detail for mentioned parameters to determine which parameters will have significant effects.

Keywords: Nanofluid, Stretching, Flat Surface and Stream Conditions.

المعاملات الاكثر فعالية على عملية تدفق الموائع النانوية على سطح مستو متمدد عند وجود حالات تيار متغيرة

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الخلاصــة

حللت عملية انتقال الحرارة والكتلة والتركيز في الطبقة الحدودية لتدفق الموائع النانوية (وهو المائع ممزوجا مع ذرات متناهية الصـغر) في حالة الجريان المستقر والناتج عن تمدد سطـح مستو تحليلا عدديا. يتضمن الأنموذج الرياضي عشرة متغيرات والتي من الممكن أن تؤثر على هذا النوع من التدفق ومنها الحركة البراونية (التي هي عبارة عن حركة غير متناسقة تنتج عن تصادم ذرات المائع مع بعضها البعض ومع ذرات النانو)، التأثير المغناطيسي والسرعة غير الخطية والسماكة المتغيرة والرحلان الحراري والتفاعل الكيميائي والوسط المسامي وعدد رينولد وعدد براندل وأخيرا مصدر الحرارة. حولت المعادلات التفاضلية الجزئية إلى معادلات اعتيادية باستخدام التحويل التماثلي ثم بعد ذلك حللت المسألة عدديا باستخدام تقنية (شوتنك تكنيك) لنحصل بعدها على دراسة تفصيلية لملامح السرعة ودرجة الحرارة والتركيز لجميع المتغيرات سابقة الذكر لتحديد المتغيرات التي سيكون لها تأثير ملحوظ.

الكلمات المفتاحية: مائع نانوي، متمدد، سطح مستوي وحالات التيار المتغير.

Introduction

Nanofluids are defined as fluids containing nanometer-sized particles, called nanoparticles (1-100) nm suspended in conventional heat transfer basic fluid (Choi and Eastman, 2013). The new mixture has an enhancement for thermal conductivity and the convective heat transfer coefficient compared to the base fluid. For example, it develops the water's ability to conduct temperature from 30 to 40 percent. So that, many studies were presented to enhance the thermal conductivity of fluids using this method (Kleinstreuer and Feng, 2011). Nanofluid enhancements were studied by many works to determine the parameters that could affect it, some researchers studied the parameters on slip flow (Bhattacharyya, *et al*., 2011) while some research focused on exponentially stretching sheet (Kala, 2014). Some researchers focused on the effect of parameters on moving permeable convective surface (Qasim, *et al*., 2013). While the study (Bognár and Csati, 2014) focused on the effects of parameters over moving flat. Rotating frame was studied by (Das, 2014), while stretching/ shrinking sheet of a nanofluid with suction/injection was investigated by (Naramgari and Sulochana, 2016), (Fang, *et al*., 2012). The effect of parameters in case of rotating system with permeable sheet was studied by (Sheikholeslami and Ganji, 2014). The vertical plate parameters included in research (Abdel Wahid, *et al*., 2015). The wedge Moving studied by research (Khan and Pop, 2013). Most of studies mentioned above focuses on some parameters while this research makes collection of some parameters altogether to find out the effects of these parameters in nanofluids as well as to these effect on each other of nanofluid flow on stretching of a flat surface.

Materials and Methods

Formulation of the Problem

We shall focus on this problem with following conditions steady, laminar, two-dimensional flow, incompressible conduction, over flat, existence of magnetic field, heat source, chemical reaction, the electrically conducting of the fluid and external electric field are negligible, this happens when Reynolds number was small.

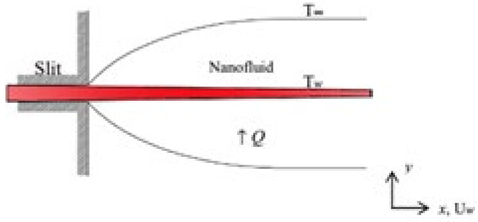

Figure (1) Physical Model and Coordinate System.

Figure (1) refers to nanofluid flow where x-axis flows along the center of the surface, and it designed as:

$$y = \delta(X + b)^{\frac{1-n}{2}}$$

that makes the surface isn't flat and thin by taking δ small.

The equations of governing BL for the case we mentioned above can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u.\frac{\partial u}{\partial x} + v.\frac{\partial v}{\partial y} = \upsilon.\frac{\partial^2 u}{\partial y^2} - \frac{\sigma.B^2(x)}{\rho}.u - \frac{\upsilon}{K} \quad (2)$$

$$u.\frac{\partial T}{\partial x} + v.\frac{\partial T}{\partial y} = \propto.\frac{\partial^2 T}{\partial y^2} + \tau[D_B.\frac{\partial C}{\partial y}.\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2] + \frac{Q(x)}{\rho.c_p}(T - T_\infty) \quad (3)$$

$$u.\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}.\frac{\partial^2 T}{\partial y^2} + k_1(C - C_\infty) \quad (4)$$

With the B.C's

$-\infty < x < \infty$, $u = U_w$, $v = 0$,

$T = T_w$, $C = C_w$, at $y = \omega(x+b)^{\frac{1-n}{2}}$

$u = 0$, $v = 0$ $T = T_\infty$, (5)

$C = C_\infty$, as $y \to \infty$

Here u and v represent the directional velocity in the direction x and y.

Where ν, ρ, σ, K, D_B,α , D_T, K_1, τ, B(x), is the kinematic viscosity, density of base fluid, electrical conductivity, permeability of surface, thermal diffusion, Brownian diffusion coefficient, thermophoretic diffusion coefficient, rate of chemical reaction, the ratio between the effective heat capacity of nanoparticle and heat capacity of the fluid is the magnetic field force respectively, where

$B(x) = B_0(x+b)^{\frac{n-1}{2}}$

$Q(x) = Q_0(x+b)^{n-1}$

Q and B are taken to get a similarity solution. The velocity, temperature, and concentration are formed as

$U_w(x) = a(x+b)^n$, $\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$, $\emptyset(x) = \frac{C-C_\infty}{C_w-C_\infty}$

a and b are constants, n represents shape parameter, and we choose

$n > -1$.

Similarity Transformation

In the equations (1-5) we want the similarity solution subject to the B.C's(5) as below:

$$\eta = y\sqrt{\left(\frac{n+1}{2}\right)\left(\frac{a(x+b)^{n-1}}{\nu}\right)} \qquad (6)$$

$$\psi = \sqrt{\left(\frac{2}{n+1}\right)(x+b)^{n+1}a\upsilon}\ F(\eta) \qquad (7)$$

The variable η represents the similarity variable, ψ represents stream function and can be written as $u = \frac{\partial\psi}{\partial x}$ and $v = \frac{\partial\psi}{\partial x}$

This makes equation (1) substituting equation (7) in equations (2-4) we will get the ODE's.

$$F''' + FF'' - \left(\frac{2n}{n+1}\right)F'^2 - \left(\frac{2}{n+1}\right)(M+\delta)F' = 0 \qquad (8)$$

$$\frac{1}{Pr}\theta'' + F\theta' + \left(\frac{2}{n+1}\right)\lambda\theta + Nb\theta' + Nt\theta'^2 = 0 \qquad (9)$$

$$\phi'' + Le.d.F.\phi' + \left(\frac{Nt}{Nb}\right)\theta'' = 0 \qquad (10)$$

And B.C,s

$F(\alpha) = \alpha\left(\frac{1-n}{1+n}\right)$, $F'(\alpha) = 1$,
$\theta(0) = 1$, $\phi(0) = 1$, and $F'(\infty) = 0$,
$\theta(\infty) = 0$, $\emptyset(\infty) = 0$ (11)

The prime indicates to differential with respect to (η), and $\alpha = \delta\sqrt{\frac{1+n}{2}\frac{a}{\nu}}$ indicates to the surface thickness parameter while $\eta = \alpha = \delta\sqrt{\frac{1+n}{2}\frac{a}{\nu}}$ refers the flat surface.

We assume $F(\eta) = f(\eta-\alpha) = f(y1)$.

So, the equations (8-10) and B.C's (11) will written as

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 - \left(\frac{2}{n+1}\right)(M+\delta)f' = 0 \qquad (12)$$

$$\theta'' + Pr\left[f\theta' + \left(\frac{2}{n+1}\right)\lambda\theta + Nb\theta' + Nt\theta'^2\right] = 0 \qquad (13)$$

$$\phi'' + Le.d.F.\phi' + \left(\frac{Nt}{Nb}\right)\theta'' = 0 \qquad (14)$$

And the B.C's

$f(0) =$, $f(0) = 1$, $\theta(0)$ 1

$f'(\infty) = \theta(\infty) = 0, \emptyset(\infty)0$ (15)

Where the prime indicates to derivative with respect to y1.

$Pr = \frac{\nu}{\alpha}$, $Le = \frac{\upsilon}{D_B}$, $M = \frac{B_0^2\sigma}{a\rho}$, $\lambda = \frac{Q_0}{a\rho c_p}$, $Nb = \frac{\tau D_B}{\upsilon}(C_w - C_\infty)$,

$$Nt = \frac{\tau D_T}{\nu T_\infty}(T_w - T_\infty) \quad \text{and} \quad \tau = \frac{(\rho c_p)_p}{(\rho c_p)_f}$$

The parameters Pr, Le, M, λ, Nb and Nt represent Prandtle number, Lewis number, magnetic field parameter, heat source parameter, Brownian motion parameter and thermophoresis parameter respectively.

The variables skin friction coefficient, the Nusselt and Sherwood numbers will be written as:

$$C_f = \frac{\tau_w}{\rho u_\infty^2}, N_u = \frac{x q_w}{K(T_w - T_\infty)},$$

$$sh = \frac{x q_m}{D_B(C_w - C_\infty)}$$

While τ_w; qw; qm represent the surface shear stress, heat, and mass flux at the surface respectively, and can be written as:

$$\tau_w = \mu(\frac{\partial u}{\partial y})_{y=0}, \; q_w = -k(\frac{\partial T}{\partial y})_{y=0},$$

$$q_m = -D_B(\frac{\partial C}{\partial y})_{y=0}$$

The dimensionless Skin friction, Nusselt number and Sherwood number can be defined as:

$$cfr = Re_x^{\frac{-1}{2}} c_f = f''(0)$$

$$Nur = Re_x^{\frac{-1}{2}} Nu = -\theta''(0)$$

$$Shr = Re_x^{\frac{-1}{2}} Sh = -\emptyset''(0)$$

Formulate Solution Using Shooting Technique

Here we will use "Shooting technique" to solve ODE system (11), (12) with the B.C's (15) by considering the equations below:

$$f'(y1) = u(y1) \quad (16)$$

$$u'(y1) = v(y1) \quad (17)$$

$$v'(y1) + f(y1).v(y1) - \left(\frac{2n}{n+1}\right).(u(y1))^2 - \left(\frac{2}{n+1}\right)(M + \delta).u(y1) = 0 \quad (18)$$

$$\theta'^{(y1)} = r(y1) \quad (19)$$

$$r'(y1) + Pr\{Nb.r(y1).h(y1) + Nt.r(y1))^2 + f(y1).r(y1) - \left(\frac{2}{n+1}\right)(\lambda.\theta(y1)\} = 0 \quad (20)$$

$$\phi'(y1) = h(y1) \quad (21)$$

$$h'(y1) + Le.f(y1).h(y1) - \left(\frac{2}{n+1}\right).$$

$$Le.d.\phi(y1) - \left(\frac{Nt}{Nb}\right)Pr\{(f(y1).r(y1) + (Nb.r(y1).h(y1)) + Nt.(r(y1))^2 + \left(\frac{2}{n+1}\right).\lambda.\theta(y1)\} = 0 \quad (22)$$

The initial conditions will be:

$$\alpha\left(\frac{1-n}{1+n}\right); v(0) = \epsilon; r(0) = h(0) = \varsigma \quad (23)$$

$$u(\infty) = 0; \quad \theta(\infty) = 0; \emptyset(\infty) = 0$$

Here $f''(0), \theta'(0)$ and $\phi'(0)$ refers to skin friction, rate of heat transfer and rate of mass transfer respectively where ϵ, τ and ς represent the numerical $f''(0), \theta'(0)$ and $\phi'(0)$ respectively. The model of (16) to (22) are additional conditions with B.C's (23) altogethe will be solve using Shooting technique by determining approximate value of $f''(0), \theta'(0)$ and $\phi'(0)$ and then by evaluate the profiles of velocity, heat and concentration.

Results and Discussion

Firstable to prove the validity of this study we will make a comparison between our outcomes and two previous studies by evaluate $f''(0)$ according to some values of surface thickness parameter and shape parameter as in Table (1).

Here we need to refer that we studied the ten parameters and we founded they have different effects on profiles of velocity, temperature or concentration with varying intensity, so we classified this effect according to the strength of the effect of the parameter into 4 types s means significant effect m means middle effect w means a slight effect n means it doesn't effect see Table (12).

Table (1) Comparison between Values of f''(0) in this Research and Previous Researches.

β	n	f''(0)	Abdelwahid, *et al.* (2015)	Fang, *et al.* (2012)
0.25	0.5	0.9339	0.9264	0.9338
0.25	1	1.0001	1.000	1.0000
0.25	5	1.1187	1.1262	1.1186
0.5	0.5	0.9800	0.9633	0.9799
0.5	1	1.0001	1.0000	1.0000
0.5	2	1.0235	1.0333	1.0234

Analysis of Shape Parameter n

Beside the thickness parameter, shape parameter determines the kind of moving such that when n = 0, the moving converted to linear with constant velocity and the motion converted to deceleration moving when n < 1 and acceleration moving when n > 1. Figure (2), indicates that any increase in n implies increasing velocity. Also Figure (3) shows that increase n leads increasing in temperature of nanofluid, while Figure (4) indicates that the concentration of nanofluids decrease by increasing n at the beginning for 0<η<s, such that s increases when n increases, the concentration after that increases with increase of n. Table (2) explains the skin friction increases as shape parameter n increase, also it refers decreasing in rate of heat transfer when n increasing, it also shows increasing in mass transfer as increasing shape parameter. We need to indicate that η in the graphs refers to similarity variable while f ''(0), θ'(0) and ϕ (0) represent the rate of skin friction heat transfer and the rate of mass transfer respectively. Here we need to refer that we studied the ten parameters and we founded they have different effects on profiles of velocity, temperature or concentration with varying intensity as shown in Table (12). From Table (12) we can see that the shape parameter effects strongly on concentration profile while it has normal effect on temperature and velocity profiles.

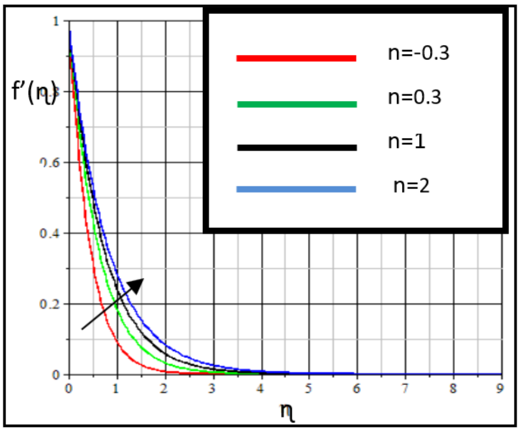

Figure (2) Velocity Profile Where f'(η) is Dimensionless Velocity, η Similarity Variable.

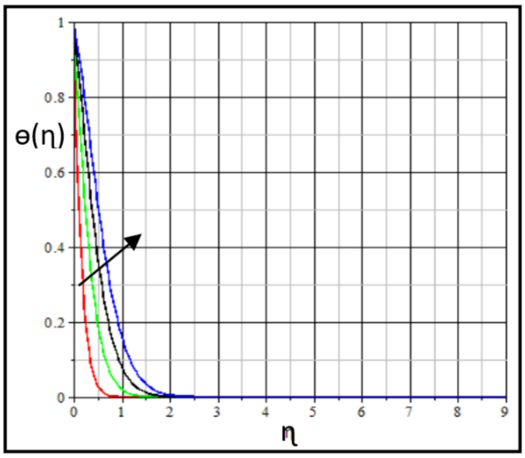

Figure (3) Temperature Profile Where θ(η) Dimensionless Temperature η Similarity Variable.

In this study we focused on the decelerating moving of nanofluid by taking the value of n = - 0.3 and we studied the accelerating moving by taking the value of n=0.3, 1 and 2.

Table (2) f '' (0), θ' (0) and Ø' (0) for some Values of n, Pr=7.0, Le=0.62, λ=γ=Nb=Nt=0.1, δ=M=ß=0.5.

n	f ''(0)	- θ'(0)	-ϕ' (0)
-0.3	-2.181138	6.030542	-4.78573
0.3	-1.622804	2.418331	-1.54212
1	-1.414213	1.267901	-0.551244
2	-1.293145	0.740107	-0.126311

Table (3) f '' (0), θ' (0) and Ø'(0) for some Values of ß, Pr=7.0, Le=2, λ==Nb=Nt=0.1, δ=n= γ =M=0.5.

ß	f ''(0)	- θ'(0)	-ϕ' (0)
0.1	-1.471600	0.920689	1.21660
1	-1.633396	2.146566	0.43389
2	-1.830487	3.741941	-0.7137
3	-1.830487	3.741941	-0.7137
4	-2.274643	7.226717	-3.2556

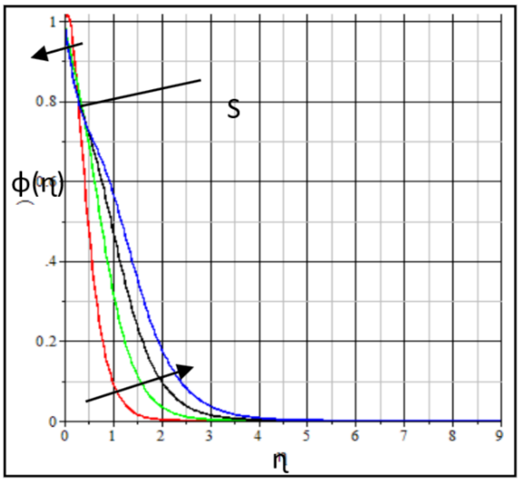

Figure (4) Concentration Profile where ϕ(η) Dimensionless Concentration η is the Similarity Variable.

Analysis of Thickness Parameter ß

The effects of thickness parameter on the three profiles f, ɵ and φ shown in Figures (5, 6 and 7), we can see from these figures the increasing of thickness parameter makes decreases in velocity and this decreasing would be more significant on the interval 0.5<η<1.5. Figure (6) refers to decrease in temperature as increase of ß, but the concentration will be increasing at the interval (0<η<s, Such that s Increases with Decreases of ß) and then it is decreasing (When η>s) see Figure (7). Table (3) indicates decreasing in skin friction as increasing in heat transfer and decreasing in mass concentration. Table (3) refers that the effect of thickness on Skin Friction, Rate of Heat and Mass doesn't change when ß changes from 1 to 2.

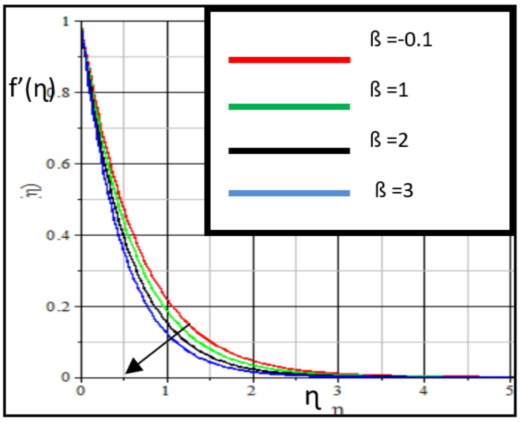

Figure (5) Velocity Profile, η Similarity Variable.

Table (12) Shows that the shape parameter can effect strongly on concentration profile while it's effect on temperature profile and velocity profile was normally.

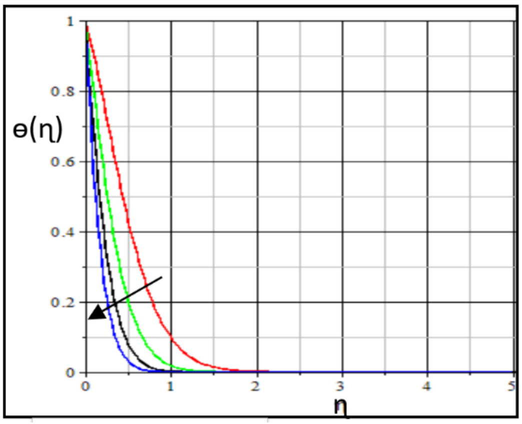

Figure (6) Temperatur Profile θ(η), η is Similarity Variable.

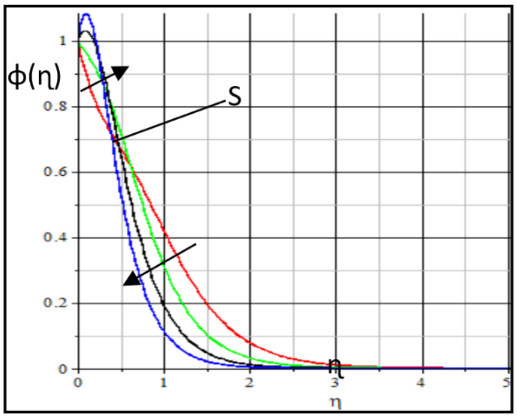

Figure (7) Concentration Profile, η is the Similarity Variable.

Analysis of Magnetic Parameter M

In the existence of magnetic field, the velocity profile shown in Figure (8) decreases as the parameter M increases, while the temperature increases as shown in Table (4) and the reason is because of that the nanoparticle dissipates energy in the form of heat which implies to the thermal boundary layer thickness to increase and eventually a localized rise in temperature of nanofluid happens see Figure (9). Whereas the concentration shows two different behaviors such that, it decreases when ($\eta<s$) and increases when ($\eta>s$) as illustrated in Figure (10) s increases as M increases. Table (5) refers to decreasing in skin friction, decreasing in heat transfer and increasing in mass transfer. Table (12) refers that the magnetic parameter can effect significantly on temperatutr while it has normal effect on velocity and slightly on temperature.

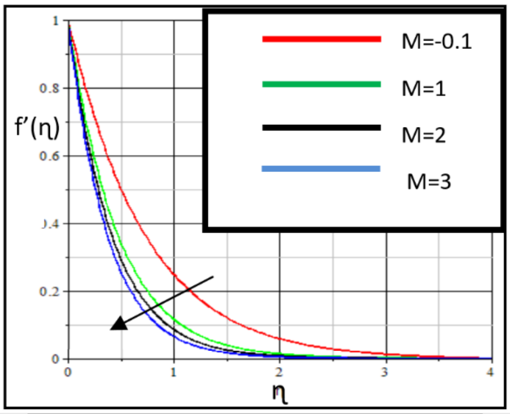

Figure (8) Velocity Profile, η Similarity Variable.

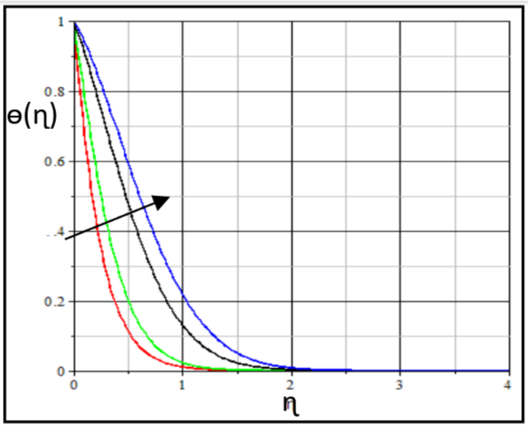

Figure (9) Temperatur Profile, η is the Similarity Variable.

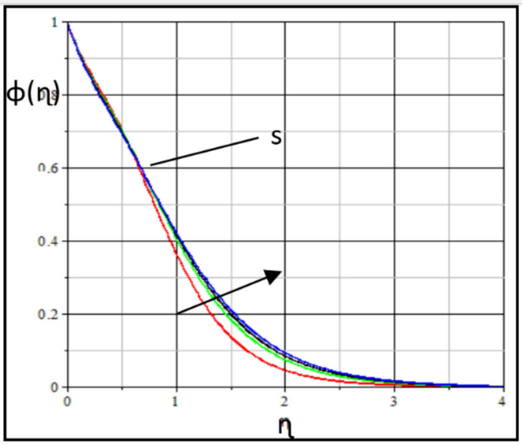

Figure (10) Concentration Profile, η is the Similarity Variable.

Table (4) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2, λ=Nb=Nt=0.1, δ=n= γ =ß =0.5.

M	f ''(0)	- θ'(0)	-ϕ' (0)
0.1	-1.346847	1.463633	0.910146
2	-2.113931	1.332412	0.935679
3	-2.419079	1.278066	0.955353
4	-2.688795	1.229623	0.975883

Table (5) Skin Friction, Rate of Heat and Rate of Mass Transfer for Different Values of Nt, Pr=7.0, Le=2, λ=Nb=ß =0.1, δ=n= γ =M=0.5.

Nt	f ''(0)	- θ'(0)	-ϕ' (0)
-0.5	-1.47169303	3.0634059	13.256781
-0.3	-1.47169303	2.2313500	6.0176905
0.2	-1.47169303	0.8442010	0.1718923
0.4	-1.47169303	0.5293875	0.3615862

Analysis of Thermophresis Nt

Thermophoresis is phenomena, which causes nanoparticles to drive away from a hot surface toward a cold one. Nanoparticles experience a force in the direction opposite to temperature gradient. It is noticed that increasing in thermophoresis implies to increasing in temperature as shown in Figure (11) regarding to the concentration Figure (12) shows that any increasing in Nt and Nt <0 implies to increasing in concentration on the interval $0<\eta<S1$, whereas it decreases on the interval $S1<\eta<\infty$. But when Nt>0 It has reverse behavior such that the concentration decreases at the interval $0<\eta<S2$ and increase at the interval $S2<\eta<\infty$. While Table (5) indicates that the skin friction increases, heat transfer decreases as Nt increase. while the rate of mass transfer increases as Nt increases when Nt>0 but when Nt<0 mass transfer decreases as Nt increases. Table (12) shows that the thermophoresis parameter can affect strongly on both concentration and temperature profiles.

Analysis of Brownian Motion Nb

Brownian motion can effect on the boundary layer subject to its value. This research shows that any increasing in Nb leads increasing in boundary layer temperature but decreasing in concentration of nanoparticles. Figure (13) shows that as moving away from the wall the temperature increases as Nb increase. Figure (14) shows that the concentration decreases by increase Nb such that the concentration near the wall reduces as Nb increases, also by taking a closer look on the graph shows that when Nb high the concentration near the surface does not affect or has a slight effect also the decrease of concentration was significant when Nb increased from 0.1 to 0.3, whereas this decrease was slight when Nb increased from 0.3 to 0.5. On the other hand, Table (6) refers to decreasing in rate of heat and mass transfer as Nb increases, increasing in mass concentration as Nb increases.

Table (6) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2 of Nb, Pr=7.0, Le=2, λ==Nt=0.1, δ=n= γ = ß =M=0.5.

Nb	f ''(0)	- θ'(0)	-ϕ' (0)
0.1	-1.5416482	1.4313901	0.9126212
0.3	-1.5416482	0.6619010	1.6180948
0.5	-1.5416482	0.2268206	1.6942749
0.7	-1.5416482	0.0225446	1.6963155

Also it shows Nb has no effect on velocity profiles. While Table (12) refers to that the Brownian motion has no effect on velocity and has normal effect on temperature and slight effect on concentration.

Analysis of Porous Medium Parameter δ

Figure (15) explains the velocity decreases as increase in δ and this decreasing was more significant while δ increased from η in (0.1 to 1) rather than η in (1 to 2) or η in (2 to 3). Figure (16) shows increasing on temperature as δ increases especially when η>0.5 .For Figure (17) the concentration experiences a slight decrease on the interval η<s before it increases when η>s. Moreover, for Table (7) increasing in δ shows decreasing in skin friction and heat transfer but slight increase in the rate of mass concentration.

In this study we prove that porous medium has a significant effect on velocity profile but it has a slight effect on temperature profile and concentration profile as shown in Table (12).

Analysis of Chemical Reaction γ

Chemical reaction effects on boundary layer of nanofluid makes significant change on concentration and slight change (Can be Neglected) on temperature but no change on velocity. Such that increasing chemical reaction will increase the temperature slightly and decrease the concentration drastically as illustrated in Figure (18). Table (8) shows decreasing in the rate of heat transfer, increasing in the rate of mass transfer as chemical reaction parameter increases. Table (12) indicates that the effect of chemical reaction will be obvious only on concentration profile but it will be very slight on temperature profile and hasn't any effect on velocity profile.

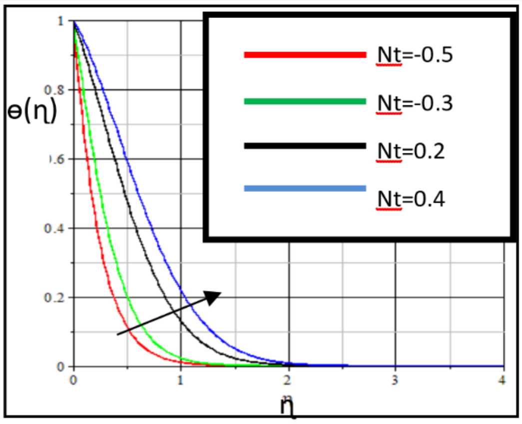

Figure (11) Temperature Profile, η is the Similarity Variable.

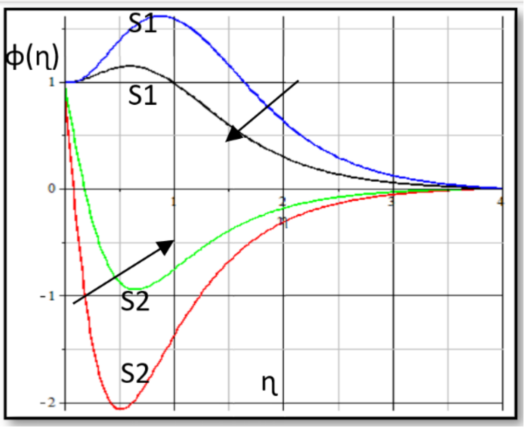

Figure (12) Concentration Profile, η is the Similarity Variable.

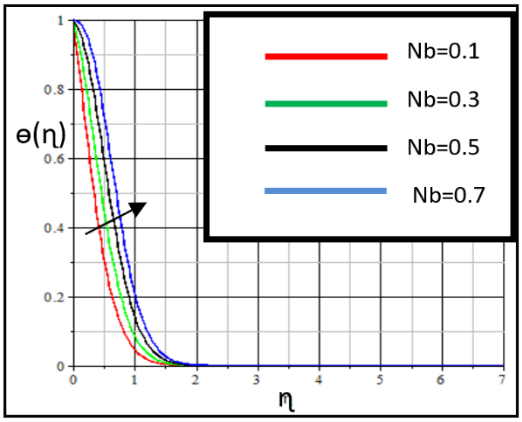

Figure (13) Temperature Profile, η Similarity Variable.

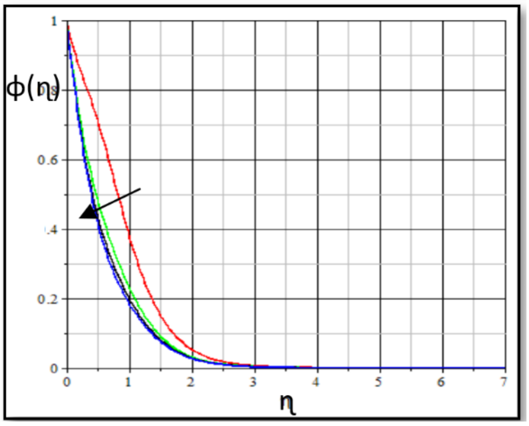

Figure (14) Concentration Profile, η is the Similarity Variable.

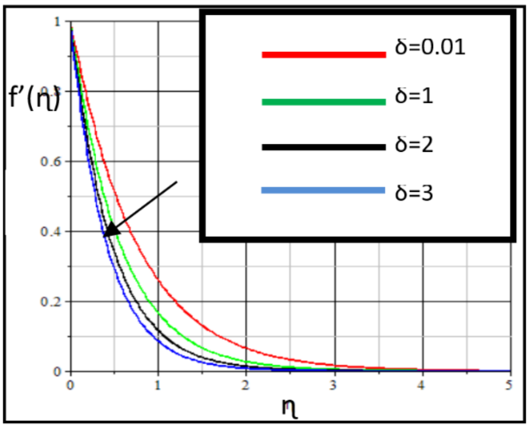

Figure (15) Velocity Profile, η Similarity Variable.

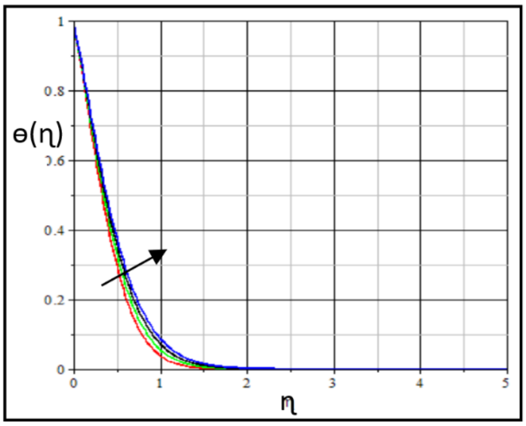

Figure (16) Temperature Profile, η Similarity Variable.

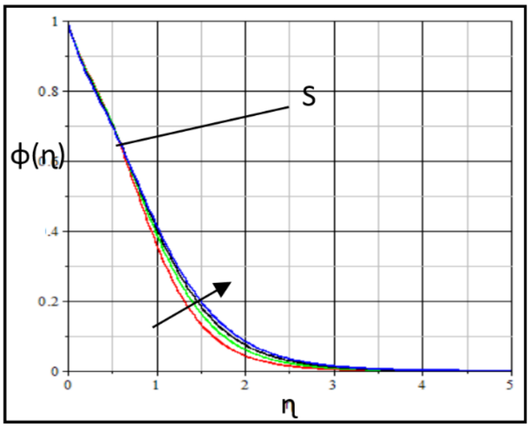

Figure (17) Concentration Profile ϕ(η).

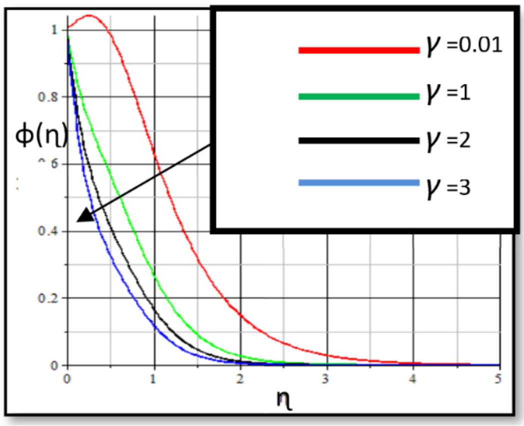

Figure (18) Concentration Profile, η is the Similarity Variable.

Table (7) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2, Values of δ, Pr=7.0, Le=2, λ=Nt=Nb=0.1, n= γ = ß =M=0.5.

δ	f ''(0)	- θ'(0)	-ϕ' (0)
0.01	-1.298415	1.471586	0.909941
1	-1.754182	1.395264	0.918616
2	-2.113930	1.332490	0.935422
3	-2.419079	1.278163	0.955032

Table (8) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2 γ, Pr=7.0, Le=2.0, λ=Nt=Nb= 0.1, M=n=ß= δ=0.5.

γ	f ''(0)	- θ'(0)	-ϕ' (0)
0.01	-1.54165567	1.64614597	-0.209315
1	-1.54165567	1.33040496	1.5794614
2	-1.54165567	1.22392347	2.4675056
3	-1.54165567	1.16462017	3.1034302

Table (9) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2, Le, Pr=7.0, λ=γ=Nt=Nb=0.1, n=δ= ß =M=0.5.

Le	f ''(0)	- θ'(0)	-ϕ' (0)
0.62	-1.5416556	1.9288012	-1.081169
1	-1.5416556	1.8075841	-0.705900
2	-1.5416556	1.5868968	0.0720384
3	-1.5416488	1.4515370	0.6580131

Analysis of Lewis Number Le

The Lewis Number is defined as the ratio of the Schmidt Number and the Prandtl Number (Pr). The Lewis Number is also the ratio of thermal diffusivity and molecular diffusivity. It is found that when increasing Le, a slight increase in temperature and significant decrease in concentration occur as represented in Figure (19), this is since there is a decrease in nanoparticle volume with decrease of Le. It is clear in Table (9) slight decreases in heat transfer as Lewis number increases, and significant increase in mass transfer as Lewis number increases.

Also, the results in Table (12) showed that Lewis number has only effects on concentration profile.

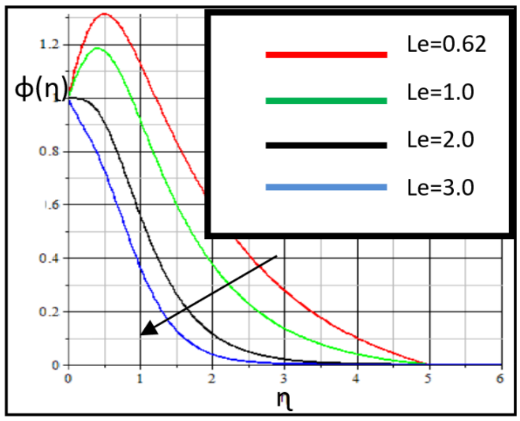

Figure (19) Concentration Profile, η is the Similarity Variable.

Table (10) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2, Pr, Le=2.0, λ=Nt=Nb=γ= 0.1, M=n=ß= δ=0.5.

Pr	f ''(0)	- θ'(0)	-ϕ' (0)
6.2	-1.5416556	1.4818009	0.162432
7	-1.5416556	1.5868968	0.072038
9	-1.5416556	1.8196260	-0.131283
10	-1.5416556	1.9234351	-0.223204

Table (11) f ''(0), θ' (0) and Ø'(0) for some Values of M, Pr=7.0, Le=2, λ, When Le=2, Pr=7,Nt=Nb=0.1, n=δ= ß =M=γ=0.5.

λ	f ''(0)	- θ'(0)	-ϕ' (0)
-0.5	-1.5416556	2.598707	-0.145916
0.1	-1.5416556	1.431385	0.9126285
0.3	-1.5416556	0.835535	1.4365728
0.5	-1.5416556	-0.150634	2.2771462

Analysis of Prandtl Number Pr

For velocity, temperature and concentration profiles subject to increasing Prandtl number this study shows that increasing Pr makes no change in velocity as illustrated in Table (12), but it will be increasing in the temperature because taking larger values of Pr are equivalent to smaller values of thermal conductive, and for concentration it has showed initial increasing near the wall over the interval 0<η<s and then decreasing on the interval s<η<∞, such that s decreases as Pr increases as shown in Figures (20) and (21). Table (10) explains that the rate of skin friction almost fixed the rate of heat transfer increases, the rate of mass transfer decreases as increasing in Pr.

Analysis of Heat Source λ

As expected, the increasing of λ leads to increasing of boundary layer temperature as demonstrated in Figure (22) and decreasing of nanoparticles concentration near the interval 0<η<s. but the concentration increases in interval s<η<∞, such that s increases as λ increases. Table (11) shows the effect of this parameter on the rate of heat transfer, rate of mass transfer, furthermore, the results represented in Table (11) indicate that the increasing of heat source parameter makes decreasing of the rate of heat transfer and increase of the mass concentration as represented in Figure (23).

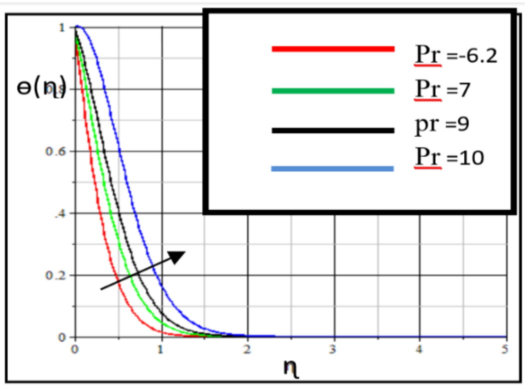

Figure (20) Temperature Profile, η Similarity variable.

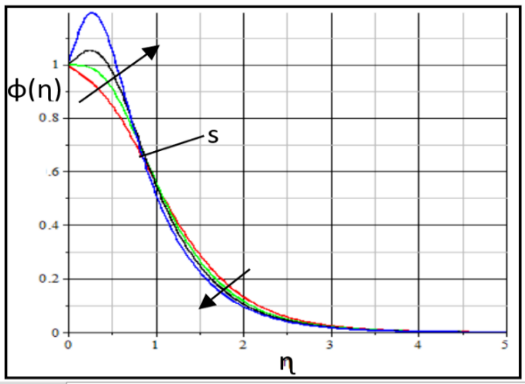

Figure (21) Concentration Profile, η is the Similarity Variable.

Table (12) Classification of the Effect of Parameters on the Velocity, Temperature, and Concentration such that "s" Means Strong "m" Means Middle "w" Means Weak "n" Means it Hasn't Effect.

No	parameter	Velocity				Temperature				Concentration			
		S	M	W	N	S	M	W	N	S	M	W	N
1	Shape		✓				✓			✓			
2	Thickness		✓			✓				✓			
3	Magnetic		✓			✓						✓	
4	Thermophorsis				✓	✓				✓			
5	Brownian motion						✓					✓	
6	Porous Medium	✓						✓				✓	
7	Chemical Reaction				✓			✓		✓			
8	Lewis Number				✓			✓		✓			
9	Prandtl Number				✓		✓				✓		
10	Heat Source				✓	✓					✓		

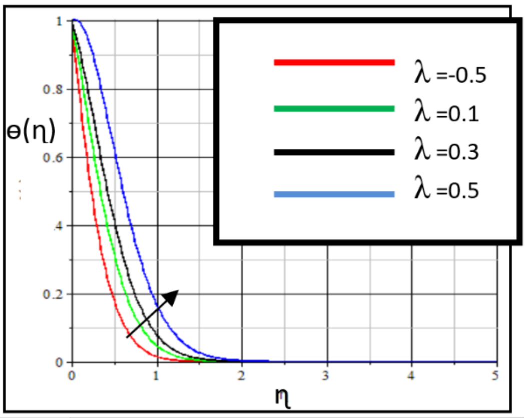

Figure (22) Temperature Profile, η Similarity variable.

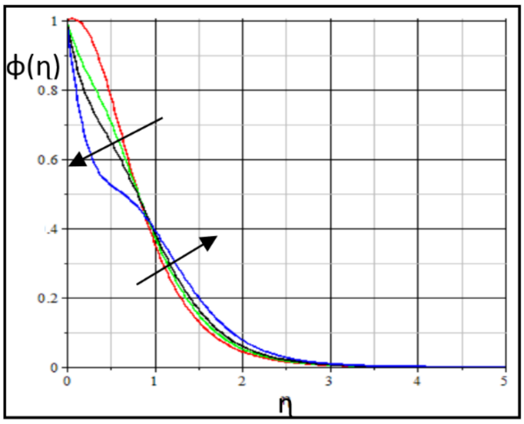

Figure (23) Concentration Profile, η is the Similarity Variable.

In this study we proved that the heat source can effect on the velocity slightly and it also can effect on the temperature strongly while it effects on concentration normally as shown in Table (12).

Conclusion

• The rate of skin friction increases with increase of shape parameter decreases with thickness, magnetic parameters and porous medium parameter the other parameters did not effect on the rate of skin friction.

• The rate of heat transfer increases by increases in thickness, heat source parameters, or by decreasing shape, magnetic field, thermophoresis, Brownian motion, porous medium, chemical reaction parameters, Lewis number and Prandtl number.

• The rate of mass transfer increases as increasing shape, magnetic, Brownian motion, porous medium, chemical reaction, Lewis number, heat source and thermophoresis parameter when Nt>0 or decreasing thickness, Prandtl number and thermophoresis parameters when Nt<0.

• Boundary layer velocity increases with increase of shape, decrease of thickness, magnetic field and porous medium parameters.

• Boundary layer velocity was not affected with heat source, thermophoresis parameter, Brownian motion, Prandtl Number, Lewis number and chemical reaction parameters.

• Temperature of Nanofluid was increasing by increasing Brownian motion, heat source, porous medium parameters, Lewis number or by decreasing thickness parameter and Prandtl number.

• Boundary layer concentration near the surface increases and decreasing away from the surface with increasing in thickness parameter, Prandtl number and thermophoresis parameter when Nt>0.

• Boundary layer concentration near the surface decreases and increasing away from the surface with increasing in shape, magnetic field, porous medium, heat source and thermophoresis parameters when Nt<0.

• Boundary layer concentration increases by decreasing Brownian motion parameter, Lewis number and chemical reaction parameter.

• We classified the parameters into three set strong (s) middle (M) and weak (W) according to its effects on the velocity, temperature and concentration profiles, so we got the following table.

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