Finite Element Analysis of Suppression the Vibration of Dish Using Closed Loop Control System

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ABSTRACT
A closed loop simulation is used for minimizing of undemanding vibration caused in the dish system. The finite element method is used to model the closed loop control via ANSYS- APDL. In this paper the Bezier function for surface modeling to get the best modeling points for dish is used. A designed surface is represented by sufficient control points, using these control points, the surface has been represented depending on Bezier technique to generate reliable and near-optimal dish surface. The required equations are generated to apply the surfaces and curves efficiently using MATLAP program, then exported to ANSYS to perform closed loop vibration analysis. It can be concluded that the closed loop control system with gain (Kp=4, Ki=1, Kd=0.1) suppression the vibration of the dish with 98% with different thickness and materials of dish. Also the natural frequencies and the mode shapes of the dish is evaluated. Three materials (pure copper, pure aluminum and steel) each with different thickness is taken for the dish (0.8, 0.9, 1, 1.1 and 1.2) mm and for each thickness the responses and the natural frequencies are determined for six modes. The effect of the thickness's variation on natural frequencies for each material was studied. It can be observed that natural frequency is direct proportional with the thickness of the dish.
Keywords: Bezier surface, Dish, MATLAB, control points, ANSYS, Natural frequency, response, close loop vibration, mode shape, active vibration.

INTRODUCTION
The actuators on beam are represented by piezo-electric for sensing the oscillation levels, can be indicated as an active vibration of beam. Kim *et al,* (1996)[1] studied the finite element analysis of the aluminum's beam with actuator. Halim and Mheimani (2002)[2] studied the damping's oscillation of the structure using feedback controller . Xianmin *et al.,* (2002)[3] developed a link between 4 bars to study the control of them via the finite element analysis, it was found that the oscillations are suppressed. Dong *et al*.,(2006)[4] developed the ANSYS code to achieve the simulation of closed loop with the controller type LQG. In our studied the dish can be modeled and simulated with ANSYS to examine the dynamic behavior of it, the finite element method using the control of closed loop actions are executed via ANSYS. The dish is generated by Bezier surface technique and closed loop finite element simulation are studied with ANSYS software to reduce vibration amplitudes and three materials (pure copper, pure aluminum and steel) each with different thickness is taken for the dish to observe it's effect, and the natural frequencies are determined for six modes.

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Generation of Dish Surface

The complexity of any designing and made the objects, depending on the representation of the curves and/or surfaces to that objects, the accuracy of the design depended on the curves and/or surfaces kind that made the designing objects. The indication of any curves and/or surfaces depends on the degree of the equations to that curve and/or surface and to the number of the control points which are used to specify the curve and/or surface. The most used surfaces to describe the complex parts are the third degree surfaces because of their high accuracy, but the disadvantage of using this degree of surfaces is its limitation to represent the desired parts [5].In implicit method, a surface is defined as the locus of points whose coordinates (x, y, z) satisfy an equation of the form $f(x, y, z)$ =0 is linear in variables x, y and z, it represents a plane. If it is of the second degree in the variables x, y, z, it represents quadrics. In explicit method, a plane surface can be solved the equation f(x, y, z)=0. For one of the variables as a function of the other two, say z is solved in terms of x and y, *then* $z= F(x, y)$ [6].One solution to the problem of efficiently saving and redrawing a smooth curve was independently developed in the late 1960s by two French automobile engineers, Pierre Bezier, who worked for Renault automobile company and P. de Casteljau, who worked for Citroen. Originally, the solutions were considered industrial secrets, but Bezier's work was eventually published first. The curves that caused using Beziers method are called Bezier curves [7]. The Bézier curve, derived by the Pierre Bézier, is an elementary and beneficial curve. It is a best conduct curve with good specifications. The Bézier surface is a 3-dimension growth of a Bézier's curve. It is made by dragging a Bézier's curve via the domain to make a surface. There are a number of different parametric forms such as Hermit, Bezier and B-splin, other more advanced forms are the rational forms such as: Rational Bezier and NURBS [8]. In this paper, parametric method (Bezier technique) to represent curve and surface were selected for creating dish. [9-10].

Mathematical Representation of Bezier Curve

A Bezier curve is a parametric curve that uses the Bernstein polynomials as basis. Curve of Bezier of rank n is represented by[11-12]:

$$p(t) = \sum_{i=0}^{n} b_i B_{i,n}(t) \quad \dots (1)$$

$0 \leq t \leq 1$; $i = 0,\ldots,n$.

$$B_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i} \quad \dots (2)$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \dots \quad \dots (3)$$

The coefficients, b_i, are the control points or Bezier points together with the basis $B_{i,n}$ (t) determine the shape of the curve. Lines drawn between consecutive control points of the curve form the control polygon. For instance for n = 3 the binomial coefficients are:

$$\binom{3}{0} = 1, \quad \binom{3}{1} = 3, \quad \binom{3}{2} = 3, \quad \binom{3}{3} = 1$$

$b_0 = (x_0, y_0)$, $b_1 = (x_1, y_1)$, $b_2 = (x_2, y_2)$, $b_3 = (x_3, y_3)$,

$$x(t) = 1\times x_o \times t^0(1-t)^3 + 3\times x_1 \times t^1(1-t)^2 + 3\times x_2 \times t^2(1-t)^1 + 1\times x_3 \times t^3(1-t)^0 \quad \ldots (4)$$

$$y(t) = 1\times y_o \times t^0(1-t)^3 + 3\times y_1 \times t^1(1-t)^2 + 3\times y_2 \times t^2(1-t)^1 + 1\times y_3 \times t^3(1-t)^0 \quad \ldots(5)$$

$$P(t) = (x(t), y(t)) \ldots\ldots \quad \ldots(6)$$

For the Bezier curve, when n=3 is rewritten in matrix form as:-

$$\left.\begin{aligned} B_{0,3}(t) &= \frac{3!}{3!*0!}*(1-t)^3 = 1 - 3t + 3t^2 - t^3 \\ B_{1,3}(t) &= \frac{3!}{2!*1!}*t*(1-t)^2 = 3t - 6t^2 + 3t^3 \\ B_{2,3}(t) &= \frac{3!}{1!*2!}*t^2*(1-t) = 3t^2 - 3t^3 \\ B_{3,3}(t) &= \frac{3!}{0!*3!}*t^3 = t^3 \end{aligned}\right\} \quad \ldots(7)$$

$$P(t) = [(1 - 3t + 3t^2 - t^3)\ (3t - 6t^2 + 3t^3)\ \ (3t^2 - 3t^3)\ t^3] \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad .. (8)$$

$$P(t) = [t^3 t^2 t \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \ldots(9)$$

Mathematical Representation of Bezier Surfaces

The Bezier surface is a direct extension of a Bezier curve. The underlying principle in defining a Bezier surface was making a point trace out of Bezier curve and then let this curve sweep out a Bezier surface. The simple extension for three dimensional free–form surfaces from 3-dimensional free-form curve is by incorporating another parameter (s) to the vector equation of the curve to obtain the surface equation:

$$P(t,s) = [x(t,s)\ y(t,s)\ z(t,s)] \quad (10)$$

Where: $0 \le (t,s) \le 1$ and they are independent variables. This equation is called bivaraite parametric equation since it includes two variable parameters in two different directions [9]. The extended of Bezier curve can be represented as:

$$P(t,s) = \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij} B_{i,n}(t) B_{j,m}(s) \quad \ldots (11)$$

where $0 \le (t,s) \le 1$; i,j = 0 ,…., n,m.

The b_{ij} comprises an (n+1) x (m+1) the vertices of the control points saving in a rectangular array. $B_{i,n}(t)$ and $B_{j,m}(s)$ are the functions defining the way of Bezier curves. Hence the matrix equation of a Bezier surface is[14]:

$$P(t,s) = T_{1*n} M_{B,n*n} b_{n*m} M^T_{B,m*m} \ S^T_{B,m*1} \quad \dots (12)$$

where the size of the matrices depend on the dimensions of the control point array. The Bezier patch need not be described by a square array of control points, although the bi-cubic Bezier patch is defined by a (4x4) array of control points. [5].

$$P(t,s) = T_{1*4} M_{B,4*4} b_{4*4} M^T_{B,4*4} \ S^T_{B,4*1} \quad \dots (13)$$

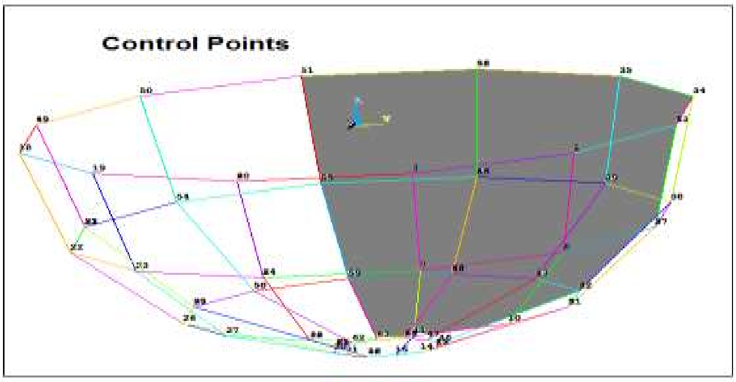
Where: the subscripts on the matrices indicate their dimensions.
Expanding equation (13) to get:

$$P(t,s) = [(1-t)^3 \ 3t(1-t)^2 \ 3t^2(1-t) t^3] * [b] * \begin{bmatrix} (1-s)^3 \\ 3s(1-s)^2 \\ 3s^2(1-s) \\ s^3 \end{bmatrix} \quad \dots (14)$$

$$P(t,s) = [t^3 t^2 t \ 1] * \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} b_{13} b_{14} \\ b_{21} & b_{22} b_{23} b_{24} \\ b_{31} & b_{32} b_{33} b_{34} \\ b_{41} & b_{42} b_{43} b_{44} \end{bmatrix} * \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} s^3 \\ s^2 \\ s \\ 1 \end{bmatrix} \quad \dots (15)$$

Curve and Surface Model in the Simulation Process

The mathematical formulation of **Bezier** techniques have been implemented and integrated with MATLAB program to generate the curves and surfaces depending on initial control points as shown in Fig.(1) [15].



Figure(1) Control points of the dish

The control points of the surface represented the Bezier surface are:

b00 = (0, 100,0),b01 = (50,86.6,0), b02 = (86.6,50,0)
b03 = (100,0,0), b04 = (86.6,-50,0), b05 = (50,-86.6,0), b06 = (0,-100 , 0)
b07 = (-50,-86.6,0), b08 = (-86.6, -50, 0), b09 = (-100,0,0)
b010 = (-86.6, 50 , 0), b011 = (-50, 86.6, 0)
b10 = (0, 89.1, -45.3), b11 = (44.5, 77.1, -45.3), b12 = (77.1,44.5, -45.3)
b13 = (89.1, 0, -45.3), b14 = (77.1 , -44.5, -45.3), b15 =(44.5,-77.1, -45.3)
b16 = (0 ,-89.1, -45.3), b17 = (-44.5, -77.1, -45.3), b18=(-77.1,-44.5, -45.3)
b19 = (-89.1, 0 , -45.3), b110 = (-77.1, 44.5 , -45.3),b111= (-44.5,77.1 , -45.3)
b20 =(0, 58.7 , -80.9), b21 = (29.3, 50.9 , -80.9), b22 = (50.9,29.3 , -80.9)
b23 = (58.7 , 0 , -80.9), b24 = (50.9, -29.3, -80.9), b25 = (29.3,-50.9, -80.9)
b26 = (0, -58.7 , -80.9), b27 = (-29.3, -50.9, -80.9), b28 =(-50.9,-29.3, -80.9)
b29 = (-58.7, 0, -80.9), b210 = (-50.9, 29.3, -80.9), b211 = (-29.3, 50.9, -80.9)
b30 = (0, 15.6, -98.7),b31 = (7.8,13.5 , -98.7), b32 = (13.5,7.8, -98.7)
b33 = (15.6, 0 , -98.7), b34 = (13.5,-7.8,-98.7), b35 = (7.8, -13.5 , -98.7)
b36 = (0, -15.6 , -98.7), b37 = (-7.8 , -13.5 , -98.7), b38 = (-13.5,-7.8 ,-98.7)
b39 = (-15.6, 0 , -98.7) , b310 = (-13.5 ,7.8 , -98.7), b311= (-7.8,13.5 , -98.7)

The proposed algorithm was coded in MATLAB Ver. (6.5) [16]. Several bi-cubic Bezier patches were designed and tested to generate the dish.

Finite Element Analysis of Dish

ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of engineering problems. These problems include: static/dynamic structural analysis (both linear and non-linear), heat transfer and fluid problems, as well as electro-magnetic problems [17-18]. To analyze the dish, the data is imported from MATLAB to generate dish with respect to Bezier surface then Shell181 element is used for meshing the dish. It is a four-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. Fig.(2) shows the element used.

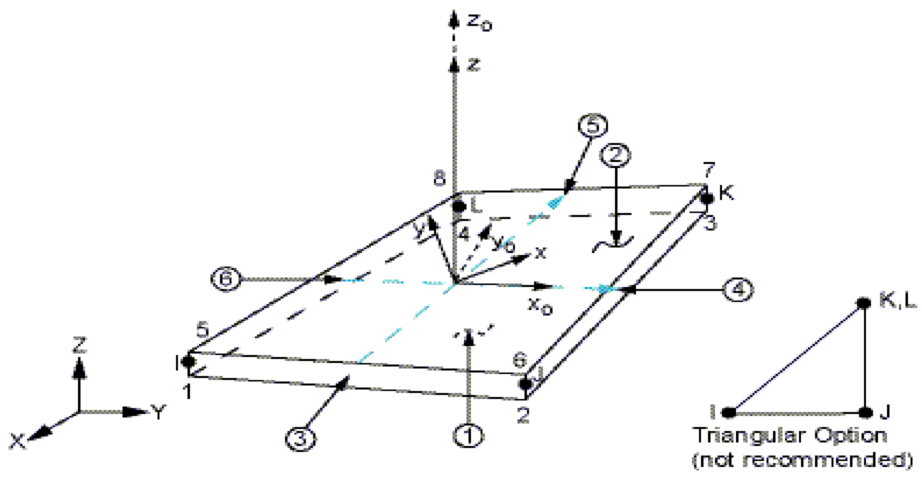
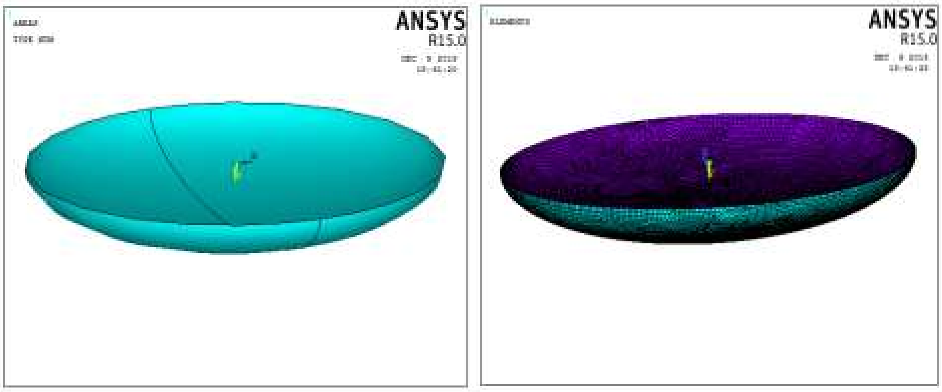


Figure (2) Shell181 Geometry [17]

Fig.(3) shows the dish and its mesh.



Figure(3) Modeling of Dish

Frequencies and Mode Shapes of the Dish

In ANSYS program, the analysis type MODAL is used to determine the natural frequencies and mode shapes. A structure is said to be undergoing free vibration when it is disturbed from its static equilibrium position and then allowed to vibrate without any external dynamic excitation or support motion. The first step in the dynamic analysis of any structural system is to determine the free vibration response such as natural frequencies and mode shapes, which are important in calculating the response of the dish. The undamped free vibration response of any system can be obtained by analyzing the governing equation of motion for the undamped system [19]. This can be obtained from Eq.(16):

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{0\} \quad \ldots(16)$$

Where: [M] and [K] are the mass and stiffness matrices of the dish, respectively; and $\{\delta\}$ is displacement vector. Therefore, the natural frequencies (ω) and the mode shapes {Ø} of any system governed by Eq.(16) are solutions of the Eigen value problem represented by:

$$\left[[K] - \omega^2[M]\right]\{\emptyset\} = \{0\} \quad \ldots(17)$$

For non trivial solution of Eq. (17),

$$\left[[K] - \omega^2[M]\right] = \{0\} \quad \ldots(18)$$

Eq.(18) is called the frequency equation of the system. Expanding the determinant will give an algebraic equation on the m^{th} degree in the frequency parameter ω^2 for a system having m degrees of freedom. The m roots of this equation ω_1^2 , ω_2^2,ω_3^2,……., ω_m^2) represent the frequencies of m modes of vibration which are possible in the system [20].The dynamic response of any structural idealization procedure is validated first, by computing the natural frequencies and the corresponding mode shapes of vibration for general structures and by comparing the resulting response with the available experimental or analytical work. In this section, the proposed idealization scheme is validated by analyzing the free vibrational response of certain dish that has already been considered in this study.

Three materials (pure copper, pure aluminum and steel) each with different thickness (0.8, 0.9, 1, 1.1, and 1.2) mm is taken for the dish, the natural frequencies are determined for six modes as illustrated in tables(1),(2) and (3). The effect of the thickness's variation on natural frequencies for each material was studied. It can be observed that natural frequency is direct proportional with the thickness of the dish.

Table(1) : Natural frequencies for pure copper's dish

Thickness (mm)	Natural frequency (Hz)					
	Mode1	Mode2	Mode3	Mode4	Mode5	Mode6
0.8	129.708	133.67	194.751	254.92	351.621	351.798
0.9	142.551	146.695	201.919	265.36	385.277	385.489
1	155.159	159.48	209.032	275.525	417.84	418.091
1.1	167.546	172.035	216.097	285.46	449.424	449.72
1.2	179.723	184.371	223.12	295.197	480.126	480.471

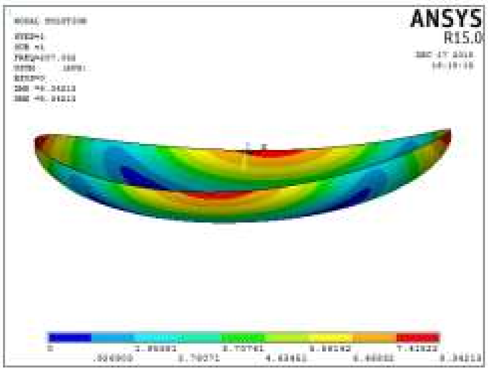
Table(2) : Natural frequencies for steel's dish

Thickness (mm)	Natural frequency (Hz)					
	Mode1	Mode2	Mode3	Mode4	Mode5	Mode6
0.8	176.338	181.557	264.306	344.635	478.494	478.735
0.9	193.876	199.333	273.785	358.519	524.546	524.832
1	211.105	216.793	283.195	372.041	569.121	569.464
1.1	228.041	233.95	292.546	385.258	612.379	612.786
1.2	244.699	250.817	301.846	398.215	654.448	654.921

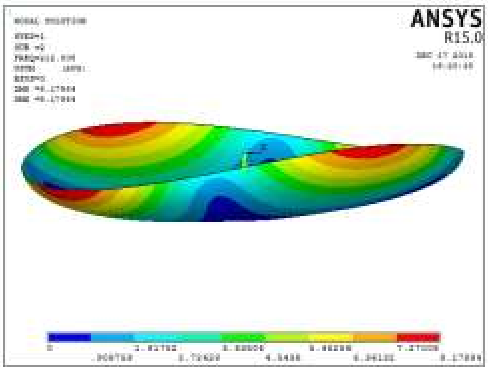
Table(3) : Natural frequencies for aluminum's dish

Thickness (mm)	Natural frequency (Hz)					
	Mode1	Mode2	Mode3	Mode4	Mode5	Mode6
0.8	173.077	178.371	259.885	340.229	469.17	469.406
0.9	190.211	195.748	269.461	354.173	514.07	514.35
1	207.032	212.805	278.963	367.749	557.505	557.839
1.1	223.557	229.554	288.4	381.017	599.635	600.031
1.2	239.801	246.011	297.782	394.021	640.59	641.049

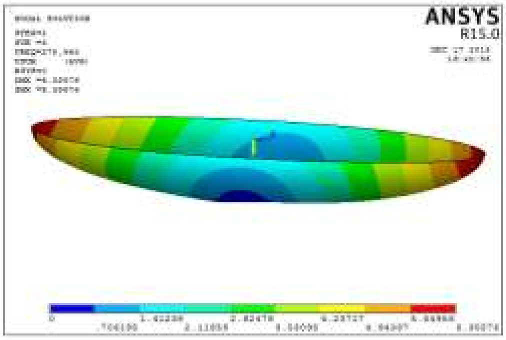
Six mode Shapes of the dish are investigated as shown in Fig.(4).

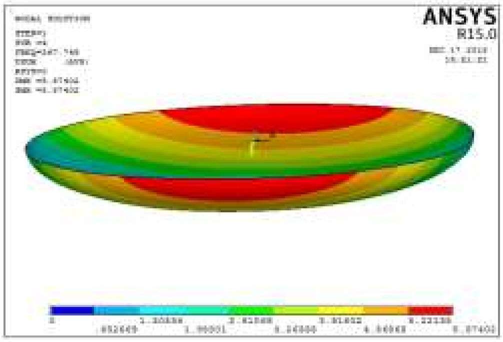


Mode-1-

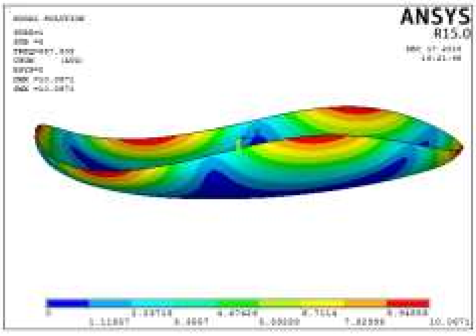


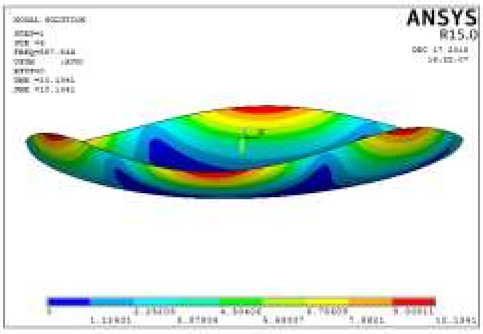
Mode-2-





Mode-3- Mode-4-



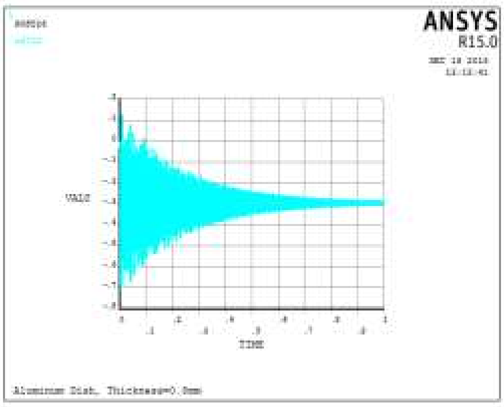


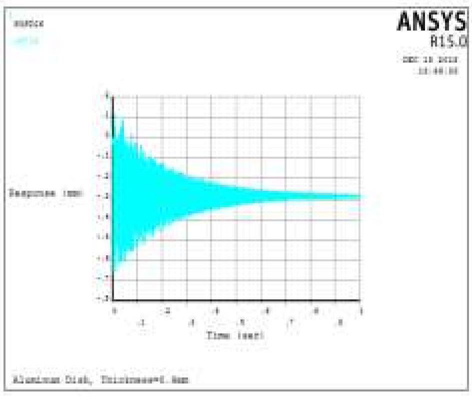
Mode-5- Mode-6-

Figure (4) Mode shapes of Dish

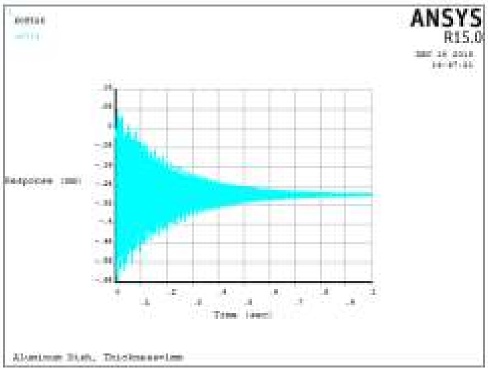
Response of Dish

The response of the dish is predicted using finite element method via ANSYS program by Transient Analysis at maximum natural frequency [21]. The detailed dimensions of the dish are (R=100mm, Thickness =0.8,0.9,1,1.1 and 1.2 mm) with the pure copper , pure aluminum and steel materials. Figs. (5), (6) and (7) illustrated the response for aluminum, steel and copper material with different thickness of dish.

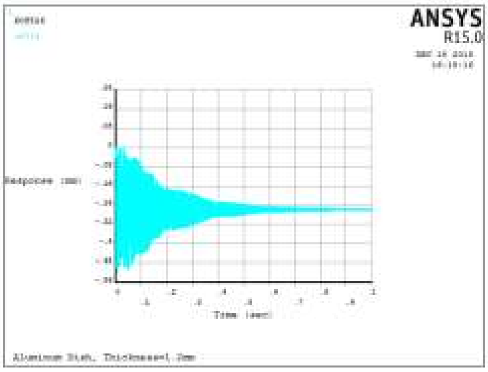




Thickness=0.8mm Thickness=0.9mm

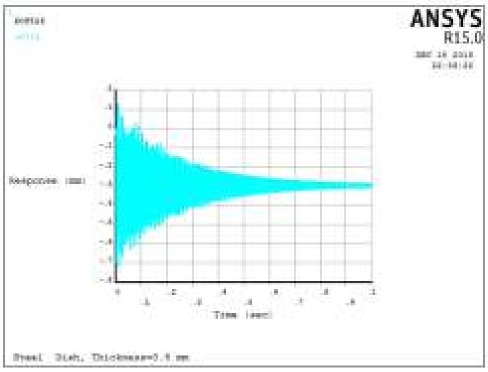


Thickness=1mm

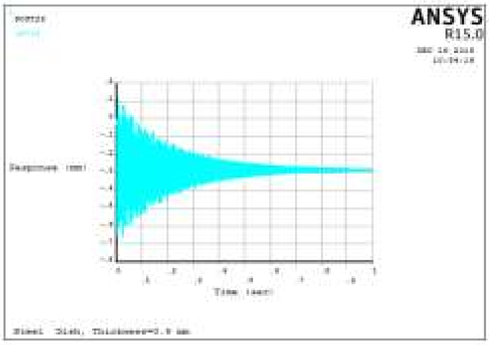


Thickness=1.2mm

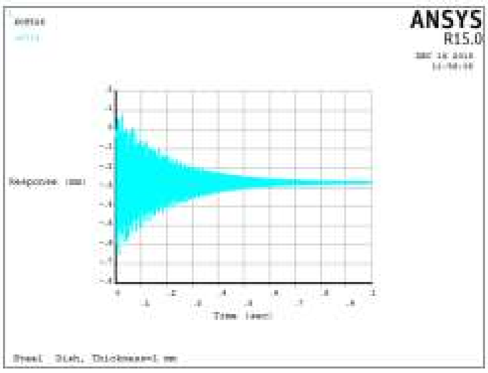
Figure(5) Response of the Aluminum's Dish



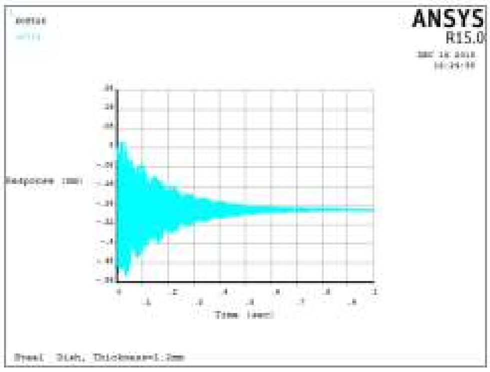
Thickness=0.8mm



Thickness=0.9mm

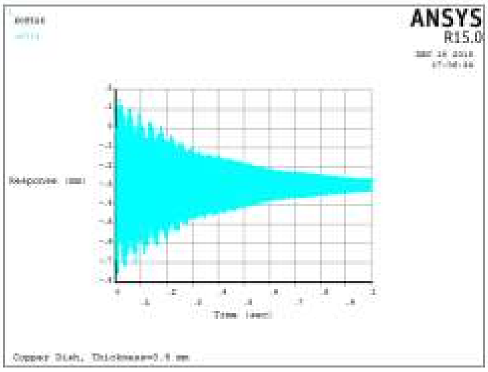


Thickness=1mm

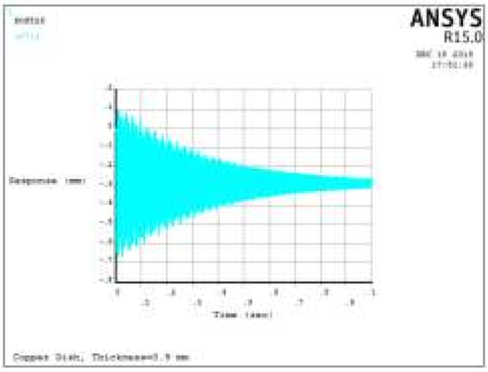


Thickness=1.2mm

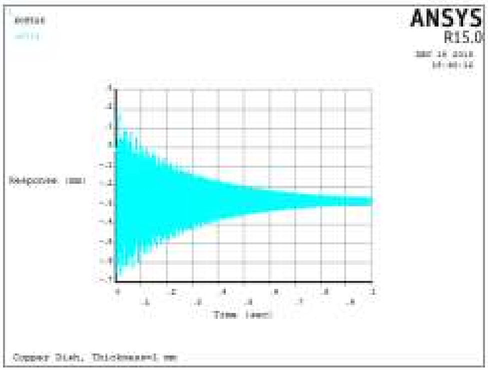
Figure (6) Response of the Steel's Dish



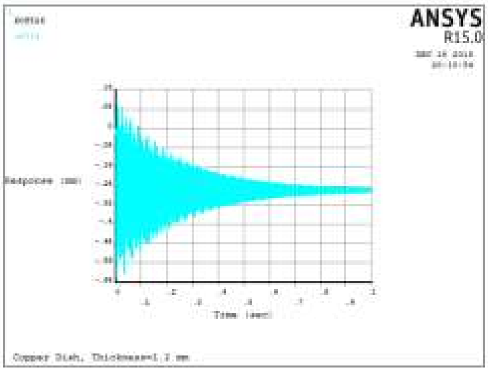
Thickness=0.8mm



Thickness=0.9mm



Thickness=1mm

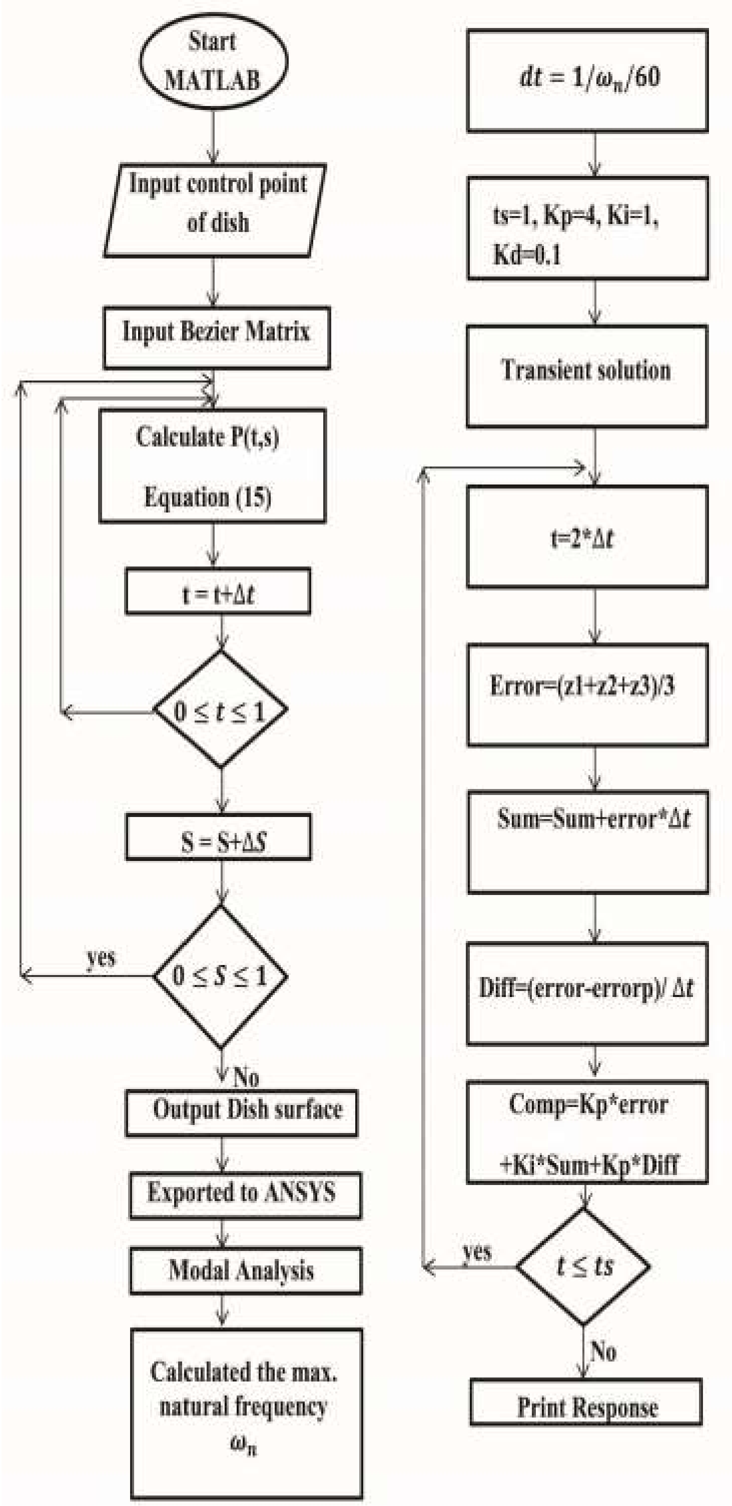


Thickness=1.2mm

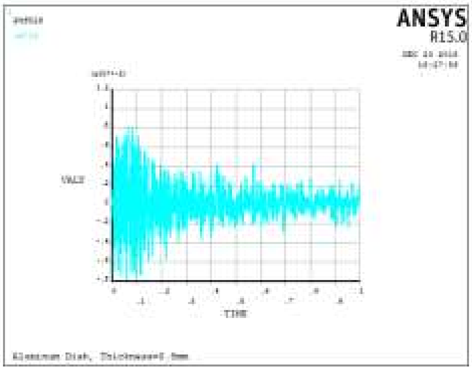
Figure (7) Response of the Copper's Dish

Active Control of Free Vibration

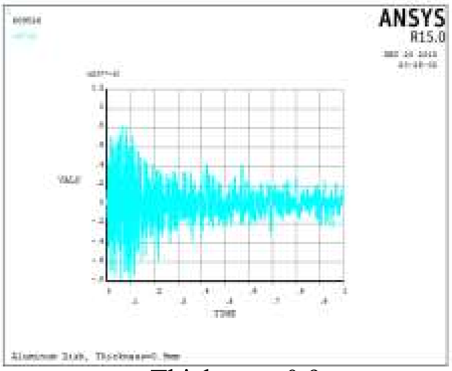
ANSYS is used to achieve the control of the closed loop [22]. To illustrate the effect of the active control, damping effect is neglected. The tip dish's displacement is determined as a criterion of PID gain. Natural frequencies and mode shapes of the dish are found using Modal Analysis in order to specify the dt (time increment). Fig.(8) shows the flowchart of the closed loop control for the dish. Hence the time increment dt is a very important factor in the analysis and it is taken dt=1/60.ω, where ω is the maximum natural frequency, which had get from the Modal analysis and its values are illustrated in Table(1), (2) and (3). Active control is executed with Do loop and ended when it's reached to steady state. The unit load input is used for node 235, after the solution is initiation, the error is determined for the next stage of the loop. The controlled response obtained by closed loop are shown in Figs.(9),(10) and (11). Tables (4),(5) and (6) illustrated the maximum response for the dish, and from that it can be deduced that the closed loop control system with gain (Kp=4, Ki=1, Kd=0.1) suppression the vibration of the dish with 98%.



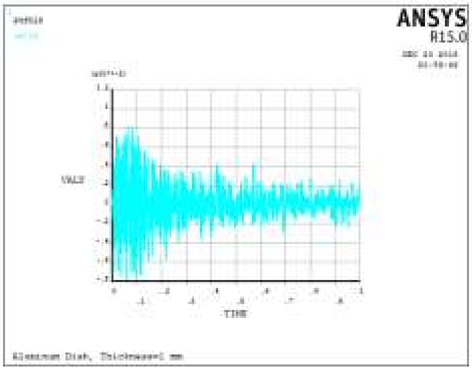
Figure(8) Flowchart for closed loop control analysis of dish



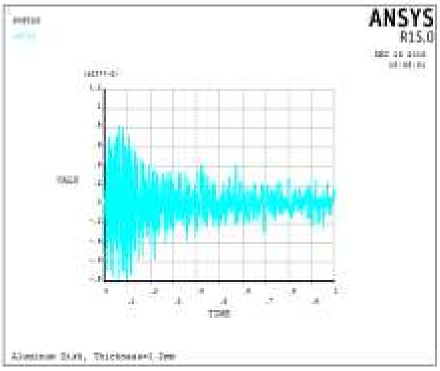
Thickness=0.8mm



Thickness=0.9mm

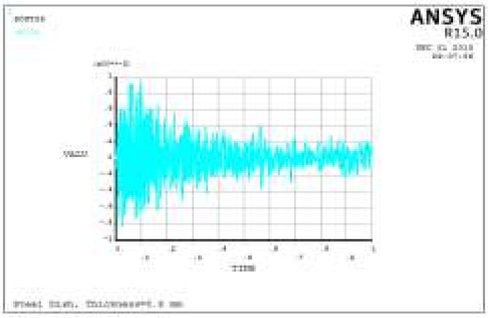


Thickness=1mm

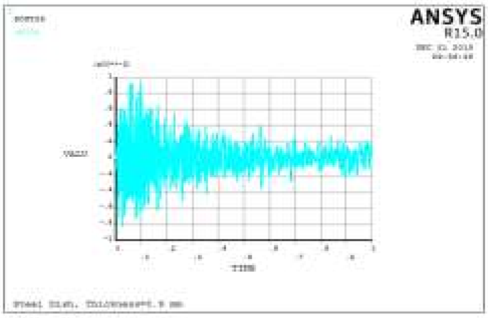


Thickness=1.2mm

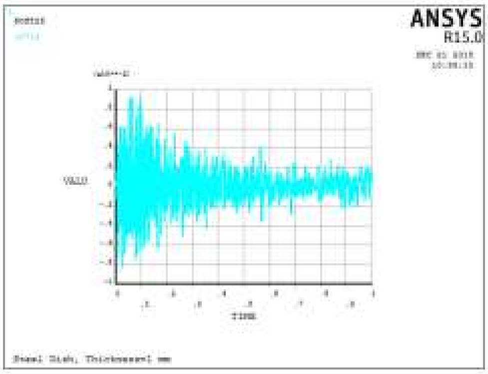
Figure(9) Response of the Aluminum's Dish with close loop control



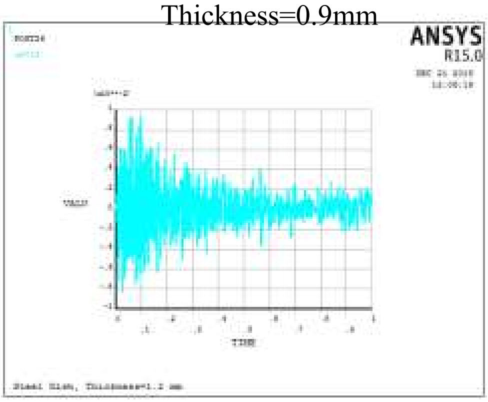
Thickness=0.8mm



Thickness=0.9mm

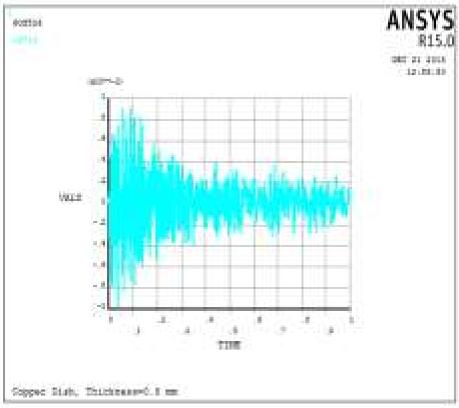


Thickness=1mm

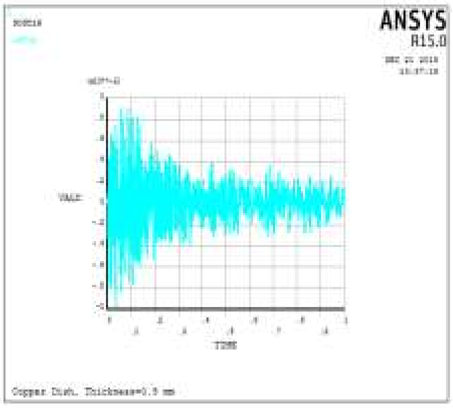


Thickness=1.2mm

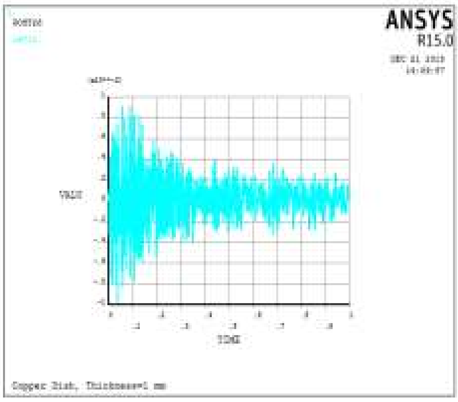
Figure(10) Response of the Steel's Dish with close loop control



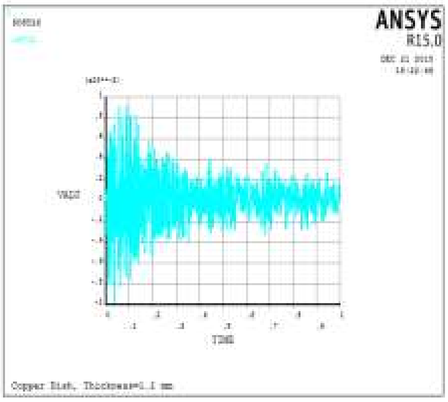
Thickness=0.8mm



Thickness=0.9mm



Thickness=1mm



Thickness=1.2mm

Figure (11) Response of the Copper's Dish with close loop control

Table (4): The amplitude of the Aluminum dish's response with and without closed loop control

thickness (mm)	closed-Loop (max. amplitude)(mm)	max. amplitude (mm)
0.8	0.0082	0.7
0.9	0.0082	0.66
1	0.0082	0.64
1.1	0.0082	0.64
1.2	0.0082	0.52

Table (5): The amplitude of the Copper dish's response with and without closed loop control

thickness (mm)	closed-Loop (max. amplitude)(mm)	max. amplitude (mm)
0.8	0.009	0.75
0.9	0.009	0.68
1	0.009	0.69
1.1	0.009	0.69
1.2	0.009	0.63

Table (6): The amplitude of the Steel dish's response with and without closed loop control

thickness (mm)	closed-Loop (max. amplitude)(mm)	max. amplitude (mm)
0.8	0.0092	0.71
0.9	0.0092	0.67
1	0.0092	0.65
1.1	0.0092	0.65
1.2	0.0092	0.54

CONCLUSIONS

Vibration causes in many fields such as machines, equipment..etc, and which may be damaged the system. It had been concluded that the best method to reduce the vibration via closed loop control system as illustrated in the results, which the optimum reduction of vibration is got at different thickness and materials for dish. The control point's generation for a dish is developed. Curved three dimensional surfaces such as the dish are particularly difficult to model because as with most things found in nature of dish has unique and complex features. Applying Bezier surface patch controls points for an existing set of collected three dimensional data points is introduced to describe such features. The proposed algorithm for dish generation was developed and implemented successfully through the integration of mathematical modeling surface depending on Bezier form. The MATLAB is used to obtain the mathematical model, and then exported to ANSYS to perform closed loop vibration control analysis. The suppression of vibration is achieved by displacement feedback with 98% using gain (Kp=4, Ki=1, Kd=0.1) with different thickness and materials of dish. The natural frequencies and the mode shapes of the dish is evaluated. Three materials (pure copper, pure aluminum and steel) each with different thickness is taken for the dish (0.8, 0.9, 1, 1.1, and 1.2) mm and for each thickness, the responses and the natural frequencies are determined for six modes. The effect of the thickness's variation on natural frequencies for each material was studied. It can be observed that natural frequency is direct proportional with the thickness of the dish.

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