An Experimental Study on the Shear Strength of High-performance Reinforced Concrete Deep Beams without Stirrups

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ABSTRACT

High-performance fiber-reinforced concrete is a new class of concrete that has been developed in recent decades. It exhibits enhanced properties such as high compressive strength and improved tensile strength. Three types of concrete with different compressive strengths, namely, normal-strength concrete, high-strength concrete, and high-performance concrete, were used in this study. The experimental program included casting and testing sixteen reinforced concrete deep beams without stirrups to study the shear strength and behavior of these beams under two-point loading. The variables considered were the compressive strength of concrete (f_c) (40–120 MPa), shear span-to-depth ratio (1, 1.5, 2, 2.5, and 3), and the ratio of the amount of flexural steel bar ratio (1.35%, 2.40%, 3.76%, and 6.108%). Experimental results showed that increasing concrete compressive strength and flexural steel bar ratio increased ultimate shear capacity. By contrast, increasing shear span-to-depth ratio and span-to-depth ratio reduced ultimate shear capacity. Based on the test results of this investigation (16 beams) and those of available literature (233 deep beams), an equation that considered the parameters affecting shear stress (f_c , l/d, a/d, and ρ_w) was proposed using SPSS software. The proposed equation was compared with predictions made by the American Concrete Institute (ACI) and the works of other researchers, including that of Zsutty and Aziz. The ACI predictions were conservative and the proposed equation had a lower coefficient of variation.

Keywords: shear strength, high-performance fiber-reinforced concrete, deep beams.

INTRODUCTION

Reinforced concrete deep beams are used as load-distributing structural elements such as transfer girders, pile caps, foundation walls, and offshore structures. The shear strength evaluation of reinforced concrete beams has been the subject of several studies to determine the influences of major parameters. Given the small value of the span-depth ratio of a deep beam, its strength is typically controlled by shear strength rather than by flexural strength if the normal amount of longitudinal reinforcement is applied [1, 2]. Reinforced concrete deep beams are used as common structural elements in many structures, from tall buildings to offshore gravity structures. They are used as panel beams and, more recently, as deep grid walls in offshore gravity-type concrete structures. The term *deep beam* is applied to any beam with a depth-to-span ratio that is sufficiently large to cause nonlinearity in the elastic flexural stresses over the beam depth as well as to make the distribution of shear stress non-parabolic [3]. The combination of stresses (bending and shear) in the shear span results in inclined cracks, which transform the beam into a tied arch.

According to American Concrete Institute [4], deep beams are defined as members loaded on one face and supported on the opposite face, and thus, compression struts can develop between loads and supports. Moreover, deep beams exhibit either of the following:

- a) $l_n/h \le 4.0$ (for the distributed load case) or
- b) $a/d \le 2.0$ (for the point load case)

 l_n : beam clear span ; h : beam height ; a : shear span ; d : effective depth

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Several experimental studies have been conducted to understand various failure modes that can occur because of the possible combination of shear and bending moments acting on a given section. The main challenge in the shear problem is the considerable number of parameters involved.

In general reinforced concrete deep beams should have adequate shear reinforcement to prevent sudden and brittle failure after formation of the diagonal cracks, and also to keep crack width at an acceptable level. However, there are no established quantitative criteria for reserve strength required beyond cracking strength and limits for the crack width. The minimum shear reinforcement is also required to provide somewhat ductile behavior prior to failure [5, 6]. To estimate the shear resistance of deep beams, standard codes and researches all over the world have been specified different formulae considering different parameters into consideration. Leading to disagreement between researchers, making it difficult to choose an appropriate model or code for predicting shear resistance of a beam subjected to symmetrically placed two equal concentrated loads 'P' at distance 'a' (shear span) from the supports. It has the advantage of combining two different test conditions, viz, pure bending, that is, no shear force is present between the two loads P, and constant shear force in the two end regions or shear spans [8].

American concrete institute concrete terminology [9],defines high-performance concrete (HPC) as "a concrete meeting special combinations of performance and uniformity requirements that cannot always be achieved routinely using conventional constituent and normal mixing, placing, and curing practices". UHPC consists of fine sand, cement, silica fume and quartz flour in a dense, low-water cementitious materials ratio (0.15 to 0.19) mix. Compressive strengths of (124- 206) MPa can be achieved depending on the mixing and curing process, and it has tensile strengths of 6.3 to 11.9 MPa. The material has low permeability and high durability, to improve ductility; steel fibers are added [10].

Objectives

The main objectives of this investigation are as follows:

1. To investigate the effects of the compressive strength of concrete (f_c) , span-to-depth ratio (l/d), shear span-to-depth ratio (a/d), flexural reinforcement of shear strength (ρ_w) , and the behavior of HPC deep beams without web reinforcement;

2. To propose an equation for predicting shear stress based on data from this investigation (16 beams) and from existing literature (233 deep beams); and

3. To compare the applicability of the proposed equation and existing methods (drawn from the ACI code and other studies) for predicting the ultimate shear stress (v_u) of normal, high-strength, and high-performance reinforced fiber concrete deep beams without shear reinforcement.

Experimental Program

Three types of concrete mixes, namely, normal-strength concrete (NSC, f_c of approximately 40 MPa), high-strength concrete (HSC, f_c of approximately 60 MPa and 80 MPa), and HPC (f_c of over 100 MPa), were used. Several trial mixes were prepared for each type before the beams were cast. Nine samples (100 mm × 100 mm × 100 mm) were cast for each beam or group. Three samples were tested at the age of 7 days, three at age of 28 days, and three at the age of 90 days.

The specifications of the materials used in the experiments are as follows:

- Cement: The cement used for this study was ordinary Portland cement (CEM-I 42.5R).

- Silica Fume: The silica fume used had a diameter that was sufficiently small to fill in the interstitial voids between cement particles.

- Fine Aggregate (Sand): The sand was thoroughly washed, air-dried, and separated according to the standard set of sieves. Afterwards, the sieved aggregate was remixed into two different

grades to conform to the standard limitations of ASTM C33[13]. The grade of sand used for HPC was different from those used for HSC and NSC.

- **Coarse Aggregates (Gravel):** Two types of coarse aggregates were used for the investigation. Natural river gravel with a maximum grain size of 12.5 mm was used for HSC and NSC, whereas crushed granite rocks with a maximum grain size of 9.5 mm were used for HPC.

- Super plasticizer: A high-dosage super plasticizer was used to obtain workable HPC mixes with extremely low water-to-cement ratios (w/c).

- Water: For all types of concrete, tap water was used for mixing, curing, and washing. The water was clean and free from contaminants.

- Steel Fibers: The steel fibers used were straight steel wire fibers (undeformed). The fibers had an aspect ratio (l/d) of 80, a nominal diameter of 0.2 mm, and a length of 40 mm.

- **Reinforcement Steel:** Deformed steel bars were used as longitudinal reinforcement. The steel bars had diameters of 25, 20, 16, and 12 mm and a yield strength of approximately 416 MPa to satisfy the specific longitudinal reinforcement ratio (ρ_w).

Sixteen reinforced concrete deep beams were tested under two symmetrically placed concentrated loads; the overall cross section was 100 mm x 200 mm. All the tested specimens were simply supported. The tested beams were divided into four groups. The properties and details of the tested specimens are provided in Fig. 1 and Table 1. All the specimens were designed to fail under shear and with the parameters listed in Table 1.



Figure (1). Details of tested specimens

Tuble (1) Details of tested beams							
Beam des	signation	<i>l</i> (mm)	a(mm)	d	a/d	ρ _w %	Concrete type
Series1	G11	1000	334	167	2.00	6.108	NSC
	G12	1000	334	167	2.00	6.108	HSC
	G13	1000	334	167	2.00	6.108	HSC
	G14	1000	334	167	2.00	6.108	HPC
	G15	1000	334	167	2.00	6.108	HPC
Series2	G21	1000	167	167	1.00	6.108	HPC
	G22	1000	250	167	1.50	6.108	HPC
	G23	1000	418	167	2.50	6.108	HPC
	G24	1000	500	167	3.00	6.108	HPC
Series3	G31	668	334	167	2.00	6.108	HPC
	G32	835	334	167	2.00	6.108	HPC
	G33	1169	334	167	2.00	6.108	HPC
	G34	1326	334	167	2.00	6.108	HPC
Series4	G41	1000	334	167	2.00	1.35	HPC
	G42	1000	334	167	2.00	2.4	HPC
	G43	1000	334	167	2.00	3.76	HPC

Table (1) Details of tested beams

Use 2-25mm as main reinforcement. b= 100mm, d= 167mm for series 1,2,3. Use 2-12mm, 2-16, 2-20 as main reinforcement. b= 100mm, d= 167mm for series 4. **Fabrication**

A rotary mixer with a capacity of 0.80 m³ was used. The fine and coarse aggregates were initially poured into the mixer, followed by 25% of the mixing water (water and admixture) to wet the aggregates. Afterwards, the cement was added, and the materials were mixed until a uniform color was obtained. The remaining water was then gradually added to the mix. Lastly, the steel fibers were introduced, and mixing was continued until a homogenous concrete was obtained.

Testing

The specimens were simply supported and tested under two symmetrical point loads. Loads and reactions were applied through rollers and bearing plates to allow free rotation and horizontal movement at the end supports. Deflections were measured at the center of the span using a dial gauge with 0.01 mm accuracy and 30 mm maximum movement.

Incremental stage loading was applied to realize continuous monitoring of the performance of each beam. At each load stage, deflection was recorded, and cracks and their extensions were searched. The first cracking load was recorded, and loading was continued until failure occurred. The failure load was recorded, and finally, photographs were taken to show the crack patterns.

Results and Discussion

The test results of the sixteen HPC deep beams and their crack patterns were included to study the effects of concrete compressive strength, shear span-to-depth ratio, span-to-depth ratio, and the amount of flexural reinforcement on the ultimate shear stress. The results of the tested beams are provided in Table 2.

Group No.	Name of Beam	Concrete Compressive strength, f c MPa	First Flexural Cracking Load (kN) (Experimental)	First Shear Cracking Load (kN) (Experimental)	Failure Load (kN) (Experimental)	Shear Stress MPa	Mode of Failure
	G11	43	43	51	130	3.89	Diagonal Tension
G1	G12	61	51	90	201	6.02	Diagonal Tension
	G13	79	52	104	264	7.90	Diagonal Tension
	G14	100	53	90	311	9.31	Diagonal Tension
	G15	119	55	97	345	10.33	Diagonal Tension
	G21	97	117	160	716	21.44	Diagonal Tension
G2	G22	101	88	120	520	15.57	Diagonal Tension
	G23	101	44	44	241	7.22	Diagonal Tension
	G24	101	48	48	145	4.34	Diagonal Tension
	G31	101	47	83	380	11.38	Diagonal Tension
G3	G32	101	50	70	340	10.18	Diagonal Tension
	G33	101	48	44	284	8.50	Diagonal Tension
	G34	122	48	50	273	8.17	Diagonal Tension
G4	G41	122	49	73	170	5.09	Diagonal Tension
	G42	122	49	74	225	6.74	Diagonal Tension
	G43	118	60	80	257	7.69	Diagonal Tension

Table ((2)	Results	of	the	tested	beams
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Crack Patterns and Modes of Failure

The cracks in the concrete beams generally formed in the regions where tensile stresses existed and exceeded the specified strength. Two types of cracks were observed in the tested beams. Flexural cracks resulted from the flexural tensile stresses in the region of the cross section of the beam below the neutral axis for positive bending, whereas shear cracks formed as a result of the inclined or principal tensile stresses acting on the web of the beam in the combined bending and shear region. The typical crack patterns in the tested beams are shown in Fig. 2. The beams failed under shear tension according to the following sequence.

1- Vertical shear-flexural cracks formed at the shear span.

2- Crack propagation continued in a curved path toward the point load and approached the compression zone.

3- As load increased, the cracks extended in two directions: toward the compression zone and along a horizontal path at the reinforcement level toward the supports.

4- Crack propagation continued until the crack reached the point load region, after which the beam carried further loads with minimal cracking. Finally, the crack extended either in the compression zone toward the pure moment region and beyond the point load or in the tension zone toward the supports, which caused failure.



Figure (2). Crack patterns in the tested beams

Load-Deflection Relationships and the Effects of Parameters

During the testing process, load was applied in increments, and the behavior of the beams was observed. The first flexural cracking load, first shear cracking load, failure load, and failure mode were recorded and listed in Table 2. The load–deflection relationships for all groups are shown in Figs. (3) to (6). The ductility of the beams increased by increasing f'_c from 40 MPa to 120 MPa (beams in group 1); a/d from 1 to 3, which increased the moment over the span (beams in group 2); and ρ_w from 1.35% to 6.108% (beams in group 4). By contrast, increasing l/d from 4 to 8 decreased the ductility of the beams, which decreased the moment over the span (beams in group 3).

In general, the curves can be divided into two stages. The first stage is the elastic (linear) stage, in which no flexural or shear crack appear. After increasing the load, the beam proceeds to the second stage, which is the inelastic (nonlinear) stage, in which flexural cracks are

noticeable. In these relationships, the slope of the curve at the first stage is steeper than that at the second stage. The division between these two stages is more pronounced at higher a/d. An increase in the compressive strength of the concrete from NSC to HSC, then to HPC, leads to a decrease in the central deflection of beams. For a specific applied load at the second stage, the deflection of the HSC beams is less than that of the NSC beams, and the deflection of the HPC beams is less than that of the HSC beams.



Figure (3). Load-deflection relationship for beams in group 1



Figure(4). Load–deflection relationship for beams in group 2



Figure (5). Load-deflection relationship for beams in group 3



Figure(6). Load-deflection relationship for beams in group 4

The results in Table 2 showed that concrete compressive strength had no obvious effect on the formation of the first flexural cracks when the strength increased from 43 MPa to 119 MPa. However, the first shear cracking load increased by 76% and 8% when strength was increased from 43 MPa to 61 MPa and from 61 MPa to 119 MPa, respectively. The ultimate (failure) load for the tested beams was increased by increasing compressive strength. In general, the results showed that the failure load increased by 55% and 72% when the compressive strength was increased from 43 MPa to 61 MPa and from 61 MPa to 119 MPa, respectively. The effect of concrete compressive strength on the ultimate shear stress of the deep beams is shown in Fig. 7. When the shear span-to-depth ratio of the beams increased from 1 to 3, a significant decrease in ultimate shear stress was noticeable (from 21.44 MPa to 4.34 MPa), as shown in Fig. 8.

The failure load decreased by about 79.7% when (a/d) ratio increased from 1 to 3.0; increasing of (a/d) ratios leads to a significant decreasing in ultimate load, in the previous researches which studying the effect of (a/d) ratios on the behavior of deep beams showed that the decreasing in the failure load is limited beyond values of 2 or $2.5^{[6]}$.



Figure. 7. Ultimate shear stress versus concrete compressive strength



Figure. 8. Ultimate shear stress versus concrete shear span-to-depth ratio

Meanwhile, Fig. 9 shows the relation between ultimate shear stress and span-to-depth ratio. Increasing span-to-depth ratio from 4 to 8 decreased ultimate shear stress from 11.38 MPa to 8.17 MPa. The effect of the amount of steel reinforcement is shown in Fig.10. When the amount of steel reinforcement ratio increased from 1.35% to 6.108%, the ultimate shear stress improved from 5.72 MPa to 9.32 MPa.



Figure. 9. Ultimate shear stress versus concrete span-to-depth ratio

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Figure. 10. Ultimate shear stress versus concrete reinforcement ratio

Cracking and Ultimate Shear Stresses

Cracking shear strength or diagonal cracking strength is defined in this study as the shear strength at which an inclined crack was formed within the shear span traversing the centroidal axis of the beam. As shown in Table 2, shear stresses increased nearly linearly with an increase in concrete compressive strength from NSC to HSC; however, increasing concrete compressive strength from HSC to HPC had a negligible effect.

Proposed Equation

In addition to the results of the present work (16 beams), the experimental results of 233 reinforced concrete deep beams without shear reinforcement (from literature) that failed under shear tension were also included in the analysis[14-30]. Nonlinear multiple stepwise regression was adopted to relate ultimate shear stress to the influencing parameters. The general models can be written as follows:

$$v_{u} = 0.85 v_{c} \qquad \dots (1)$$

$$v_{c} = k_{o} \left[((X_{1})^{k1}/(X_{2})^{k2}) + (X_{3})^{k3}*(X_{4})^{k4} \right]$$

where

 v_c : Predicted shear stress, N/mm².

 $X_1 = (\rho_w \cdot d_b \cdot d_{agg}), X_2 = (l/d \cdot a/d), X_3 = f_c, X_4 = 1/(d/b).$

 $k_{0,}k_{1,}k_{2,}k_{3,}k_{4}$: constants; ρ_{w} : web reinforcement ratio

d_b:web bar diameter; d_{agg}: max. aggregate size

Constants were calculated using SPSS software and the proposed equation could be written as follows:

 $v_{c} = 0.165[(\rho_{w.} d_{b} \cdot d_{agg})^{0.209}/(a/d \cdot l/d)^{0.577}) + f_{c}^{0.862} * (1/(d/b))^{0.392}] \qquad \dots (2)$

Evaluation of the Experimental Results Shear Design Equations

Many design equations were proposed to predict the ultimate shear stress for reinforced concrete deep beams. The ACI code method [4], the Zsutty method [11], the Aziz method [12], and the proposed equation were applied to the 233 deep beams without stirrups that failed under shear tension to investigate the most suitable design shear equation for reinforced concrete deep beams with and without steel fibers and for NSC, HSC, and HPC. Ultimate shear stresses (v_u) were used to compare the design methods. The relative shear strength values ($v_{u exp}/v_{u pred}$) were obtained using these equations, and then the values of the standard deviation (SD), coefficient of

variation (COV), and correlation coefficient (R^2) were also calculated for each equation Figs.(11) to (14).

ACI Code Method $v_u = 0.85 v_c$...(3) $v_c = [0.16 \lambda \sqrt{f_c} + 17 \rho_{w*}(Vu d/Mu)] < 0.29 \lambda \sqrt{f_c}$ 1/(a/d) = Vu d/Mu, Vu d/Mu < 1.0. λ :constant for normal weight concrete=1 Zsutty Method $v_u = 0.85 v_c$...(4) $v_c = 2.51 [f_c \cdot \rho_w/(a/d)]^{0.33}.$ Aziz Method $v_u = 0.85 [v_c + v_s]$ $v_c = 1.51 [(f_c \rho_w (1+F) b \cdot d)/(1\cdota)]^{0.46}.$ (5)

Comparison between design methods

Table 3 compares the four design methods for the 233 reinforced concrete deep beams without shear reinforcement (NSC, HSC, and HPC, with and without steel fibers). If the minimum $v_{u exp}/v_{u pred}$ being equal to or greater than 1 is considered a measure of conservatism, then no design equation passes all the tests. The ACI code method is more conservative, with a COV of 65%. As a representation of the measure of shear capacity, the proposed equation has the lowest COV value (45%) and mean. The Zsutty equation clearly has the lowest SD among the four equations, and its predictions are not conservative.

Table(3)Comparison of the four design methods

		V _{exp} /V _{ACI}	V _{exp} /V _{Aziz}	V _{exp} /V _{zsutty}	V_{exp}/V_{prop}
Mean		2.8838	0.9367	0.3912	1.0421
Standard Dev		1.8761	0.4875	0.2402	0.4692
COV%		0.6506	0.5204	0.6140	0.4503
Range	High	11.5532	4.9549	1.5727	4.3907
	Low	0.5261	0.1929	0.1043	0.2053
Number < 1*		6	152	226	126



Figure(11). Experimental shear stress versus prediction by the proposed equation





Figure(12). Experimental shear stress versus prediction by ACI 318



Figure(13). Experimental shear stress versus prediction by the Aziz method



Figure(14). Experimental shear stress versus prediction by the Zsutty method

Influences of Major Parameters

The aforementioned 233 beam test results were used to investigate the reasons behind the weak representation of design equations for predicting the shear stress of the reinforced concrete deep beams without stirrups. To do this, a series of graphs Figs. (15) to (18) were plotted using the major parameters affecting shear stress (f_c , a/d, l/d, and ρ_w) in the x axis, and the $v_u \exp l/v_u \exp l/v_u$ pred. values in the y axis using the estimation of Equations (2–5).



Figure. 15. Experimental to predicted shear capacity by the proposed equation versus l/d, a/d_{fc} , $\rho_{w\%}$, d/b, d_b , and d_{agg}





Figure(16). Experimental to predicted shear capacity by ACI 318 equation versus l/d, a/d, $f_{c,\rho_{W}\%,d/b}$, d_b, and d_{agg}

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 0
 5
 10
 15
 20
 25
 30

 dagg. (mm)

 Figure(17). Experimental to predicted shear capacity by the Aziz equation versus l/d, a/d,

 $f_{c}, \rho_{w \%}, d/b, d_{b}, and d_{agg}$

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Figure (18). Experimental to predicted shear capacity by the Zsutty equation versus l/d, a/d, f_c , ρ_w %, d/b, d_b , and d_{agg}

CONCLUSIONS

1-Under a specific applied load after the formation of flexural cracks, the central deflection of beams decreased by increasing the compressive strength of the concrete from NSC to HSC, and from HSC to HPC.

2-By increasing the compressive strength of concrete from 43 MPa to 61 MPa, and then to 119 MPa, diagonal cracking load increased by 69% and 8%, respectively. Failure load increased by approximately 55% and 72% when the compressive strength of concrete increased from 43 MPa to 61 MPa, and then to 119 MPa.

3-Shear span-to- depth ratio had a highly significant effect on failure load. Increasing the ratio from 1 to 3 resulted in a decrease in the failure load from 716 kN to 145 kN.

4-Shear span-to- depth ratio, except if the ratio was 1.0, had minimal effect on the formation of the first flexural cracks. The ratio of the first diagonal cracking load to failure load was 16%, 17%, 17%, 18%, and 33% for span-to-depth ratios of 1, 1.5, 2, 2.5, and 3, respectively.

5-The formation of flexural cracks occurred before the formation of diagonal cracks at approximately 0-10% failure load.

6-When the span-to-depth ratio increased from 4 to 8, the ultimate load decreased from 380 kN to 273 kN.

7-Increasing the amount of flexural reinforcement improved resisting failure loads. When the amount of steel ratio increased from 0.0134 to 0.06108, failure load increased by 63%.

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Notations

The following Symbols are used in this paper:

l or ln = beam length;

а	= shear span;
b	= beam width;
d	= beam effective depth;
a/d	= shear span to depth ratio;
f`c	= concrete compressive strength;
l/d	= span to depth ratio;
ρ_w %	= flexural steel bar ratio;
l_n / h	= span to height ratio;
w/c	= water to cement ratio;
v_u	= ultimate shear stress;
v_c	= Predicted shear stress;
k_0, k_1, k_2	$_2, k_3, k_4 = \text{constants};$
d_{agg} .	= maximum aggregate size;
d_b	= flexural bar diameter;
d/b	= effective depth to width ratio;
λ	= modification factor reflecting the reduced mechanical properties
	of lightweight concrete
M_{μ}	= ultimate moment;

 v_s = stirrup shear stress