Approximate solution of integral by taylor expansion method

By

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Abstract

In this paper we translate some functions where have the ability of differentiation infinity of time and difficult finding integration to them or their integral take long time to solve, therefore we thought by taylor series which essentially depends on differentiation.

We integrated this function definite integral founded the nearly integral after make a cutting in series, then integrate the the function of two variable twice definite integral to find the nearly integral after make a cutting in series and gave some examples, after that, we proved the uniqueness of the reminder of the series.

Last, we showed the shape of these functions using matlab program.

الخلاصة

في هذا البحث قمنا بتحويل الدالة القابلة للاشتقاق مالانهاية من المرات والتي من الصعب ايجاد التكامل لها او ان تكاملها ياخذ وقتا طويلا في الحل الى متسلسلة تيلر التي تعتمد اساسا على الاشتقاق وبعد عمل قطع جزء من المتسلسلة قمنا بتكاملها تكامل غير محدد لايجاد اتكامل التقريبي ثم قمنا بنفس الشئ الى دالة لمتغيرين واعطينا بعض الامثلة ثم اثبتنا وحدانية المتبقي للمتسلسلة. اخيرا اوضحنا شكل هذه الدوال باستخدام برنامج ماتلاب

Introduction

If the function f(x) has a difficult integration, and if it is can be a differentiable function this function can be transform to Taylor expansion which has the form :

$$f(x) = f(a) + f(a)'(x-a)^1 + f(a)^{(2)}\frac{(x-a)^2}{2!} + f(a)^{(3)}\frac{(x-a)^3}{3!} + \cdots$$

Then we integrated it. Also, the function of two variables f(x,y) that can be written by f(x,y)=g(x).h(y) can be written by taylor expansion, we translated it then integrated it. In [1]George B.Thomas, Maurice D.Weir and Joel R.Hass found by taylor expansion method the limits of functions and the integral of sin x^2, [3] G. B. Folland estimate the reminder of taylor.here we'll find the numerically integral by taylor expansion method and prove the uniqueness of the reminder.

Definition. A function f defined on an interval I is called k times differentiable on I if the derivatives f ',f '',…,$f^{(n)}$ exist and are finite on I, and f is said to be of class C^k on I if these derivatives are all continuous on I,[2] . Lemma(1)(Bainov D. and Simeonov P.,1992)

Let u(x) and b(x) be nonnegative continues functions for x$\geq \alpha$ and let

u(x)$\leq a + \int_\alpha^x b(t)u(t)dt, \quad x \geq \alpha,$

where a is a nonnegative constant, then;

u(x)$\leq ae^{\int_\alpha^x b(t)dt}$, x$\geq\alpha$.

Theorem (1) (uniqueness of reminder)

Taylor series is written by:

F(x)=f(b)+(x-b)f'(b)+$(x-b)^2$f''(b)/2!)+$(x-b)^3$f'''(b)/3!+…+$(x-b)^n f^{(n)}$ (b)/n!

$+\int_c^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)dt.$

The last part $r_n(x)=$ $\int_c^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)dt$ is called the reminder of f, this reminder is unique.

Proof:

Suppose there are two reminder

$r_n(x)= \int_c^x \frac{(x-t)^n}{n!} f_n^{(n+1)}(t)dt$ and

$r_m(x)= \int_c^x \frac{(x-t)^n}{n!} f_m^{(n+1)}(t)dt$

$r_n(x)= \int_c^x \frac{(x-t)^n}{n!} f_n^{(n+1)}(t)dt \leq$

$\epsilon_1 + \int_c^x \frac{(x-t)^n}{n!} f_n^{(n+1)}(t)dt \leq \epsilon_1 +$

$\int_c^x \frac{(x-t)^n}{n!} r_n(t)dt$

$r_m(x)= \int_c^x \frac{(x-t)^n}{n!} f_m^{(n+1)}(t)dt \leq$

$\epsilon_2 + \int_c^x \frac{(x-t)^n}{n!} f_m^{(n+1)}(t)dt \leq$

$\epsilon_2 + \int_c^x \frac{(x-t)^n}{n!} r_m(t)dt$

where ϵ_1 and ϵ_2 are very small positive numbers.

$|r_n(x) - r_m(x)| \leq |\epsilon_1 - \epsilon_2| + \int_c^x \frac{(x-t)^n}{n!} |f_n^{(n+1)}(t) - f_m^{(n+1)}(t)|dt \leq |\epsilon 1 - \epsilon 2| + \int_c^x \frac{(x-t)^n}{n!} |r_n(t) - r_m(t)|dt$

By lemma (1)

$|r_n(x) - r_m(x)| \leq |\epsilon_1 - \epsilon_2| e^{\int_c^x \frac{(x-t)^n}{n!} dt}$.

Since $|\epsilon_1 - \epsilon_2| \to 0$ this implies:

$|r_n(x) - r_m(x)| \to 0$.

In the following example, we'll give a function of on variable in which difficult integrable and has a taylor expansion.

Example(1) this function of one variable:

f=exp(x^2)

the exact integral of this function is:

$\int_0^{\frac{\pi}{2}} e^{x^2} dx$ =4.8128.

The taylor expansion to this function at a=0 with 12 part before cutting is:

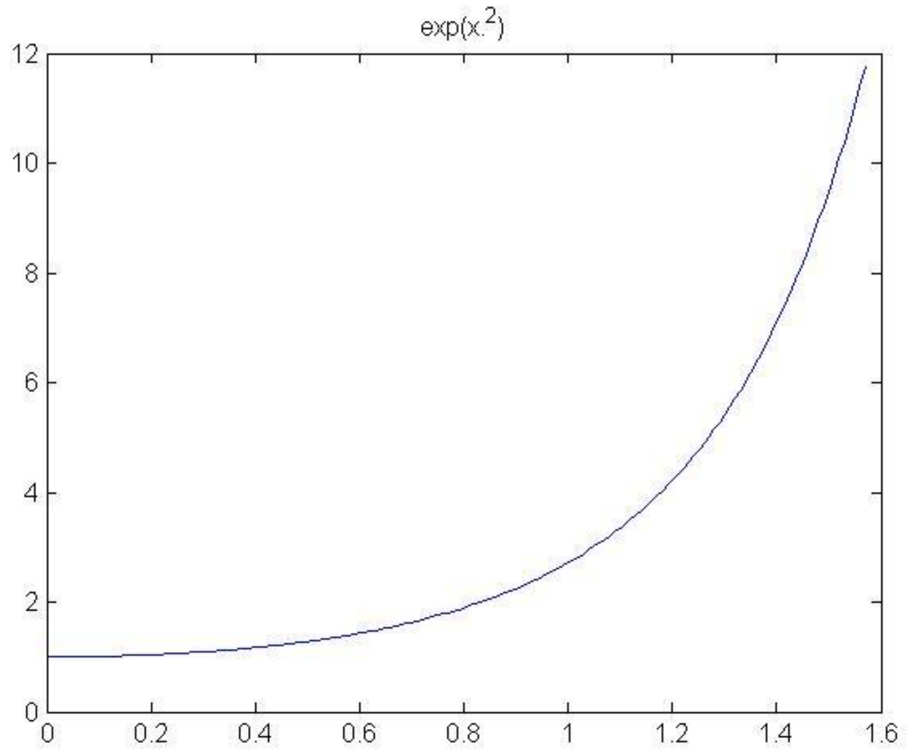
f(x)=1+x^2+1/2*x^4+1/6*x^6+1/24*x^8+1/120*x^10

The integral of the taylor series f(x) from 0 to $\frac{\pi}{2}$ is: 4.7592.

The reminder of the series is:

R(x) = exact value - approximate value = 4.8128 – 4.7592 =0.0 536

The following shape of the function.



In the following, we'll give a function of two variables it also difficult integrable and has a taylor expansion.

Example(2) this function of two variables:

f=exp(x^2)*cos(y)

the exact integral is:

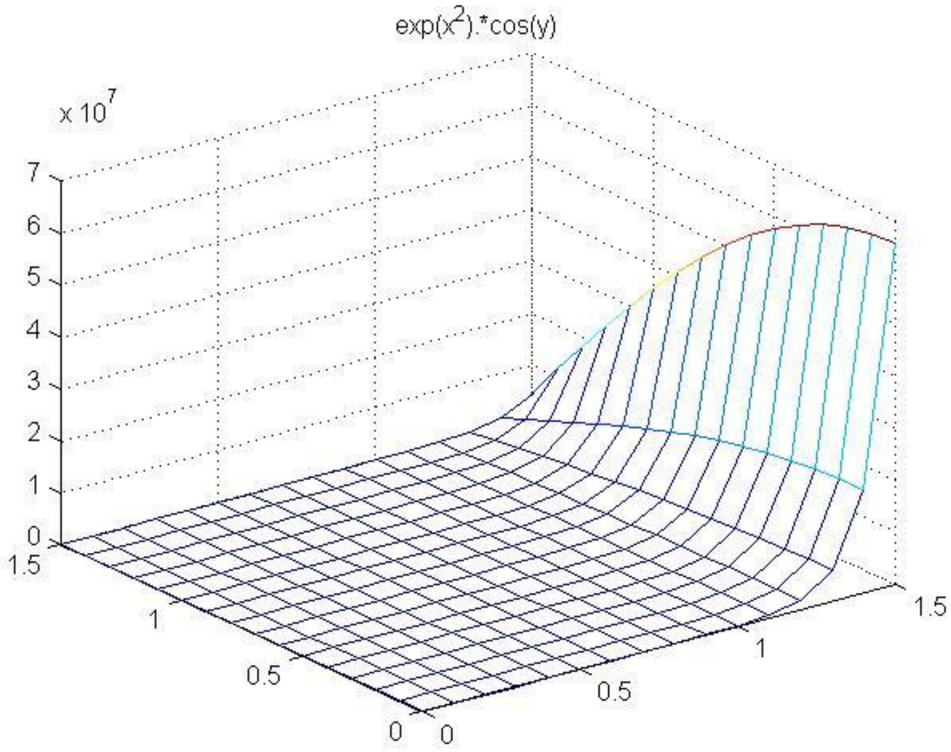
$$\int_{y=0}^{pi/2}\int_{x=0}^{pi/2} \exp(x^2) * \cos(y)dx\,dy = 4.8128.$$

the taylor expansion to this function about x and y is:

f(x)=cos(y)+cos(y)*x^2+1/2*cos(y)*x^4+1/6*cos(y)*x^6+1/24*cos(y)*x^8+1/120*cos(y)*x^10.

The integral of this series is: 4.7592

The reminder of the series is:
R(x) = exact value - approximate value = 4.8128 – 4.7592 =0.0 536
The following shape of the function.



References:

[1] George B.Thomas, Maurice D.Weir and Joel R.Hass,"Thomas'Calculus", twelfth edition,2010.

[3] G. B. Folland,"Remainder estimates in Taylor's theorem, " American Mathematical Monthly 97 (1990), 233-235.

[2]Bainov D. and Simeonov P.,"Integral inequalities and applications", Dordrecht, Boston, London, 1992.