A comparison and an application of reliability, reliability system, hybrid reliability system functions for Rayleigh, Pareto, and Rayleigh Pareto distributions

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Abstract:
This study dealt with the analysis of the reliability function for hybrid systems with asymmetric components, where the operating times of these components follow Rayleigh, Pareto, and Rayleigh Pareto distributions, furthermore, we introduced a method of operation based on hybrid systems in order to increase the reliability of the machines operating times. Different simulation experiments were done to estimate the reliability function of these systems using the ML method, then a comparison was made between those estimators, the simulation results proved that the Rayleigh-Pareto distribution gives the best estimate for the reliability functions, also the estimated values for the reliability function for the hybrid parallel-series system greater than the estimated values of the reliability function for the hybrid series-parallel system, finally, the estimated reliability function values of the proposed system are better than the estimated values of the two other systems. In the application part, it was chosen the operating times for the machines of the University House for printing, publishing ,and translation in Baghdad, where the factory management depends on the system hybrid serial-parallel system to operate the factory machines, this system leads to a decrease in the reliability of the operating times of the factory machines to 50% at the first operating hour, but if the blistering hybrid system is followed, i.e. the similar machines are run in parallel, and the different machines are run in series, then the probability that the machinery system will continue to operate for four hours will be 50%.

Keywords: Rayleigh, Pareto, Rayleigh - Pareto distributions, reliability functions for the hybrid systems: series–parallel and parallel – series.

Introduction:

After the wide spread of industry and the increase in mechanical, electrical and electronic complexities in devices and equipment in the last century, the interest in reliability has increased in modern research and studies. This rapid technology and the complex systems in various areas of life, have imposed an increasing interest in studying the causes of disruptions of all types of the devices and machines, as the failure that occurs in the work of the devices leads to a decrease in production and losses in material. The previous studies conducted in this field focused on studying the reliability of simple systems or on the application of the serial-parallel hybrid system, among these studies, we mention the study of (Raheem, 2014) [10], which focused on estimating the reliability function of hybrid parallel-series systems. In another study, the researcher (Al-Saady, 2016) [4] modeled the reliability function of the hybrid series-parallel system in the case of asymmetric components, while there are many systems production consisting of complex work systems and asymmetric in their behavior. One of these problems is the permanent breakdowns in the machines of the University House of Printing, Publishing, and Translation in Baghdad, which contain different asymmetric machines that are connected as hybrid series-parallel, this research will shed light on that problem as well as present a proposal for an optimal reliability system of the printing system. To achieve this goal, various simulation experiments and the maximum likelihood method were employed in estimating the reliability function for failure times which follows different distributions, and estimating the reliability functions of the series and parallel systems, as well as estimating the reliability function of asymmetric hybrid systems, whether it was the hybrid series-parallel system or the hybrid series-parallel system, then a comparison was made between these methods estimators.

Rayleigh, Pareto, and Rayleigh Pareto distributions:

Rayleigh and Pareto distributions are important continuous distributions, which commonly used in economic, failure and survival times studies, the Rayleigh random variable S with scale parameter α has the following reliability function:[14]

$$\mathbf{R}(\mathbf{s},\boldsymbol{\alpha}) = \mathbf{e}^{-\left(\frac{S^2}{2\alpha^2}\right)} \qquad (1)$$

The random variable **Z** which follows one parameter Pareto distribution have the following reliability functions:[13]

$$\mathbf{R}(\mathbf{z},\boldsymbol{\varphi}) = \left(\frac{1}{z}\right)^{\varphi} \qquad (2)$$

The Rayleigh-Pareto (**RP**) distribution is useful in studies of real life fields such as the biostatistics, the probability density function of the RB distribution is given by: [2]

$$\mathbf{f}(\mathbf{v},\theta,\beta,\mathbf{b}) = \frac{b}{2\theta^2\beta}\left(\frac{v}{\beta}\right)^{b-1} e^{-\frac{1}{2\theta^2}\left(\frac{v}{\beta}\right)^{b}} \quad ,z > 0\ ,\ \ \theta,\beta,b > 0 \qquad (3)$$

Where θ,β are scale parameters, b is a shape parameter. The **RP** distribution reliability function can be defined as: [2]

$$\mathbf{R}(\mathbf{v},\theta,\beta,\mathbf{b}) = e^{-\frac{1}{2\theta^2}\left(\frac{v}{\beta}\right)^{b}} \quad ,z > 0\ ,\ \ \theta,\beta,b > 0 \qquad (4)$$

Maximum Likelihood Estimation(MLE):

Let ($s_1, s_2, \ldots, s_n$) a sample from the Rayleigh distribution, then MLE for α is given as:[5]

$$\hat{\alpha}_{ML} = \left(\frac{\sum_{i=1}^{n} s_i^2}{2n}\right)^{\frac{1}{2}} \quad (5)$$

Also the MLE for the scale parameter **φ** of the Pareto random sample ($z_1, z_2, \ldots, z_n$) **i**s given as:[1], [7]

$$\hat{\varphi}_{ML} = \frac{n}{\sum_{i=1}^{n} \mathrm{Ln}\, z_i} \quad (6)$$

The reliability function for Rayleigh and Pareto distributions are given as: [5], [7], [14]

$$\hat{R}(s)_{ML} = e^{-\left(\frac{n\, s^2}{\sum_{i=1}^{n} s}\right)} \quad (7)$$

$$\hat{R}(z)_{ML} = \left(\frac{1}{z}\right)^{\frac{n}{\sum_{i=1}^{n} \mathrm{Ln}\, z_i}} \quad (8)$$

To estimate MLE of the unknown parameters for RP distribution, we have: [2]

$$L(v_i; \theta, \beta, b) = \prod_{i=1}^{n} f(v_i;\ \theta, \beta, b) = \left(\frac{b}{2\theta^2\beta}\right)^{n} \left(\frac{\prod_{i=1}^{n} v_i}{\beta}\right)^{b-1} e^{-\frac{1}{2\theta^2}\left(\frac{\sum_{i=1}^{n} v_i}{\beta}\right)^{b}}$$

So the log-likelihood function:

$$\ln L(v_i; \theta, \beta, b) = n\ln(\theta) - \ln(2\theta^2) + (b-1)\sum_{i=1}^{n} \ln(v_i) - n(b-1)\ln(\beta) - e^{-\frac{\sum_{i=1}^{n} v_i{}^{b}}{2\theta^2\, \beta^{b}}} \quad (9)$$

The derivatives of the lnL with respect to the two parameters are given by:

$$\frac{\partial \ln L}{\partial \theta} = -\frac{2n}{\theta} + \frac{1}{\theta^3\beta^b}\sum_{i=1}^{n} v_i{}^{b} \quad , \quad \frac{\partial \ln L}{\partial \beta} = -\frac{n}{\beta} - \frac{n(b-1)}{\beta} + \frac{b\sum_{i=1}^{n} v_i{}^{b}}{2\theta^2\beta^{b+1}}$$

$$\frac{\partial \ln L}{\partial b} = \frac{n}{b} - \sum_{i=1}^{n} \ln(v_i) - n\ln(\theta) - \frac{1}{2\theta^2}\sum_{i=1}^{n}\left(\frac{\sum_{i=1}^{n} v_i}{\beta}\right)^{b} \ln\left(\frac{\sum_{i=1}^{n} v_i}{\beta}\right)$$

The MLEs of θ, β, b **say** $\hat{\theta}_{ML}, \hat{\beta}_{ML}, \hat{b}_{ML}$ are the simultaneous solutions of the above equations $\frac{\partial \ln L}{\partial \theta} = 0, \frac{\partial \ln L}{\partial \beta} = 0, \frac{\partial \ln L}{\partial b} = 0,$ maximizing of (9) is performed using (optimize) in R statistical package. From the above, we can get the MLE of the reliability function for RP distribution by substituting simultaneous solutions for parameters θ, β, b in equation (4): [2], [9], [15]

$$\hat{R}(v)_{ML} = e^{-\frac{1}{2\hat{\theta}_{ML}{}^2}\left(\frac{v}{\hat{\beta}_{ML}}\right)^{\hat{b}_{ML}}} \quad (10)$$

Systems Reliability:

Reliability system is defined as the probability that the system will work successfully after time (t), or the probability that the system will not fail during the period $[0, t]$. [6],[8] The importance of the system reliability lies in the nature of the relationships that connect those components, i.e. the system type based on the nature of the link between its components, so it is necessary to know the pattern of behavior of these components and then their impact on this system, thus the systems can be classified into two main types: [11]

Series System: In this system type any component leads to the failure of the entire system because the components are connected respectively. Let the system consists (m) components, then the reliability function of the series system can be expressed mathematically as follows: [3], [11]

$$\mathbf{R_s(t) = R_1(t).R_2(t)\dots R_m(t) = \prod_{j=1}^{m} R_j(t)} \quad , \quad \mathbf{j = 1, 2, \dots, m} \qquad (11)$$

Parallel System: Parallel System is a system in which components are interconnected such that a malfunction of any component does not lead to system failure. The mathematical formula for the reliability function of this system is given by: [11]

$$\mathbf{R_p(t) = 1 - \prod_{j=1}^{m}\left[1 - R_j(t)\right]} \quad , \quad \mathbf{j = 1, 2, \dots, m} \qquad (12)$$

Hybrid system: A system consisting of many partial systems and each partial system consists of many compounds, these partial systems can be linked together series or in parallel, also within each partial system can be linked the components in series or in parallel. [11]

Hybrid Parallel - Series System: This type of hybrid systems consists of (r) partial systems connected series, each partial system has (m) components that are connected in parallel. This system is fail if one partial system fails. The reliability function for hybrid parallel – series system (R_{HPS}) can be calculate as:[11]

$$\mathbf{R_{HPS}(t) = \prod_{k=1}^{r}\left[1 - \prod_{j=1}^{m}\left\{1 - R_{kj}(t)\right\}\right], j = 1, 2, \dots, m, k = 1, 2, \dots, r} \qquad (13)$$

To clarify the mechanism for estimating the reliability function of this system, assuming that six components say **A** to **D** distributed into three partial systems, where that **A** and B have Rayleigh distribution, **C** and D have Pareto distribution, **E** and **F** followed Rayleigh - Pareto distribution, then **j=3, k=2** and the reliability function $\mathbf{R_{HPS}(t)}$ is give as: [3], [11]

$$\mathbf{R_{HPS}(t) = \prod_{k=1}^{3}\left[1 - \prod_{j=1}^{2}\left\{1 - R_{kj}(t)\right\}\right]} \qquad (14)$$

$$\mathbf{R_{HPS}(t) = [1 - \{1 - R_{1A}(t)\}\{1 - R_{1B}(t)\}] * [1 - \{1 - R_{2C}(t)\}\{1 - R_{2D}(t)\}] * [1 - \{1 - R_{3E}(t)\}\{1 - R_{3F}(t)\}]} \qquad (15)$$

The MLE for the reliability function of the Hybrid Parallel - Series System $\widehat{\mathbf{R}}_{\mathbf{HPS}}(\mathbf{t})$ is given by: [3]

$$\widehat{\mathbf{R}}_{\mathbf{HPS}}(\mathbf{t}) = \left[1 - \left\{\left(1 - e^{-\frac{S^2}{2\hat{\alpha}_{ML1}{}^2}}\right)\left(1 - e^{-\frac{S^2}{2\hat{\alpha}_{ML2}{}^2}}\right)\right\}\right] * \left[1 - \left\{\left(1 - \left(\frac{1}{z}\right)^{\varphi_{ML1}}\right)\left(1 - \left(\frac{1}{z}\right)^{\varphi_{ML2}}\right)\right\}\right] * \left[1 - \left\{\left(1 - e^{-\frac{1}{2\hat{\theta}^2_{ML1}}\left(\frac{v}{\hat{\beta}_{ML1}}\right)^{\hat{b}_{ML1}}}\right)\left(1 - e^{-\frac{1}{2\hat{\theta}^2_{ML2}}\left(\frac{v}{\hat{\beta}_{ML2}}\right)^{\hat{b}_{ML2}}}\right)\right\}\right] \quad (16)$$

Hybrid Series – Parallel System: This type of hybrid systems consists of (**r**) partial systems connected as parallel, within each partial system there are (**m**) components that are connected as a series system. This system is fail if any partial system fails, i.e. if any component is fails. The reliability function for a hybrid series – parallel system ($\mathbf{R}_{\mathbf{HSP}}$) can be calculated as: [11]

$$\mathbf{R}_{\mathbf{HSP}}(\mathbf{t}) = 1 - \left[\prod_{k=1}^{r}\left\{1 - \prod_{j=1}^{m} R_{kj}(t)\right\}\right], j = 1, 2, \dots, m, k = 1, 2, \dots, l \qquad (17)$$

For k=2, j=3, we have:

$$\mathbf{R}_{\mathbf{HSP}}(\mathbf{t}) = 1 - \left[\prod_{k=1}^{2}\left\{1 - \prod_{j=1}^{3} R_{kj}(t)\right\}\right] \qquad (18)$$

$$\mathbf{R}_{\mathbf{HSP}}(\mathbf{t}) = 1 - \langle[1 - \{R_{1A}(t) * R_{1B}(t) * R_{1C}(t)\}] * [1 - \{R_{2D}(t) * R_{2E}(t) * R_{2F}(t)\}]\rangle \quad (19)$$

Then the MLE of reliability function for the Hybrid Parallel - Series System can be obtained as:

$$\hat{R}_{HSP}(t) = 1 - \left[\left[1 - \left\{ e^{-\frac{S}{2\hat{\alpha}_{ML1}{}^{2}}} \left(\frac{1}{z}\right)^{\varphi_{ML1}} e^{-\frac{1}{2\hat{\theta}^{2}_{ML1}}} \left(\frac{v}{\hat{\beta}_{ML1}}\right)^{\hat{b}_{ML1}} \right\}\right] * \left[1 - \left\{ e^{-\frac{S}{2\hat{\alpha}_{ML2}{}^{2}}} \left(\frac{1}{z}\right)^{\varphi_{ML2}} e^{-\frac{1}{2\hat{\theta}^{2}_{ML2}}} \left(\frac{v}{\hat{\beta}_{ML2}}\right)^{\hat{b}_{ML2}} \right\}\right]\right] \quad (20)$$

Simulations and Results:

Simulation experiments were conducted to estimate the reliability functions for the series, parallel and hybrid systems using R program [12] according to the following steps:

Step1: Select the default parameters values for Rayleigh, Pareto and RP distributions as shown in table (1).

Table (1) Simulation Experiments

1st	α_1	12	φ_1	0.3	β_1	θ_1	b_1	1
2nd	α_2	15	φ_2	0.4	β_2	θ_2	b_2	2
3rd	α_1	18	φ_3	0.5	β_3	θ_3	b_3	3

Step2: Select different failure times $\mathbf{t = 2, 3, 4, 5}$ and sample sizes $n = 25, 50, 100$.

Step3: Based on equations 7, 8, 10, 16 and 20, we estimate the reliability function for each component and (series, parallel, hybrid) systems reliability functions, the simulation replicated ($L = 5000$) times, where:

$$\hat{R}_j(t) = \frac{1}{L}\sum_{l=1}^{L}\hat{R}_{jl}(t) \quad , \quad MSE\left[\hat{R}_{jl}(t)\right] = \frac{1}{L}\sum_{l=1}^{L}\left[\hat{R}_{jl}(t) - R_j(t)\right]^2$$

Based on the R 4.1.2 program, simulation results for estimation by ML method, are shown in tables 2 to 6 below.

Table (2) Estimated reliability functions for 1st experiment

Dist		Rayleigh			Pareto			RP		
t	n	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE
2	25	0.8948	0.8516	0.0174	0.8123	0.7669	0.0157	0.9060	0.8651	0.0139
	50		0.9359	0.0121		0.7692	0.0110		0.8680	0.0095
	100		0.8558	0.0085		0.8524	0.0076		0.8707	0.0066
3	25	0.7788	0.7386	0.0172	0.7192	0.6770	0.0154	0.8007	0.7627	0.0137
	50		0.7406	0.0119		0.6791	0.0108		0.8360	0.0093
	100		0.8143	0.0083		0.6827	0.0074		0.7685	0.0063
4	25	0.6412	0.6018	0.0171	0.6598	0.6185	0.0153	0.6736	0.6364	0.0136
	50		0.6046	0.0118		0.6982	0.0106		0.6398	0.0092
	100		0.6072	0.0082		0.6249	0.0072		0.7044	0.0062
5	25	0.4994	0.5379	0.0171	0.6170	0.5766	0.0153	0.5394	0.5030	0.0136
	50		0.4637	0.0117		0.5795	0.0106		0.5063	0.0092
	100		0.4662	0.0081		0.5829	0.0072		0.5094	0.0062
8	25	0.1690	0.1314	0.0170	0.5359	0.5754	0.0152	0.2059	0.1703	0.0135
	50		0.1341	0.0117		0.4992	0.0105		0.1736	0.0091
	100		0.1366	0.0081		0.5026	0.0071		0.1766	0.0062
10	25	0.0622	0.0254	0.0169	0.5012	0.4626	0.0151	0.0847	0.1194	0.0135
	50		0.0281	0.0116		0.4654	0.0104		0.0532	0.0091
	100		0.0306	0.0080		0.4687	0.0071		0.0561	0.0061

Table (3) Estimated reliability functions for 2nd experiment

Dist		Rayleigh			Pareto			RP2		
t	n	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE
2	25	0.8752	0.8358	0.0149	0.7579	0.7992	0.0158	0.8768	0.8396	0.0140
	50		0.8378	0.0141		0.7186	0.0149		0.8422	0.0130
	100		0.9107	0.0133		0.7214	0.0137		0.8446	0.0120
3	25	0.7408	0.7054	0.0146	0.6444	0.6072	0.0154	0.7439	0.7104	0.0138
	50		0.7079	0.0138		0.6099	0.0145		0.7135	0.0127
	100		0.7103	0.0130		0.6130	0.0134		0.7162	0.0117
4	25	0.5866	0.5520	0.0146	0.5743	0.5379	0.0154	0.591	0.5583	0.0137
	50		0.5544	0.0137		0.6081	0.0145		0.5613	0.0126
	100		0.5568	0.0130		0.5436	0.0134		0.5640	0.0117
5	25	0.4346	0.4008	0.0145	0.5253	0.4898	0.0153	0.4396	0.4076	0.0137
	50		0.4032	0.0137		0.5583	0.0144		0.4105	0.0126
	100		0.4055	0.0129		0.4954	0.0133		0.4132	0.0116
8	25	0.1184	0.0853	0.0145	0.4353	0.4700	0.0153	0.122	0.0908	0.0136
	50		0.1491	0.0137		0.4031	0.0144		0.0936	0.0126
	100		0.0900	0.0129		0.4061	0.0133		0.0963	0.0116
10	25	0.0357	0.0034	0.0144	0.3981	0.3642	0.0152	0.0374	0.0679	0.0136
	50		0.0656	0.0136		0.4295	0.0143		0.0097	0.0125
	100		0.0080	0.0128		0.3696	0.0132		0.0123	0.0116

Table (4) Estimated reliability functions for 3rd experiment

Dist		Rayleigh			Pareto			RP2		
t	n	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE
2	25	0.8465	0.8815	0.0124	0.7071	0.6703	0.0131	0.8007	0.7676	0.0117
	50		0.8132	0.0118		0.6722	0.0124		0.7699	0.0108
	100		0.8149	0.0111		0.6746	0.0115		0.7721	0.0100
3	25	0.6873	0.6575	0.0121	0.5774	0.6087	0.0127	0.6065	0.5784	0.0114
	50		0.7149	0.0114		0.5484	0.0120		0.5809	0.0105
	100		0.6617	0.0108		0.5511	0.0111		0.5833	0.0097
4	25	0.5134	0.4843	0.0121	0.5	0.4695	0.0127	0.4111	0.3836	0.0113
	50		0.4864	0.0114		0.4717	0.0120		0.4360	0.0105
	100		0.4884	0.0107		0.4743	0.0111		0.3885	0.0097
5	25	0.3529	0.3245	0.0120	0.4472	0.4174	0.0127	0.2494	0.2226	0.0113
	50		0.3266	0.0113		0.4196	0.0119		0.2251	0.0104
	100		0.3286	0.0107		0.4222	0.0110		0.2274	0.0096
8	25	0.0695	0.0972	0.0120	0.3536	0.3245	0.0126	0.0286	0.0024	0.0113
	50		0.0439	0.0113		0.3267	0.0119		0.0049	0.0104
	100		0.0458	0.0107		0.3292	0.0110		0.0071	0.0096
10	25	0.0155	0.0208	0.0112	0.3162	0.3107	0.0118	0.0039	0.0022	0.0106
	50		0.0112	0.0107		0.3117	0.0112		0.0027	0.0099
	100		0.0123	0.0101		0.3130	0.0104		0.0030	0.0092

Table (5) Estimated reliability series systems functions

Exp.		1st			2nd			3rd		
t	n	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE	Real	$\hat{R}$	MSE
2	25	0.6585	0.5650	0.0982	0.5816	0.5608	0.0218	0.4793	0.4536	0.0270
	50		0.6249	0.0347		0.5070	0.0768		0.4209	0.0602
	100		0.6352	0.0236		0.5549	0.0270		0.4245	0.0554
3	25	0.4485	0.3814	0.0705	0.3551	0.3043	0.0534	0.2407	0.2315	0.0097
	50		0.4205	0.0289		0.3081	0.0485		0.2277	0.0133
	100		0.4272	0.0215		0.3118	0.0437		0.2127	0.0283
4	25	0.2850	0.2369	0.0505	0.1991	0.1658	0.0350	0.1055	0.0872	0.0192
	50		0.2701	0.0153		0.1892	0.0102		0.1000	0.0057
	100		0.2673	0.0179		0.1707	0.0287		0.0900	0.0157
5	25	0.1662	0.1560	0.0107	0.1004	0.0800	0.0214	0.0394	0.0302	0.0097
	50		0.1361	0.0311		0.0924	0.0082		0.0309	0.0088
	100		0.1384	0.0281		0.0830	0.0175		0.0316	0.0079
8	25	0.0187	0.0129	0.0061	0.0063	0.0036	0.0066	0.0007	0.0001	0.0016
	50		0.0116	0.0072		0.0056	0.0023		0.0001	0.0013
	100		0.0121	0.0066		0.0035	0.0012		0.0001	0.0011

Table (6) Estimated reliability parallel systems functions

Exp.		1st			2nd			3rd		
t	**n**	**Real**	$\hat{R}$	**MSE**	**Real**	$\hat{R}$	**MSE**	**Real**	$\hat{R}$	**MSE**
2	25	0.9981	0.9953	0.0098	0.9963	0.9947	0.0086	0.9910	0.9909	0.0034
	50		0.9981	0.0011		0.9928	0.0035		0.9859	0.0018
	100		0.9973	0.0009		0.9961	0.0003		0.9863	0.0012
3	25	0.9876	0.9800	0.0230	0.9764	0.9665	0.0297	0.9480	0.9435	0.0135
	50		0.9864	0.0036		0.9674	0.0253		0.9460	0.0055
	100		0.9864	0.0026		0.9682	0.0172		0.9367	0.0051
4	25	0.9602	0.9448	0.0462	0.9280	0.9086	0.0584	0.8567	0.8314	0.0761
	50		0.9570	0.0088		0.9234	0.0130		0.8470	0.0273
	100		0.9565	0.0074		0.9118	0.0089		0.8355	0.0116
5	25	0.9117	0.9028	0.0268	0.8496	0.8189	0.0921	0.7315	0.6941	0.0393
	50		0.8887	0.0233		0.8446	0.0274		0.6971	0.0337
	100		0.8908	0.0205		0.8240	0.0251		0.7003	0.0306
8	25	0.6937	0.6940	0.0013	0.5629	0.5592	0.0183	0.4157	0.3916	0.0362
	50		0.6416	0.0011		0.5396	0.0035		0.3594	0.0084
	100		0.6464	0.001		0.5116	0.0026		0.3645	0.0041

Table (7) Estimated reliability hybrids systems functions

		Series - Parallel			Parallel - Series			Suggested hybrid system		
t	**n**	**Real**	$\hat{R}$	**MSE**	**Real**	$\hat{R}$	**RMSE**	**Real**	$\hat{R}$	**MSE**
2	25	0.9256	0.8956	0.0180	0.9855	0.9811	0.0115	0.9824	0.9768	0.0045
	50		0.8929	0.0139		0.9769	0.0045		0.9721	0.0026
	100		0.9065	0.0057		0.9798	0.0022		0.9798	0.0006
3	25	0.7299	0.6692	0.0364	0.9142	0.8936	0.0135	0.9217	0.8985	0.0092
	50		0.6903	0.0168		0.9027	0.0045		0.9049	0.0034
	100		0.6897	0.0121		0.8945	0.0041		0.9023	0.0015
4	25	0.4878	0.4189	0.0413	0.7634	0.7136	0.0072	0.7930	0.7416	0.0136
	50		0.4674	0.0086		0.7485	0.0053		0.7767	0.0033
	100		0.4470	0.0122		0.7287	0.0045		0.7636	0.0019
5	25	0.2794	0.2470	0.0195	0.5666	0.5131	0.0289	0.5924	0.5480	0.0126
	50		0.2401	0.0167		0.5232	0.0227		0.5421	0.0058
	100		0.2349	0.0134		0.5140	0.0137		0.5375	0.0026
8	25	0.0255	0.0165	0.0054	0.1623	0.1520	0.0124	0.0853	0.0593	0.0097
	50		0.0173	0.0035		0.1244	0.0086		0.0601	0.0041
	100		0.0157	0.0029		0.1205	0.0058		0.0524	0.002

From tables 2-7 above, we noticed that when the sample size increase the estimated values approached to the default values and the MSE values are decrease and the reliability function values are decrease over time, and that the smallest MSE values were for Rayleigh-Pareto distribution, then the MSE values for Rayleigh distribution, then the MSE values for Pareto distribution. Also the MSE values for the components are decrease as the scale parameter values of the three distributions increases. Furthermore, for all simulation experiments, the default and estimated reliability function values for the parallel system were greater than the default and estimated reliability function values of the series system, and the default and estimated reliability function values for the hybrid parallel - series systems were greater than the default and estimated reliability function values for the hybrid series-parallel system, and that the estimated reliability function MSE values for the proposed hybrid system function were smaller than the MSE values for the others hybrid systems, i.e. the parallel-series and the series-parallel.

Application:

The real data for this research represent the operating times between malfunction and the other for the machines of the University House for printing, publishing and translation in Baghdad, recorded during April and May 2022. The daily working in this printing press is about 8 hours in all week day except days off in Friday and

Saturday. One of the most important production processes in this printing press has been highlighted, that is the exam notebooks printing process, this process can be described in three stages: the first stage begins by inserting raw paper into the first machine, which is responsible for the content printing , and then the resulting paper is transferred to the second stage, which is the crushing stage, as the second machine folds the paper and turns it into the form of the exam notebook, the third stage is the cutting and pressing the exam notebooks. Table (8) below represents the daily hours operating for the six machines, which consists of the production two lines for exam notebooks printing, each line contains three machines (printing, crushing, cutting and pressing).

Table (8) Daily working time (in hours)

Obs.	Machine						Obs.	Machine					
	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}		1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
1	5.7	1.5	3.6	5.7	1.4	3.7	26	3.3	0.4	1.9	3.4	0.4	2
2	3.2	0.4	1.8	3.3	0.4	1.9	27	4.7	0.8	2.8	4.8	0.8	2.9
3	1	0.2	0.6	1	0.2	0.6	28	3.9	0.5	2.2	3.9	0.5	2.3
4	3.7	0.5	2.1	3.8	0.5	2.2	29	3	0.3	1.7	3	0.3	1.7
5	0.6	0.2	0.4	0.6	0.2	0.4	30	7.7	8	7.9	7.8	7.2	7.9
6	4	0.5	2.3	4	0.5	2.4	31	2.2	0.3	1.3	2.2	0.3	1.3
7	1.3	0.2	.0.8	1.3	0.2	0.8	32	6.8	3.6	5.2	6.9	3.3	5.3
8	4.4	0.7	2.6	4.5	0.7	2.7	33	0.9	0.2	0.5	0.9	0.2	0.6
9	4.1	0.6	2.4	4.1	0.6	2.5	34	3.1	0.4	1.8	3.2	0.4	1.9
10	4.7	0.8	2.8	4.8	0.8	2.9	35	1.1	0.2	0.7	1.1	0.2	0.7
11	1.6	0.2	0.9	1.7	0.2	1	36	7	4.1	5.6	7	3.7	5.7
12	6.5	2.8	4.7	6.6	2.6	4.8	37	3.7	0.5	2.1	3.8	0.5	2.2
13	4.2	0.6	2.4	4.2	0.6	2.5	38	4.2	0.6	2.4	4.2	0.6	2.5
14	1.9	0.3	1.1	2	0.3	1.2	39	1.9	0.2	1.1	1.9	0.2	1.1
15	1.9	0.2	1.1	1.9	0.2	1.1	40	3.4	0.4	1.9	3.4	0.4	2
16	2.7	0.3	1.5	2.7	0.3	1.6	41	7.3	5.4	6.4	7.4	4.9	6.5
17	4.9	0.9	2.9	5	0.9	3.1	42	0.9	0.2	0.6	0.9	0.2	0.6
18	3	0.4	1.7	3.1	0.3	1.8	43	4.5	0.7	2.6	4.5	0.7	2.7
19	1.9	0.3	1.1	2	0.3	1.2	44	4.3	0.6	2.5	4.3	0.6	2.6
20	3.5	0.4	2	3.5	0.4	2.1	45	3.7	0.5	2.1	3.7	0.5	2.2
21	4.9	0.9	3	5	0.9	3.1	46	6	1.9	4	6.1	1.8	4.1
22	4.5	0.7	2.6	4.5	0.7	2.7	47	4.7	0.8	2.8	4.7	0.8	2.9
23	3.4	0.4	1.9	3.4	0.4	2	48	6	1.8	3.9	6	1.7	4.1
24	2.9	0.3	1.7	3	0.3	1.7	49	2.5	0.3	1.4	2.6	0.3	1.5
25	2.6	0.3	1.5	2.6	0.3	1.5	50	3	0.3	1.7	3	0.3	1.7

The Kolmogorov Smirnov test at (0.05) significance level was conducted to purpose of knowing the distribution of the six machines operating times, the hypotheses were as follows:

H_0: The operating times for the machine is follow the target distribution

The results in table (9) below indicate to accept the above null hypotheses as the p-values are greater than the level of significance,

Table (9) Kolmogorov Smirnov data tests

Machine	K-S	p-value	Machine	K-S	p-value
1^{st} Machine	0.92634	0.14731	3^{rd} Machine	0.92507	0.14995
2^{nd} Machine	0.93647	0.12716	5^{th} Machine	0.93277	0.13454
4^{th} Machine	0.92602	0.14802	6^{th} Machine	0.93904	0.12197

The reliability functions for the six machines and the reliability functions of the hybrid systems the serial-parallel and the reliability function of the proposed hybrid system, i.e. the parallel-serial system for the two stages of cutting, printing and packaging have been estimated by using the ML method, the results are shown in table (10) below.

Table (10) Estimated Reliability Functions

t	$\hat{R}_1$	$\hat{R}_2$	$\hat{R}_3$	$\hat{R}_4$	$\hat{R}_5$	$\hat{R}_6$	$\hat{R}_S$	$\hat{R}_{SS}$
2	0.80371	0.50198	0.87595	0.80545	0.51196	0.91563	0.59754	0.94813
3	0.61162	0.33543	0.71693	0.61459	0.34606	0.75208	0.28351	0.74579
4	0.41726	0.25199	0.58488	0.42087	0.26211	0.61568	0.12524	0.50731
5	0.2552	0.20185	0.46860	0.25866	0.21129	0.49523	0.05055	0.31334

Conclusions:

Through the reliability function values of the six machines, it is clear that the probability that the shredding machines works for four hours without stopping is 50%, and that the probability that the printing machines may works for three hours without stopping are 50%, while that the crushing machines works for only one hour then stopped is 50%, that indicates a clear problem of the working system, because the system that followed by the printing press management, i.e. the hybrid serial-parallel system is an useless, where the probability that machines or the system may stopped after the first operating hour is 50%, but if the blistering hybrid system is followed, i.e. two similar machines are run in parallel and different machines are run in series, then the probability that the system will works for four hours is more than 50%.

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مقارنة وتطبيق المعّولية ونظام المعّولية ووظائف نظام المعّولية المختلطة لتوزيعات رايلي وباريتو ورايلي باريتو

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المستخلص :
تناولت هذه الدراسة تحليل دالة المعّولية للأنظمة الهجينة ذات المكونات غير المتماثلة ، حيث تتبع أوقات تشغيل هذه المكونات توزيعات Rayleigh و Pareto و Rayleigh Pareto ، علاوة على ذلك ، قدمنا طريقة تشغيل تعتمد على الأنظمة الهجينة من أجل زيادة موثوقية أوقات تشغيل الآلات. تم إجراء تجارب محاكاة مختلفة لتقدير دالة المعّولية لهذه الأنظمة باستخدام طريقة ML ، ثم المقارنة بين تلك المقدرات ، وأثبتت نتائج المحاكاة أن توزيع Rayleigh-Pareto يعطي أفضل تقدير لوظائف المعّولية ، وكذلك القيم المقدرة لـ دالة المعّولية لنظام السلاسل المتوازية الهجينة أكبر من القيم المقدرة لوظيفة المعّولية للنظام الهجين المتوازي ، وأخيرًا ، فإن قيم دالة المعّولية المقدرة للنظام المقترح أفضل من القيم المقدرة للنظامين الآخرين. في الجزء التطبيقي تم اختيار اوقات التشغيل لماكينات الدار الجامعية للطباعة والنشر والترجمة في بغداد حيث تعتمد ادارة المصنع على النظام الهجين المتسلسل المتوازي لتشغيل مكائن المصنع وهذا النظام يؤدي إلى تقليل معّولية أوقات تشغيل آلات المصنع إلى 50٪ في ساعة التشغيل الأولى ، ولكن إذا تم اتباع النظام الهجين المتقرح ، أي يتم تشغيل الآلات المماثلة بالتوازي ، ويتم تشغيل الآلات المختلفة في سلسلة ، عندها يكون احتمال استمرار عمل نظام الماكينة لمدة أربع ساعات 50٪.

الكلمات المفتاحية : توزيعات رايلي ، باريتو ، رايلي - باريتو ، وظائف المعّولية للأنظمة الهجينة: متسلسلة - متوازية ومتوازية - متسلسلة.

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