On-Line Tuning Sliding Mode Controller Design for Nonlinear Inverted Pendulum System based on Bees Algorithm

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ABSTRACT
This paper proposes a modified swing control for a nonlinear inverted pendulum system by utilizing the sliding mode controller based on the on-line tuning Bees algorithm as speed of optimization and accuracy of results. The goal of the proposed nonlinear controller is to obtain the optimal force control action for the pendulum cart in order to stabilize the pendulum in the inverted position precisely and quickly. The optimal parameters of the nonlinear controller are on-line tuned by Bees algorithm and guided by Lyapunov stability criterion to reduce the amplitude of the sliding mode signum function in order to eliminate the chattering phenomena and make the smoothness control action. Matlab simulation results confirm the validity of the proposed controller algorithm in terms of fast dynamic response, minimizing the pendulum's angle tracking error to the zero radian at 2.5 second and obtaining the optimal and smooth force control action without saturation state, with the minimum number of fitness evaluation of the algorithm.

Keywords: Nonlinear Inverted Pendulum System; Sliding Mode Controller; Bees Algorithm.

INTRODUCTION
An inverted pendulum system considers a highly nonlinear dynamic and unstable system and the most fundamental problems in the control engineering is the stabilization of the inverted pendulum [1]. In general, it is still active region of research because it has evolutionary application in various industries such as rocket technology, missile guidance, air planes, ships, mobile robots and automobiles [2].
Therefore, various control algorithms are proposed to solve the stabilization problems of pendulum in the inverted position for inverted pendulum system, such as PID with sliding mode controllers [1, 2 and 3], fuzzy with sliding mode controllers [4], adaptive-fuzzy with sliding mode controllers [5], neural networks with sliding mode controller [6] and second-order sliding mode controller [7].

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The motivation for this work is [1, 5 and 7] to stabilize the pendulum in upright position; to generate optimal force control action and to overcome the chattering effect of the fast switching surface of the sliding mode control.

This paper summarizes the fundamental essences of the contribution by modifying the analytical derive for the nonlinear swing control law based on sliding mode control with Lyapunov criterion and on-line tuning control gains by using Bees algorithm. This derive is used to obtain the best force control action quickly without saturation state that leads to fast dynamic response and minimizing the tracking error of the pendulum's angle to the zero radian in inverted position.

The following section contains the description of the nonlinear inverted pendulum system in section two. Section three derives the proposed nonlinear swing controller based on sliding mode control and on-line tuning Bees algorithm. Section four shows the simulation results of the proposed controller and the last section gives the conclusions of this paper

Inverted Pendulum System

In this study, the inverted pendulum system shown in Fig. 1 consists of a pendulum that is attached to the side of a cart by means of a pivot which allows the pendulum to swing in the *xy*-plane and a force $u(t)$ that is applied to the cart in the *x* direction, with the purpose of keeping the pendulum balanced upright position [7].

Assume that the pendulum be modeled as a thin rod and applying Newton's second law to the linear and angular position, the equations of motion for nonlinear inverted pendulum system are expressed as follows [3 and 7]:

$$(M+m)\ddot{x} + b\dot{x} - mlsin(\theta)\dot{\theta}^2 + mlcos(\theta)\ddot{\theta} = u(t) \quad \ldots(1)$$

$$m\ddot{x}\cos(\theta) + ml\ddot{\theta} = mgsin(\theta) \quad \ldots(2)$$

Where:

m: is the mass of pendulum's bob (kg).

M: is the mass of the cart (kg).

l: is the pendulum's length (m).

θ: is the angle between the pendulum and its upright position (radian).

x: is the position of the card (m).

g: is the gravitation constant (m/sec^2).

b: is the friction of the cart (N/m/sec).

u: is the force applied to the cart (N).

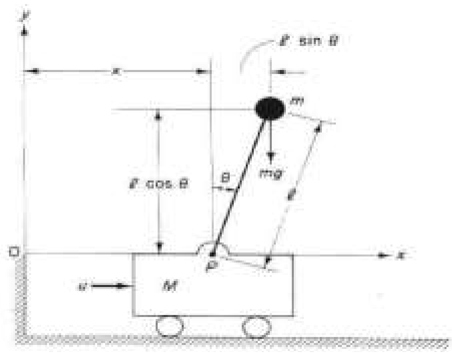


Figure (1) : Inverted Pendulum System [7].

From equations (1) and (2), a nonlinear state – space model can be derived as follows:

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix}=\begin{bmatrix}\dot{\theta}\\ f_1(\theta,\dot{\theta},x,\dot{x},u)\\ \dot{x}\\ f_2(\theta,\dot{\theta},x,\dot{x},u)\end{bmatrix} \quad ...(3)$$

In order to find the $f_1(-)$ and $f_2(-)$ equations, firstly solving equation (2) for $\ddot{x}$ and $\ddot{\theta}$ as follows:

$$\ddot{x}=\frac{gsin(\theta)-l\ddot{\theta}}{\cos(\theta)} \quad ...(4)$$

$$\ddot{\theta}=\frac{gsin(\theta)-\ddot{x}\cos(\theta)}{l} \quad ...(5)$$

Inserting equations (4 and 5) into equation (1):

$$mlcos(\theta)\ddot{\theta}=u(t)-(M+m)\ddot{x}-b\dot{x}+mlsin(\theta)\dot{\theta}^2 \quad \ldots(6)$$

$$mlcos(\theta)\ddot{\theta}=u(t)-(M+m)\frac{(gsin(\theta)-l\ddot{\theta})}{\cos(\theta)}-b\dot{x}+mlsin(\theta)\dot{\theta}^2 \quad \ldots(7)$$

$$mlcos(\theta)\ddot{\theta}-(M+m)\frac{l\ddot{\theta}}{\cos(\theta)}=u(t)-(M+m)\frac{gsin(\theta)}{\cos(\theta)}-b\dot{x}+mlsin(\theta)\dot{\theta}^2 \quad \ldots(8)$$

$$\ddot{\theta}(mlcos(\theta)-(M+m)\frac{l}{\cos(\theta)})=u(t)-(M+m)\frac{gsin(\theta)}{\cos(\theta)}-b\dot{x}+mlsin(\theta)\dot{\theta}^2 \quad ...(9)$$

$$\ddot{\theta}(mlcos^2(\theta)-(M+m)l)=u(t)cos(\theta)-(M+m)gsin(\theta)-bcos(\theta)\dot{x}+mlcos(\theta)sin(\theta)\dot{\theta}^2 \quad \ldots(10)$$

$$\ddot{\theta}=\frac{u(t)cos(\theta)-(M+m)gsin(\theta)-bcos(\theta)\dot{x}+mlcos(\theta)sin(\theta)\,\dot{\theta}^2}{(mlcos^2(\theta)-(M+m)l)} \quad \ldots(11)$$

$$\ddot{\theta}=\frac{u(t)cos(\theta)}{ml(cos^2(\theta)-1)-Ml}+\frac{-(M+m)gsin(\theta)-bcos(\theta)\dot{x}+mlcos(\theta)sin(\theta)\ \dot{\theta}^2}{ml(cos^2(\theta)-1)-Ml} \quad \ldots(12)$$

Then,

$$f_1(\theta,\dot{\theta},x,\dot{x},u) = \frac{u(t)cos(\theta)}{ml(cos^2(\theta)-1)-Ml} + \frac{-(M+m)gsin(\theta)-bco\,s(\theta)\dot{x}+mlcos(\theta)sin(\theta)\,\dot{\theta}^2}{ml(cos^2(\theta)-1)-Ml} \quad \ldots(13)$$

$$(M+m)\ddot{x} = u(t) - b\dot{x} + mlsin(\theta)\dot{\theta}^2 - mlcos(\theta)\ddot{\theta} \quad \ldots(14)$$

$$(M+m)\ddot{x} = u(t) - b\dot{x} + mlsin(\theta)\dot{\theta}^2 - mlcos(\theta)(\frac{gsin(\theta)-\ddot{x}\cos(\theta)}{l} \quad \ldots(15)$$

$$\ddot{x}((M+m) - mcos^2(\theta)) = u(t) - b\dot{x} + mlsin(\theta)\dot{\theta}^2 - mgcos(\theta)sin(\theta) \quad \ldots(16)$$

$$\ddot{x}((M+m(1-cos^2(\theta))) = u(t) - b\dot{x} + mlsin(\theta)\dot{\theta}^2 - mgcos(\theta)sin(\theta) \quad \ldots(17)$$

$$\ddot{x} = \frac{u(t)-b\dot{x}+mlsin(\theta)\;\dot{\theta}^2-mgcos(\theta)sin(\theta)}{M+m(1-cos^2(\theta))} \quad \ldots(18)$$

$$\ddot{x} = \frac{u(t)}{M+m(1-cos^2(\theta))} + \frac{-b\dot{x}+mlsin(\theta)\,\dot{\theta}^2-mgcos(\theta)sin(\theta)}{M+m(1-cos^2(\theta))} \quad \ldots(19)$$

Then,

$$f_2(\theta,\dot{\theta},x,\dot{x},u) = \frac{u(t)}{M+m(1-cos^2(\theta))} + \frac{-b\dot{x}+mlsin(\theta)\;\dot{\theta}^2-mgcos(\theta)sin(\theta)}{M+m(1-cos^2(\theta))} \quad \ldots(20)$$

From $f_1(-)$ and $f_2(-)$, the system parameters variation and the uncertainty make the inverted pendulum effect on the existence of the non-linearity degree thus, there is a necessary need for the swing control algorithm, which enhances the overall nonlinear system performance.

Swing Control Algorithm Design

The advantages of modified a swing control algorithm for the inverted pendulum can be described as follows:

- Find and tune the optimal stable control parameters of the nonlinear controller based on sliding mode and Lyapunov stability criterion in order to track the desired angle of the inverted pendulum precisely and quickly by generating optimal force control action with eliminating undesirable chattering oscillations.
- Overcome the parameter variation and the uncertainty of the nonlinear system by applying bee algorithm as a powerful, fast and stable on-line tuning with minimum number of fitness evaluation technique.

The structure of the proposed nonlinear swing controller consists of two parts:

a) The nonlinear sliding mode control.

b) On-line tuning algorithm.

Fig. 2 shows the structure of the proposed nonlinear swing controller for inverted pendulum system.

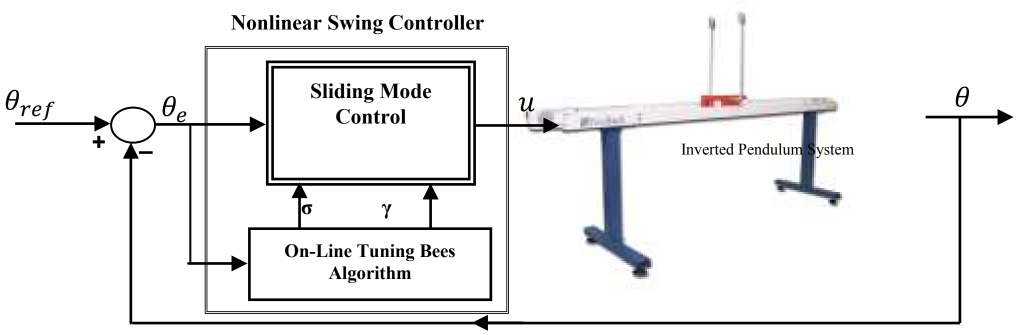


Figure (2): The Proposed Nonlinear Swing Controller Structure.

The Sliding Mode Control

The main core of the most significant matter of the nonlinear controller is to find the adaptive sliding mode control law which is responsible for generating optimal and smooth force control action with high control activity that it eliminates undesirable chattering and minimizes the tracking angle error when the pendulum's angle drifts from the desired angle in the inverted position.

The proposed sliding mode control law for the nonlinear swing controller is based on on-line tuning technique which can be derived the sliding surface which is defined as:

$$S = \sigma(\theta - \theta_{ref}) + \dot{\theta} \qquad \ldots(21)$$

Where:

σ: is the on-line positive scalar parameter tuning.

θ_{ref} : is the desired angle.

This means that the pendulum's angle converges to desired angle (θ_{ref}= 0 rad) at t=∞ and put the pendulum position to the upright position.

In order to confirm the proposed swing control law is asymptotically stable at closed loop nonlinear system, Lyapunov criterion is used because this criterion represents the simple and successful method in finding the nonlinear system stability, thus the constructive Lyapunov function is described as follows [7]:

$$V = 0.5S^2 \qquad \ldots(22)$$

The time derivative of equation (22) becomes:

$$\dot{V} = S\dot{S} = S(\sigma\dot{\theta} + \ddot{\theta}) \leq 0 \qquad \ldots(23)$$

$$\dot{S} = (\sigma\dot{\theta} + \ddot{\theta}) = \gamma sgn(S) \qquad \ldots(24)$$

Where

γ : is the on-line tuning positive scalar parameter that guarantees the system trajectory hits the sliding surface at t=∞.

$sgn(S)$: is the signum function, can be defined as:

$$sgn(S) = \begin{Bmatrix} 1 & S(\theta) > 0 \\ 0 & S(\theta) = 0 \\ -1 & S(\theta) < 0 \end{Bmatrix} \quad \ldots(25)$$

In order to derive the proposed swing control law that depends on the sign of sliding surface and the nonlinear system. Substituting equations (13) into equation (24), then the following equation can be derived:

$$\sigma\dot{\theta} + f_1(\theta, \dot{\theta}, x, \dot{x}, u) = \gamma sgn(S) \quad \ldots(26)$$

$$\sigma\dot{\theta} + f(-) + u(t)g(-) = \gamma sgn(S) \quad \ldots(27)$$

Where

$$f(-) = \frac{-(M+m)gsin(\theta) - bcos(\theta)\dot{x} + mlcos(\theta)sin(\theta)\ \dot{\theta}^2}{ml(cos^2(\theta)-1) - Ml} \quad \ldots(28)$$

$$g(-) = \frac{cos(\theta)}{ml(cos^2(\theta)-1) - Ml} \quad \ldots(29)$$

Then, the proposed nonlinear swing control law based on sliding mode technique for $\lim_{t\to\infty}(\theta_{ref} - \theta) = 0$ in order to make the nonlinear system asymptotically stable and to find a smooth force control action as follows:

$$u(t) = \frac{\gamma sgn(S) - \sigma\theta^{\cdot} - \frac{-(M+m)gsin(\theta) - bcos(\theta)\dot{x} + mlcos(\theta)sin(\theta)\ \dot{\theta}^2}{ml(cos^2(\theta)-1) - Ml}}{\frac{cos(\theta)}{ml(cos^2(\theta)-1) - Ml}} \quad \ldots(30)$$

On-Line Tuning Algorithm

In this paper, the Bees algorithm is used to find and tune the control gains (σ and γ) of the sliding mode controller for nonlinear inverted pendulum system. In general, this algorithm mimics the food foraging behavior of swarms of honey bees and this algorithm carries out by using two researches types (neighborhood and random), so it is a simple and fast optimization with accuracy results [8, 9, 10 and 11].

The main steps of the Bees algorithm as follows:

- Generated randomly values of the control gains as the initial population for (n) Scout Bees.
- Evaluated the fitness of the (n) population. The fitness equation is [12]:

$$fitness = \frac{1}{\mu + objective\ function} \quad \ldots(31)$$

Where:

μ: is a constant value and large than zero to avoid division by zero.

Objective function: is mean square error as equation (32) .

$$MSE = \frac{1}{N}\sum_{i=1}^{N}[(\theta_{ref} - \theta)^2] \quad \ldots(32)$$

N: is the number of iteration.

- Chosen the highest fitness for the Scout Bees (population) as (m) Selected Bees in order to use in neighborhood search as a local search.
- Determine the size of neighborhood (patch size) by applying the proposed equations (33 - 36) as follows:

$$\Delta\sigma = \sigma_{old} + 0.01 \times \sigma_{old} \times random(0,1) \quad \text{…(33)}$$

$$\sigma_{new} = \sigma_{old} + \Delta\sigma \quad \text{…(34)}$$

$$\Delta\gamma = \gamma_{old} + 0.01 \times \gamma_{old} \times random(0,1) \quad \text{…(35)}$$

$$\gamma_{new} = \gamma_{old} + \Delta\gamma \quad \text{…(36)}$$

- Generated Recruit Bees depend on equations (34 and 36) more Bees in order to search near to the best control gains (σ and γ).
- Choose the highest fitness for the Recruit Bees as Fittest Bees by using equation (31).
- Assign the (n-m) remaining Bees to random search as global search and new population of Scout Bees are generated.

These steps of the optimization algorithm for the control gains are repeated for each sample (on-line), where the sampling time is equal to 0.01 second in order to reduce the effect of the (ZOH) Zero Order Hold or (ADC) Analog to Digital Converter in the real-time control system depends on Shannon theorem.

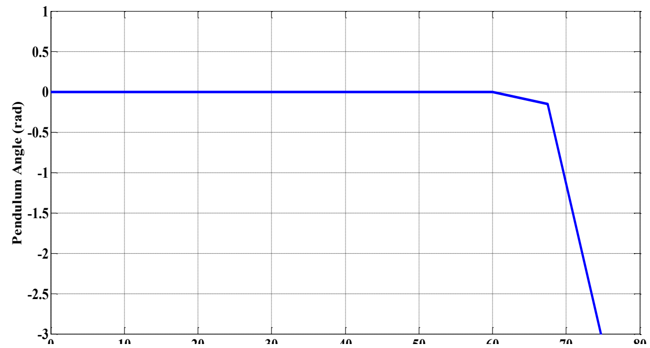
Simulation Results

The swing control algorithm for the nonlinear inverted pendulum system is verified by means of the MATLAB computer simulation package (m.file) after solving of nonlinear dynamic inverted pendulum model equations by applying the finite difference method with sampling time equal to 10mSec.The parameter values of the inverted pendulum system are taken from [13] as shown in table 1.

Table (1). The parameter values of the inverted pendulum system [13].

Description	Symbol	Value	Unit
The mass of pendulum's bob	m	0.23	kg
The mass of the cart	M	2.4	kg
The pendulum's length	l	0.36	m
The friction of the cart	b	0.1	N/m/sec
The gravitation constant	g	9.81	m/sec^2

To show the nonlinear dynamic behavior of the inverted pendulum system, Fig. 3 shows the open loop step response of the pendulum's angle for the system. The swing response of the pendulum's angle is fast and unstable; therefore, it is necessary to design the nonlinear swing controller as shown in Fig. 2, in order to stabilize the pendulum in the inverted position and prevent the pendulum from falling on the pendulum's cart and achieve the suitable dynamic behavior response of the pendulum's angle.



Figure(3): Open Loop Inverted Pendulum System Step Response.

To carry out the proposed swing control algorithm, there are many parameters of the algorithm should be defined as follows:

The number of the Scout Bees (n) is equal to 10.

The number of Selected Bees (m) is equal to 4.

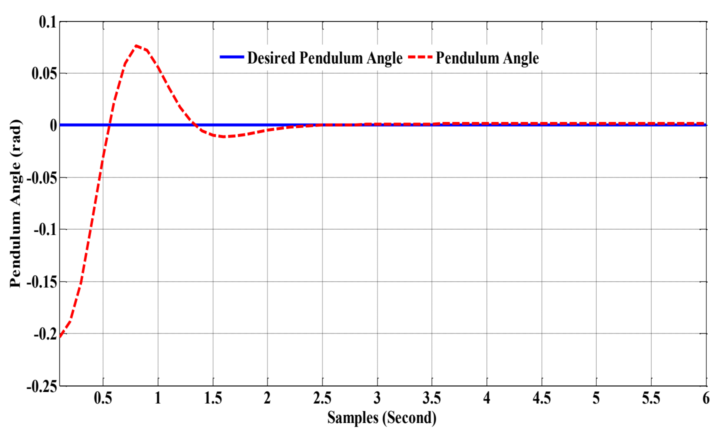
The number of Recruit Bees is equal to 20.

The number of Fittest Bees is equal to 4.

The number of weights in each Bee is equal to 2 because the nonlinear swing controller has two control gains (σ and γ).

The number of iterations (N) is equal to 25.

The response of the output pendulum's angle of the system with initial angle equal to -0.2 radian, as shown in Fig.4 ; it has a time response specification of 0.48 sec rising time, 2.5 sec settling time and at 0.79 sec the maximum overshoot at transient state equal to 0.078 radian then at steady-state the output pendulum's angle approaches to zero radian when the desired pendulum's angle is zero radian at inverted position.



Figure(4): Pendulum Angle Output for Inverted Pendulum System.

Fig. 5 represent simulation results of the closed loop time response of the pendulum's cart position that it is started from zero position with maximum movement equal to 14 cm and the cart is returned to zero position at 2.81 second.

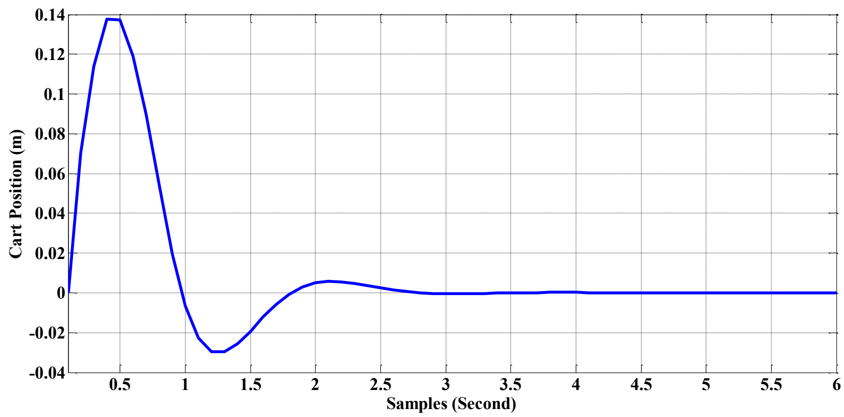


Figure (5): Pendulum Cart Output for Inverted Pendulum System.

The angle error between the desired angle of the pendulum at inverted position and the angle of the inverted pendulum system output is shown in Fig. 6. The maximum error is 0.2 radian at the transient state response, while at steady-state the error approximates to zero radian that means the pendulum is stabilized at inverted position.

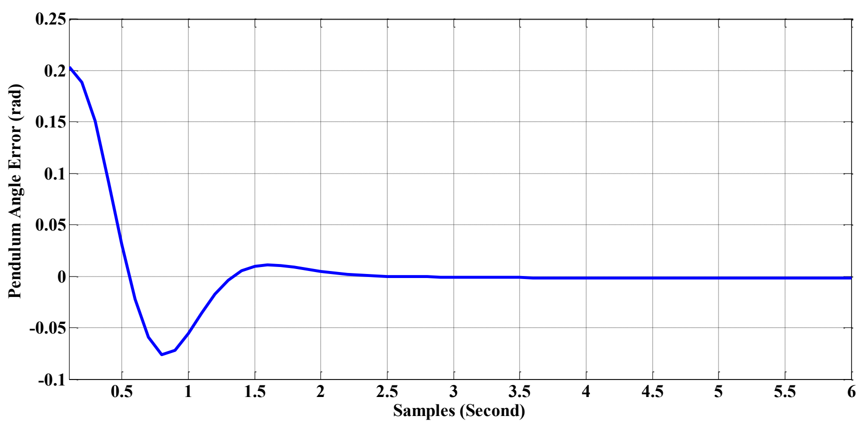
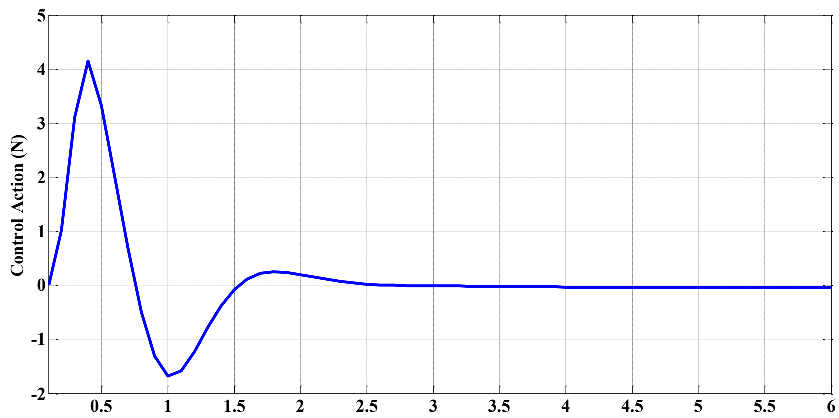


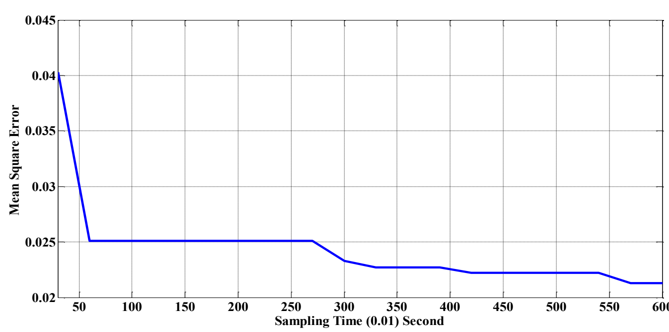
Figure (6): Pendulum Angle Error Output for Inverted Pendulum System.

The on-line tuning nonlinear swing control action response is shown in Fig. 7 that it has maximum value of 4.1 Newton and minimum value of -1.68 Newton at transient state in order to keep the position output of pendulum in the inverted position and minimize tracking angle error of the inverted pendulum system.



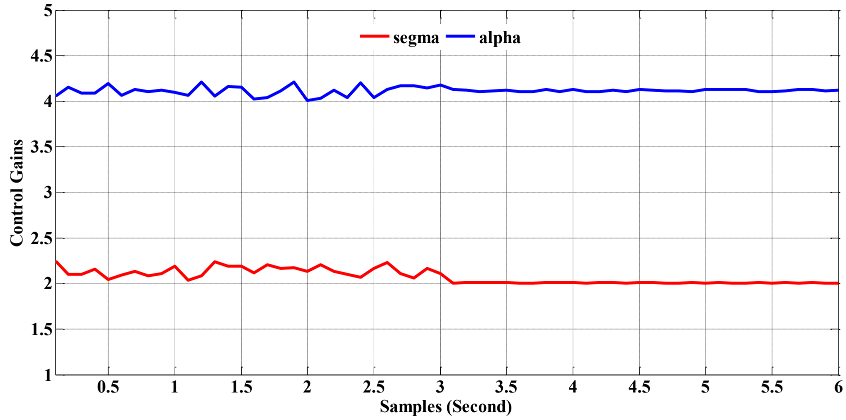
Figure(7): Force Control Action for Inverted Pendulum System.

The on-line performance index MSE for angle and angular velocity errors for the inverted pendulum motion at each sample is shown in Fig. 8.



Figure(8): The Performance Index of the On-Line Tuning Algorithm.

Fig. 9 shows the optimal control gains parameters σ and γ at each sample of the nonlinear swing controller that has been tuned on-line based on Bees algorithm.



Figure(9): On-Line Control Gains.

The effects of the on-line tuning swing control algorithm on the control gains σ and γ with MSE can be shown in Fig. 10. It is observed the best value of σ equal to 2.05 and the best value of γ equal to 4.22, these values lead to minimum mean square error which it is equal to 0.022.

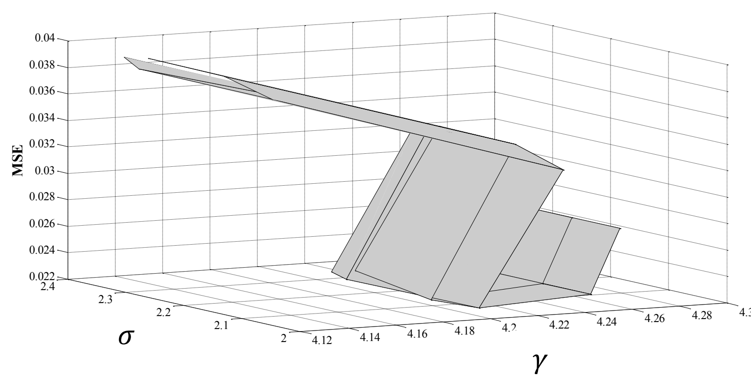
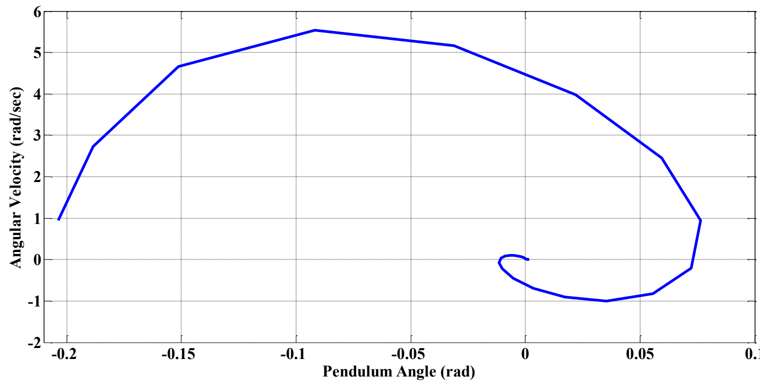


Figure (10): The Effects of the Control Gains with MSE.

The stability of the closed loop control inverted pendulum system based on nonlinear swing controller is observed by phase-plane plot, as shown in Fig. 11 and it is clearly that chattering has been eliminated.



Figure(11): Inverted Pendulum System trajectory on the (θ,$\dot{\theta}$) plane.

CONCLUSIONS

The numerical results of the Matlab simulation on the on-line tuning nonlinear swing control based on sliding mode control with Bees algorithm which has been presented in this paper for the nonlinear inverted pendulum system show that the proposed swing control algorithm has the following properties:

- It has fast, accuracy and stable on-line tuning control gains with minimum number of fitness evaluation.
- It has active to minimize the tracking angle error of the pendulum to track the inverted position.
- It has capability in generating smooth and optimum force control action without saturation state.
- It has efficiency in reduction and elimination the chattering effect or the switching surface of the sliding mode control.

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