Robust PI-PD Controller Design for Systems with Parametric Uncertainties

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ABSTRACT
This paper presents a robust design of the four parameters PI-PD controller for systems with parametric uncertainties. The Particle swarm Optimization (PSO) method is applied to tune the controller parameters such that the robust specifications are satisfied. For the robust stability and performance to be guaranteed, the Kharitonov's theorem of interval polynomials is combined with the time domain performance index. The effectiveness of the proposed controller is illustrated by two examples of uncertain systems.
Keywords: PI-PD controller, uncertain systems, parametric uncertainty, PSO, interval systems

INTRODUCTION
In process control fields, the PID controllers make up 90% of automatic controllers [1]. It is also necessary for the total energy saving system or the model predictive control system to operate appropriately, the PID control is absolutely essential. The PID controller is used for a wide range of problems: process control, motor drives, magnetic and optic memories, automotive, flight control, instrumentation, etc. The PID controller can deal with the important practical problems such as actuator saturation and integrator windup [2]. On the other hand, wide spectrums of choices for control schemes are offered by the science of automatic control because of the advances in digital technology. However, most of the industrial controllers are still implemented based on PID algorithms, particularly at lowest levels, as no other controllers match the simplicity, clear functionality, applicability, and ease of use offered by the PID controller [3]. The accurate and efficient tuning of parameters is the key issue for PID controllers. The nonlinearity, uncertainty and time delay are known features of the controlled systems, which make controller parameters tuning more complex. Thus, the goal of tuning the PID controller is to determine the parameters that meet the closed-loop system performance specifications over a wide range of operating conditions [4]. Several methods for obtaining the controllers parameters have been developed during the last years such as Ziegler-Nichols tuning method, Cohen-Coon method, Astrom-Hagglund method, Internal Model Control (IMC) design approach, gain and phase margins based, Genetic Algorithm (GA), Particle Swarm Optimization (PSO) and Ant Colony optimization (ACO).

Since, the most of real system models are uncertain, and the design procedure of the conventional PID controller is based on a plant with fixed parameters, a PI-PD controller is used. This controller can compensate the system by ensuring an appropriate location of the open loop stable system poles for resonant or integrating processes. Consequently, the PI-PD controller has an advantage over the PID controller in dealing with uncertain systems [5].

In this work, the four parameters of PI-PD controller are tuned using Particle Swarm Optimization (PSO) method subject to the performance index in time domain specification. To guarantee the system stability, the parameters uncertainties are compensated during the

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optimization process. The Kharitonov's theorem of interval polynomials is used to examine the robust stability of the closed loop characteristic polynomial.

Stabilization of Uncertain System

To bound the uncertain parameters in transfer functions, state space and differential equations, the intervals are used to characterize the uncertainties and the system is called an interval system [6]. The objective of the control design is to find a controller that stabilizes the given interval system. Consider the block diagram of the system with PI-PD controller shown in Figure 1. The transfer function of the uncontrolled system and the transfer functions of PI and PD controllers are defined respectively as [7]:

$$(s) \quad \frac{\ \ _p(s)}{\ \ _p(s)} \qquad \ldots(1)$$

$$(s) \qquad - \qquad \ldots(2)$$

$$(s) \qquad \ldots(3)$$

where $N_p(s)$, (s) are the system transfer function numerator and denominator polynomials, is the proportional feedforward gain, is the integral gain, is the proportional feedback gain and is the derivative gain.

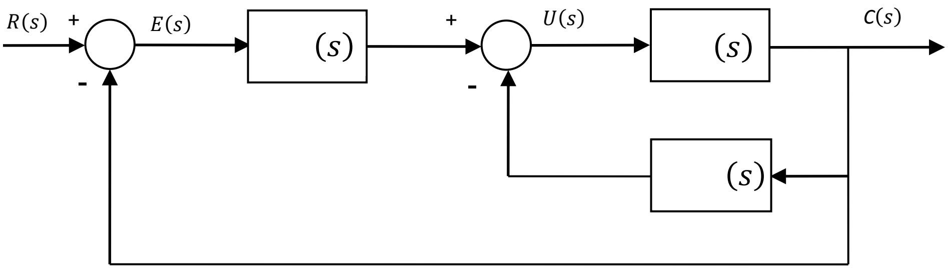


Figure (1): Block diagram of PI-PD controller with a system.

The problem is to find the parameters of the PI-PD controller that maintains the stabilization of the uncertain system described by interval polynomials with independent uncertainties in the coefficient. An interval polynomial can be expressed by [6]:

$$P(s,k) = \sum_{i=0}^{n}\left[k_i^{min}, k_i^{max}\right] s \qquad \ldots(4)$$

Thus to investigate the stability of the interval system, the Kharitonov theorem is applied. This theorem uses four Kharitonovs' polynomials which are constructed by means of upper and lower bounds of the interval coefficients. For the internal polynomial expressed by [8]:

$$P(s,k) = k_0 + k_1 s + k_2 s^2 + k_3 s^3 + \cdots + k_n s^n , \qquad \ldots(5)$$

where $k_i \in [k_i^{min}, k_i^{max}]$, i =1, 2, ..., n.

The stability of the set can be obtained by applying the Routh criterion to the following four Kharitonov polynomials:

(s)

(s)

(s)

$$(s) \qquad \ldots(6)$$

Consider the control system in Figure 1 with an uncertain transfer function of the form:

$$(s) \quad \frac{\ \ (s)}{\ \ _p(s)} \quad \frac{\sum_{i=0}^{m}\left[a_i^{min}, a_i^{max}\right] s}{\sum_{i=0}^{n}\left[b \qquad ^{max}\right] s} \qquad \ldots(7)$$

and

$(s)\quad \frac{_{PI}(s)}{_{PI}(s)}$, …(8)

$(s)\qquad (s)$ …(9)

where (s) and (s), represent the numerator and denominator of PI controller and represents the polynomial of PD controller. Consequently, the overall closed loop transfer function is:

$$\frac{C(s)}{R(s)}\quad \frac{_{PI}(s)N_P(s)}{_{PI}(s)D_P(s)+\ _{PI}(s)N_P(s)P_{PD}(s)+\ _{PI}(s)N_P(s)}\quad \frac{_{CL}(s)}{_{CL}(s)}\quad (s) \qquad \dots(10)$$

and $\frac{_{CL}(s)}{_{CL}(s)}$ ——————

where $q's$ represent the closed loop numerator coefficients and $r's$ represents the closed loop denominator coefficients. The PI-PD controller parameters (k) are included in the overall closed loop numerator and denominator coefficient ($N\ \ (s)$ and $D_{CL}(s)$). That is, all or some of the coefficients q'_s and are functions of the PI-PD controller parameters. Using the Kharitonov's polynomials in equation (6), a sixteen Kharitonov plants family (each one of the four closed loop numerators is taken with the four closed loop denominators) are obtained. For the closed loop numerator $N_{CL}(s)$ and denominator$D_{CL}(s)$, the Kharitonov polynomials are:

(s)

(s)

(s)

(s) …(11)

and

(s)

(s)

(s)

(s) ...(12)

When all combinations of $N_{CL_i}(s)$ and (s) for i, j = 1, 2, 3, 4 are taken, the following sixteen Kharitonov's plants family are obtained

$(s) = \frac{_{CL_i}(s)}{_{CL}\ (s)}$ …(13)

The interval plant described by equation (13) is robustly stable if and only if all the Kharitonov's plants are stable, that is, the polynomials roots have strictly negative real parts.

Controller Design

PSO Algorithm

The PSO method is one of the latest population based optimization method. Recently, it has attracted a lot of attention because of its computational efficiency. In this method, the individuals are called "particles" and the particle is treated as a point in an n-dimensional space. In the problem space, each particle will keep track of its coordinates which are associated with the best solution which is called *pbest* that has achieved so far. There is another best value which is obtained by any particle in the neighbors of the particle and called *gbest*. The velocity of each particle is changing toward its *pbest* and *gbest* position. The particles are updated according to the following equations of motion [9, 10]:

…(14)

$\times \left(x_i^b - x_i^k\right) + c_2 \times rand \times (x_i^g - x_i^k)$ …(15)

where is the particle velocity, x_i^k is the current particle position, w is the inertia weight, and are the best value and the global best value, is a random function between 0 and 1, and are learning factors.

Controller Parameters Tuning

In this work, the PSO algorithm is applied to tune the parameters of PI-PD controller such that the desired robust stability and performance specifications are satisfied. An optimal set of controller parameters $k_{p_1}, k_i, k_{p_2}, k_d$ can yield a system response and result in minimization of performance index in time domain. The summation of the integral time square errors of the generated plants due to the parameters variation is used to achieve a desirable time response specifications. Thus the suitable performance index (objective function) is selected to be:

$$\sum \qquad \int \quad (t)dt \qquad \ldots(16)$$

where h represents the number of plants generated through the variation of system parameters, represents estimated settling time and $e(t)$ represents the system error.

On the other hand, in order to guarantee the robust stability, the Kharitonov's robust stability criterion is applied during the optimization process to examine the robust stability of the system with parameter uncertainty. The tuning procedure using PSO algorithm can be summarized by:

1- Specify the number of particles (swarm size) and the number of iterations.
2- Initialize random particle position and velocity.
3- Find the system response for each particle then calculate the objective function *J*.
4- Apply the Kharitonov's test
 if there are positive closed loop poles, go to step 6
5- Compare the objective function of each particle with its local best then the best evaluated value among locals is set as global best.
6- Update the position and velocity of each particle according to equations (14) and (15).
7- If the maximum iteration is reached, go to step 8, otherwise go to step 3.
8- The global best is a set of optimal PI-PD controller parameters.

Illustrative Examples

To illustrate the effectiveness of the proposed controller, two illustrative examples have been considered in this section. The first example is unstable and uncertain system with time delay while the second example is an integrating system with uncertain parameters.

Example 1: Consider the control system in Figure 1 with an uncertain system transfer function represented by:

$$(s) \quad \frac{_p(s)}{_p(s)} \quad \frac{\quad}{\quad} \qquad \ldots(17)$$

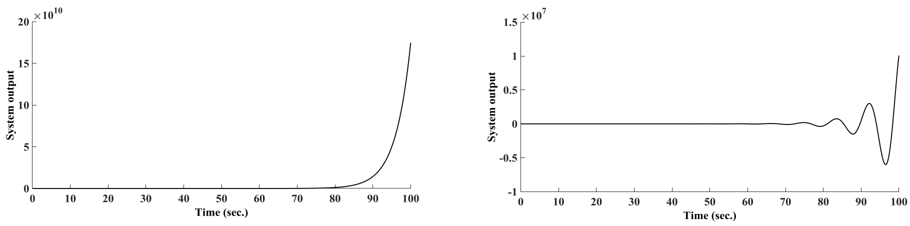
which is an unstable first order plus time delay transfer function, where $\in$ [1.5 2], $k \in$ [3.8 4.2], $T \in$ [3.8 4.2]. When a second order Pade approximation is applied to represent the system time delay, the system transfer function $G_p(s)$ becomes:

$$(s) \quad \frac{\quad}{+(6 \quad _d^2)s \; +(12 \quad _d)s} \qquad \ldots(18)$$

It is shown that the numerator and denominator coefficients of the system transfer function in equation (18) contain the product of two uncertain parameters k and T_d or T andT_d. In addition, since the parameter T_d enters in $N_p(s)$ and$D_p(s)$, this means that there is a coupling between numerator and denominator coefficients. That is, the numerator and denominator parameters uncertainties are dependent and the coefficients of the numerator and denominator are changing dependently within given intervals. Figure 2 shows the time response specifications of the system before applying the controller. It shows that the system is unstable in open loop and closed loop cases. The step response of the nominal and uncertain controlled systems and the resulting control signal is shown in Figure 3. The obtained optimal parameters of the controller are:$k_d = 0.4531$,$k_{p_2} = 0.5292$,$k_{p_1} = 0.0354$,$k_i = 0.035$. It is shown that the proposed PI-PD controller with the obtained parameters can compensate the system parameter uncertainties with a desirable time response specifications in comparison to those obtained by [5]. Moreover, It shows that a low control effort has been achieved. The minimum cost function which was achieved is 215.3065. The following parameters have been selected by experience for carrying out the design of PI-PD controller using PSO: w=2,$c_1 = c_2 = 2$. The swarm size is set to 100. It was shown that setting the number of iterations to 100 result in obtaining the best optimal cost function, where increasing the number of iterations did not improve the convergence of the PSO algorithm. Table 1 compares the resulting time response specifications with previous work.

Table (1): Performance comparison

Controller	$(se\ .)$	$(sec.)$	%
PI-PD [5]	8	23	8
Proposed PI-PD controller	15	20	0



(a) open loop (b) closed loop

Figure (2): Time response specifications of the system

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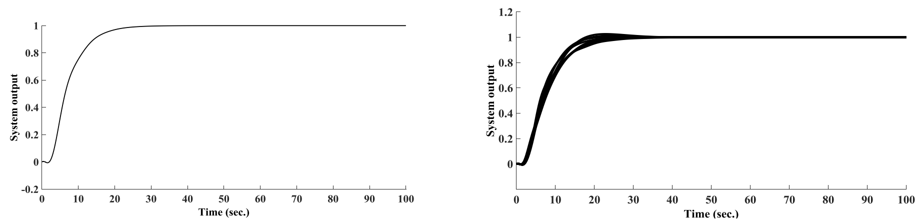
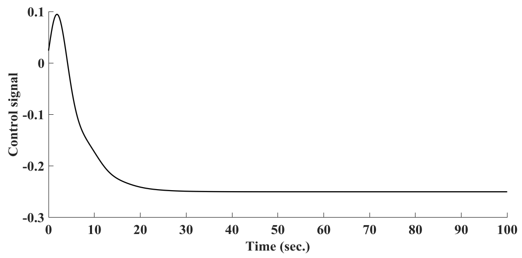


(a) nominal system (b) uncertain system



(c) control signal

Figure (3): Time response specifications of the controlled system with

5, $k_{p_2} = 0.5292$, $k_d = 0.4531$.

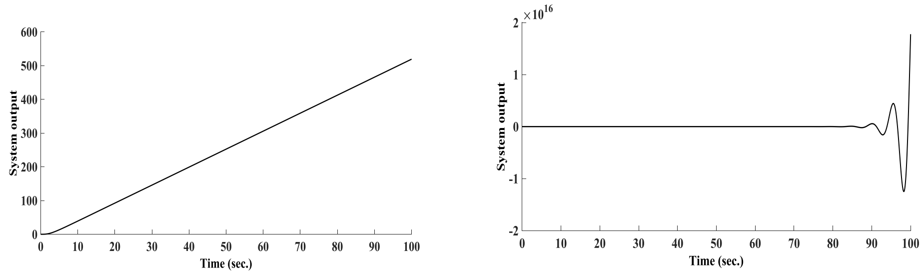
Example 2: Consider Figure 1 with an uncertain transfer function represented by:

$$(s,q) \quad \frac{p(s,q)}{p(s,q)} \quad \text{———} \qquad \dots (19)$$

where $q_0 \in [8,10]$, $q_1 = 0$, $\in [1, 2]$, $q_3 \in [3,6]$, $q_4 \in [2,4]$ and $= 1$. There are (two) Kharitonov polynomials for (s,q) and (four) Kharitonov polynomials for (s,q). Thus, there are (eight) Kharitonov transfer functions (two numerators and four denominators) for this system. Figure 4 shows the time response specifications of the system in open loop and closed loop. It is clear that the system is unstable with an integrating behavior. To stabilize this system a PI-PD controller has been applied. The PSO algorithm was applied to obtain the optimal parameters of PI-PD controller. With$k_{p_1} = 0.0247, k_d = 0.2597, k_{p_2} = 0.0828, k_i = 0.0132$, the resulting time response pecifications of the controlled system and control signal is shown in Figure 5. It is shown that the proposed PI-PD controller can compensate the integrating and uncertain system with a desirable time response specifications in comparison to those obtained by [5]. The minimum cost function which has been achieved is 756.1651. For carrying out the design of controller using PSO algorithm, the PSO parameters are set as: w =2, $c_1 = c_2 = 2$, swarm size = 100, no. of iterations=50. It is shown that increasing the number of iterations over 50 did not improve the convergence of algorithm. Table 2 compares the achieved time response specifications with previous work.

Table (2): Performance comparison

Controller	$(se\ .)$	$(se\ .)$	%
PI-PD [5]	15	50	18
Proposed PI-PD controller	12	35	2



(a) open loop **(b) closed loop**

Figure (4): Time response specifications of the system.

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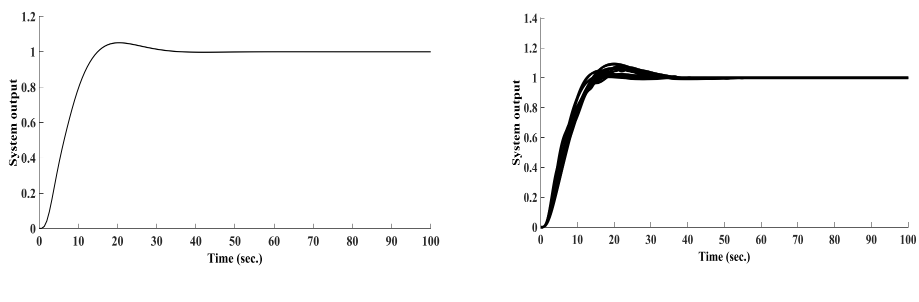
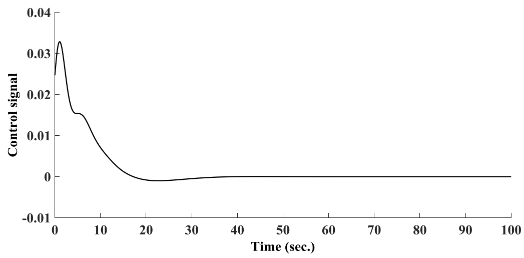


(a) **nominal plant** **(b) uncertain system**



(c) control signal

Figure (5): Time response specifications of the controlled system with $2, k_{p_2} = 0.0828, k_d = 0.2597.$

CONCLUSION

In this paper the robust tuning of the four PI-PD controller parameters has been presented. The PSO method was used to tune the parameters of the controller that achieves the robust specifications based on the Kharitonov's theorem. Two illustrative examples represented unstable, integrating and uncertain systems have been used to show the effectiveness of the proposed controller. The results show that the proposed controller is robust for the entire range of system parameter variation within the given known bounds. Moreover, the main advantage of the proposed control design is the possibility to use a low order specific controller structure (PI-PD) for robust performance objective.

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