Smoothed Artificial Potential Field (APF) Via Points Path Planning Algorithm Based On PSO

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Abstract

Finding the path planning solution considered as one of the most important aspects in the navigation of the robot, involving one of the optimization methods is the most successful way to get the best path. This paper proposed a mixing approach for robot path planning, by applying modified Artificial Potential Field (APF) to find accepted path then applying particle swarm optimization (PSO) to find the best coordinate locations for the intermediate points that chosen from the original APF path, in order to make an iteratively enhancement till reaching the shortest path. This approach has been tested in two cases, first in case of mass point and second in case of two-link robot arm. The results clearly show the effectiveness and strength where the path length cost minimized from 10.8519m to 10.2144m after optimization, which is represent the best solution for the tested environment.

Keywords: Artificial Potential Field; Particle Swarm Optimization (PSO); Path Planning algorithm.

الخلاصة

ايجاد حل خوارزمية تخطيط المسار تعتبر واحدة من اهم الجوانب المهمة في ملاحة الروبوت. والتي يجب ان تتضمن واحدة من تقنيات ايجاد الحلول المثلى لايجاد افضل مسار. هذا البحث يقترح طريقة دمج لتخطيط مسار الروبوت، بتطبيق نظرية مجال الجهد الصناعي المعدلة لايجاد مسار مقبول بعد ذلك تطبيق تقنية سرب الجزيئات لايجاد الحلول المثلى لايجاد افضل احداثيات المواقع للنقاط المختارة من المسار الاصلي الناتج من نظرية مجال الجهد الصناعي، لكي يتم تحسين المسار بشكل تكراري للوصول لافضل واقصر مسار. هذه الطريقة تم اختبارها بدراستين، الاولى بتطبيقها على الروبوت النقطة، الثانية على روبوت ثنائي الاذرع. النتائج تظهر بوضوح كفائة وقوة الطريقة المتبعة حيث ان طول المسار تم تغييره وجعلها افضل وتحسن من حيث قصر الطول من ١٠.٨٥١٩ متر الى ١٠.٢١٤٤ متر بعد تطبيق نظرية ايجاد الحلول المثلى والتي تمثل افضل حل للبيئة المختبرة.

كلمات البحث: مجال الجهد الصناعي، تقنية سرب الجزيئات لايجاد الحلول المثلى ، خوارزمية تخطيط المسار

1. INTRODUCTION

Motion planning is an important affair in robotics. In an environment which filled by obstacles, the purpose of path planning is to find an acceptable collision-free path to ensure the movement of the robot from the initial point to the final destination. Also is to locate a set of points for the robot to go away from the obstacles and prevent it from any possible collisions till reach the goal. The previous algorithms deal with path planning problems of robots by the navigation in a known environment filled with static obstacles [1-3].

The artificial potential field (APF) technique is commonly applied for path planning to many types of robots. In the APF theory, the field of forces affect on the robot. The total force came by combination of to two types of forces: first type is the field of attractive force and second type is the repulsive field force. In this method, each obstacle can create a repulsive force to the robot which its magnitude is proportional inversely to the distance measured from the robot to the obstacles. If the distance greater than the influence area the force will not affect the robot otherwise if the robot in this area, then it will be repelled. The direction of all obstacles forces is pointing away from the robot. In the same time the target point has attractive force that has an effect in all the environment to attract the robot to the goal. The summation of these two force components will create a field with whole magnitude and direction in which the robot moves to the goal with avoiding any possible collision [4-7]. The potential function used in this method has two values, a minimum value, when the robot is very near or at the final destination and a highest value on each obstacle. The function tends down and converges the robot to the final destination [8-9].

Khatib's who gave the basic definition of the method in the configuration space where the minimum forces at the target point but the highest on the obstacles so the whole environment will represent as valleys which are the goals and hills which are obstacles. The goal point in the potential field is attract the robot while it is repelling by obstacles in the environment. The gradient has followed by the robot till reaching the target point and in the same time avoid any possible collisions with the obstacles [10].

A navigation method was proposed to find suitable path. The main idea behind this method is to combine the virtual obstacle with potential field to make the cylindrical mobile robot movement flexible in totally unknown environments. The Simulation results of the experiments show acceptable performance and ability to find suitable solve to the commonly problem in Artificial Potential Field theory, especially the local minima problem. In the function of the classic artificial potential field approach, there is no optimization method was deal with [11].

An Evolutionary Artificial Potential Field (EAPF) for path planning introduced. The artificial potential field theory in this method was combined with the genetic algorithm, to reach the best potential field functions. This proposed approach had the ability of pointing the robots that deal with dynamic obstacles. The functions of potential field for obstacles and target are also known which contain a changeable factors. The performance strength of this methodology is clear after taking in consideration the dynamic obstacles and dynamic goal. [12]

2. ARTIFICIAL POTENTIAL FIELD THEORY

In case of mass point let q referred to the robot position to move in a environment with two-dimension. The robot current position represented by q=[x y] while the position coordinate of obstacle denoted by q_{obs} =(x_{obs},y_{obs}), and the goal position is q_{goal} =(x_{goal}, y_{goal}) [13].

2.1 Attractive Surface of Potential Field

The parabolic form is the very commonly style of potential field function. The attractive potential that grows up quadratic way proportional with the distance to the goal as shown in (Figure 2) [14].

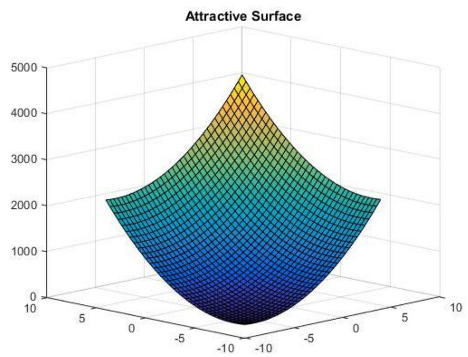


Figure (1): Attractive Surface

$$U_{att}(q) = \frac{1}{2}\ K_a\ \ d^2\ (q, q_{goal}) \qquad (1)$$

where, K_a is the relative factor of the attractive potential surface, $d\,(q, q_{goal})$ is the Euclidean distance from the robot to the desired point (goal) q_{goal}. The attractive force is measured as the negative gradient of the attractive potential field [13-14]:

$$F_{att}(q) = -\nabla U_{att}(q)$$

$$= -K_a \; d(q, q_{goal}) \qquad (2)$$

2.2 Repulsive Potential Field

The repulsive force has a relative relationship between the distances of the obstacles to the position of robot. The repulsive potential surface was introduced by the repulsive forces of all the obstacles. The equations (3-4) represent the repulsive potential function and figure (2) shows the repulsive surface [13-14].

$$U_{rep}(q) = \sum_i U_{repi}(q) \qquad (3)$$

Where, $U_{repi}(q)$ referred to the repulsive potential field created by obstacle *i*, where *i* is number of obstacles [13-14].

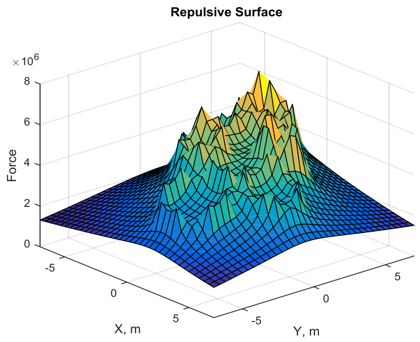


Figure (2): Repulsive Surface

$$U_{rep}(q) = \begin{cases} \frac{1}{2} K_{rep}\left(\frac{1}{d\,(q,q_{obs})} - \frac{1}{d_0}\right)^2 d\,(q,q_{goal})^n & if \;\; d\,(q,q_{obs}) < d_0 \\ 0 & if \;\; d\,(q,q_{obs}) > d_0 \end{cases}$$

(4)

Where, *q* is the robot current position, *n* is a real integer number, the position of obstacle is q_{obs} and d_0 is the positive number referred to the distance of effective obstacle, the distance between the robot and the obstacles is $d\,(q, q_{obs})$ and the factor of the repulsive potential surface is K_{rep} which is an adaptable constant. The total repulsive force is negative slope as shown in equation (5) [13-14]:

$$F_{rep}(q) = -\nabla U_{rep}(q) = \begin{cases} K_{rep}\left(\frac{1}{d\,(q,q_{obs})} - \frac{1}{d_0}\right) \frac{(q,q_{obs})}{d^3(q,q_{obs})} & if \;\; d\,(q,q_{obs}) < d_0 \\ 0 & if \;\; d\,(q,q_{obs}) > d_0 \end{cases}$$

(5)

The combination of the two surfaces of attractive potential U_{att} and a repulsive potential U_{rep} result the total potential field. Which is represented by the equation (6).

$$U(q) = U_{att}(q) + U_{rep}(q) \qquad (6)$$

All forces that applied to the robot is came by the negative gradient and use the steepest descent method to lead the robot direction to final destination.

$$F(q) = -\nabla U(q) = -\nabla U_{att}(q) - \nabla U_{rep}$$

(7)

where, the gradient vector of *U* is ΔU, the influence force that act on the robot is expressed as the summation of two components first one is the attractive vector and the second is the repulsive vectors force, F_{att} and F_{rep}, respectively.

$$F(q)= F_{att}(q) + F_{rep}(q) \quad (8)$$

3. Particle Swarm Optimization Method (PSO)

The basic ideas of particle swarms, was to find computational intelligence by employing simple analogues of social interaction, rather than purely individual cognitive abilities. Searching for corn by bird swarm were the first samples of their simulation. The method after 5 years was developed till become more powerful. The number of particles was added to the research space to some cases and each one solves the problem function at own current location. The movement of each particle should measure its position in the search field, this done by summation of some information of the previous position and new position which may considered as the best locations with one or more members of the flocks, and with some random perturbations. After the total particles have been gone, the next iteration will take a place. At the end flock totally, like a swarm of birds collectively foraging for food, it should search near the optimal place which represent the fitness function [15-16].

PSO play important role in path planning research. A Multi-robot cooperation was proposed to find the performance for some hard tasks can be developed by the collaboration between the robots in totally unknown environment. A group of robots search cooperatively to reach the goal points. The PSO fitness function is the potential function, in this research employs the PSO method to discover the unknown area, but even the appropriate cooperation not lead the robots to find the optimal path [16], Rainer Palm presence the safe navigation of multiple non-holonomic mobile robots in shared areas. Artificial potential fields used to avoid obstacle for mobile robots. The attitude of mobile robots is optimized by particle swarm optimization (PSO). [17]

4. Two-Link Robot Arm Kinematics Modeling

Kinematic analysis of the mechanical structure of a robot concerns with the description of the motion with respect to a fixed reference Cartesian frame by ignoring the forces and moments that cause motion of the structure. With reference to a robot manipulator, kinematics describes the analytical relationship between the joint positions and the end-effector position and orientation [18].

4.1 Forward Kinematics of Manipulators [13, 18]

A very basic problem in the study of mechanical manipulation is called forward kinematics, which represents a fixed problem of computing the position and orientation of the end-effector of the manipulator. Specifically, given a set of joint angles, the forward kinematic problem is to compute the position and orientation of the tool frame relative to the base frame.

Figure (3) shows the 2-DOF planar manipulator arm having l_1 and l_2 as their link lengths and θ_1 and θ_2 as joint angles with x and y as task coordinates.

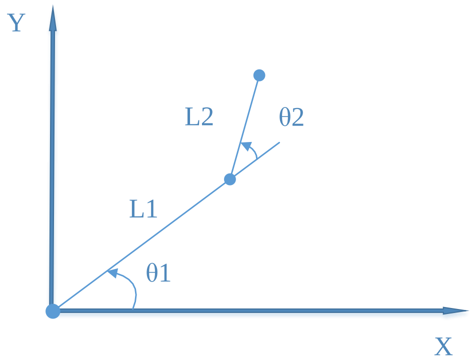


Figure (3): 2-DOF manipulator [6]

The forward kinematic equations are:

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \quad (9)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \quad (10)$$

4.2 Inverse Kinematics of manipulators

Inverse kinematics is the determination of all possible and feasible sets of joint variables, which would achieve the specified positions and orientations of the manipulator's end-effector with respect to the base frame. In contrast to the forward problem, the solution of the inverse problem is not always unique: the same end effector pose can be reached in several configurations, corresponding to position vectors.

The two main solution techniques for the inverse kinematics problem are analytical and numerical methods. There are two approaches in the analytical method: geometric and algebraic solutions. Geometric approach is applied to the simple robot structures, such as 2-DOF planar manipulator or less DOF manipulator with parallel joint axes. Geometric solution approach is based on decomposing the spatial geometry of the manipulator into several plane geometry problems. It is applied to the simple robot structures, such as, 2-DOF planer manipulator whose joints are both revolute and link lengths.

The following equations are used to compute θ_1 and θ_2 of the 2-DOF planar manipulator arm. Consider the diagram of Figure (4).

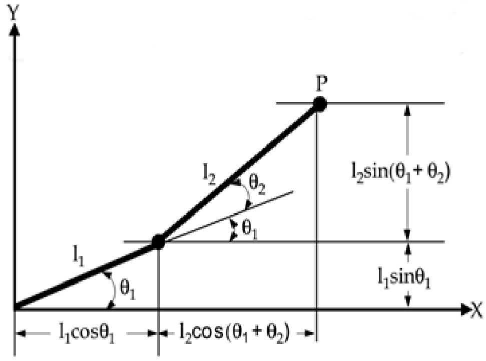


Figure (4): Solving the inverse kinematics based on trigonometry [15]

Using the law of cosines that the angle is given by:

$$\cos\theta_2 = \frac{p_x^2+p_y^2-l_1^2-l_2^2}{2l_1l_2} = D \qquad (11)$$

Since, sin θ_2 is

$$\sin\theta_2 = \mp\sqrt{1-D^2} \qquad (12)$$

We could solve for $\theta_2$using the cos^{-1}function. However, it is better to use $tan^{-1}$for reasons of numerical accuracy. The software function implementing tan-1*(b/c)*=ATAN2 (b, c).

This function has a uniform accuracy over the range of its arguments, returns a unique value for the angle depending on the signs of *b* and *c,* and gives the correct solution if *b* and/or *c* is zero.

Therefore, The two possible solutions for θ_2 can be obtained by writing as:

$$\theta_2 = \tan^{-1}\frac{\mp\sqrt{1-D^2}}{D} \qquad (13)$$

Finally, $\theta_1$can be found by:

$$\theta_1 = \tan^{-1}({}^{y}/_{x}) - \tan^{-1}\frac{l_2\sin\theta_2}{l_1+l_2\cos\theta_2} \quad (14)$$

Where, $l_1$and l_2 are lengths of of first and the second links, respectively.

5. Two-Link Robot Free Cartesian Space Analysis

In this thesis and in the case of two-link robot arm, the workspace has been compiled and analyzed based on the inverse kinematics result. The free Cartesian space of the two-link arm can be defined as a space with the set of all points that can be reached by a specific end-effector. These points are associated with joint angles (θ_1 andθ_2) which have been obtained from inverse kinematics. In an environment which contains many obstacles, the shape and volume of the free Cartesian space are varied according to the shape, size, position, and number of obstacles in addition to mechanical limits of joints. In our theoretical study, the modeling and simulation are as in table (1). These constraints influence and restrict motion of manipulator as well as separate the workspace into a reachable area and an unreachable one [6].

Table (1): Theoretical ranges of two-link robot arm

Link Number	Range of arm's joints in degree
Joint 1	$0 \leq \theta_1 \leq 360$
joint 2	$-90 \leq \theta_2 \leq 90$

Computing the free Cartesian space has been done by point's analysis in the environment and finding all possible solutions of points after checking each limitation of joints and collision with obstacles. The checking function is depending on center point as well as the radius of the circle obstacle. Moreover, there are three possible cases for each coordinates point in Cartesian space. In the first case, the point has two solutions (elbow up and elbow down) as shown in figure (5). But in the second case, the point has just one solution, either the manipulator is fully extended to reach a point at the border of manipulator's reachable workspace, or the point has one configuration (elbow up or elbow down) due to the fact that the other configuration collides the obstacle at any part of manipulator links. While in the third case, the point has no solution when coordinates point are out of reach of manipulator or in obstacles area, or the two configurations collide with an obstacle at any part of arm links [9].

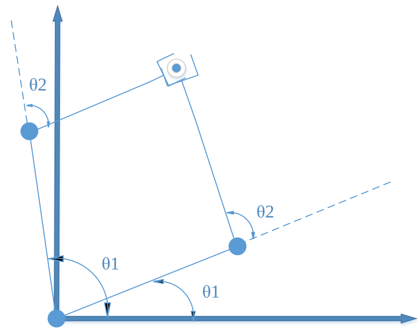


Figure (5): Two-link arm elbow up and elbow down configurations [6]

The results of Cartesian space points analysis have been formed as three spaces, the first space is representing all points that are reachable in the elbow up solution (free elbow up space), and the second space is representing all points that have elbow down solution (free elbow down space), and the third space is unreachable space which includes points outside the manipulator reachable workspace and points that collide with obstacle and points in obstacle area. Moreover, the free Cartesian space is comprising of all points which have at least one solution (elbow up solution space and elbow down solution). The length of the arm link can also be considered as a major factor to construct the free Cartesian space [6].

6. PROPOSED METHOD: APF Based on PSO for Via Points Optimization

The first modification is on the forces and their directions ,in the traditional Artificial Potential Field the environment will be a net of cells, every node in this grid has a force that comes from two sources, the attractive force from the target point and the repulsive force from the obstacle (if the points in the range of the influence). Every point has two types of forces one is towards the X-axis and the other is towards the Y-axis, some of researchers play with these forces to inforce the robot to find one lonely path with these equations:

$$F_{x\_total} = F_{x\_att} + F_{x\_rep} + F_{y\_rep} \quad (15)$$

$$F_{y\_total} = F_{y\_att} + F_{y\_rep} - F_{x\_rep} \quad (16)$$

$$F_{x\_total} = F_{x\_att} + F_{x\_rep} - F_{y\_rep} \quad (17)$$

$$F_{y\_total} = F_{y\_att} + F_{y\_rep} + F_{x\_rep} \quad (18)$$

The equations (15 and 16) result in Figure 6 and equations (17 and 18) result in Figure 7 which shows that the forces are in one direction around the obstacle as shown:

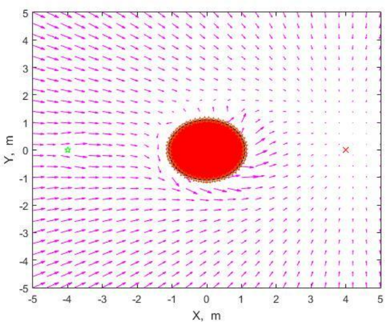


Figure (6):result of eqs (15and 16)

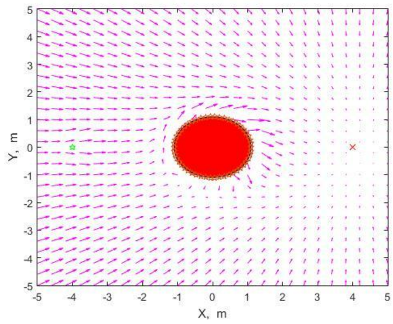


Figure (7): result of eqs. (17 and 18)

The proposed modification can be summarized as:

Step 1: Remove the third term in equations (15-18).

Step2: Add a term which is found by try and error and it is suitable to these cases.

The term was added to each equation to make the total force in X, Y-axis goes towards the goal and at the same time insures that the path will be collision-free as shown in equations (19 and 20) and figure (8)

$$F_{x\_total} = F_{x\_att} + F_{x\_rep} - 0.75 \quad (19)$$

$$F_{y\_total} = F_{y\_att} + F_{y\_rep} + 0.5 \quad (20)$$

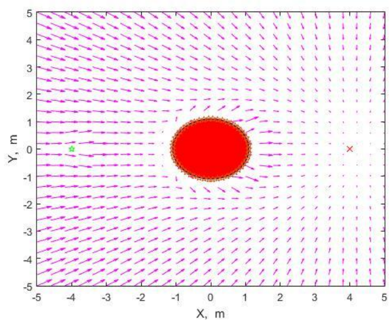


Figure (8): The adjusted forces

The second modification is mixing APF with PSO for via points optimization.

Let's suppose S represent the search domain, *n* referred to the particle number, the position of all particles represented by this vector $X_i=(x_{i1}, x_{i2}, \dots x_{iD})$,the global best position till now is $P_i= (p_{i1}, p_{i2}, \dots p_{iD})$,the best position in the all flock's represent as $P_g= (p_{g1}, p_{g2}, \dots p_{gD})$, the rate of changing the position by i_{th} particle is the vector $V_i= (v_{i1}, v_{i2}, \dots v_{iD})$. The particles update their positions according to the equation (21-22) [18]:

$$v_{id}(k+1) = w \times v_{id}(k) + c_1 \times rand() \times (P_{id}(k) - x_{id}(k)) + c_2 \times rand() \times (P_{gd}(k) - x_{id}(k)) \quad (21)$$

$$x_{id}(k+1) = x_{id}(k) + v_{id}(k+1) \quad (22)$$

Where, *c*1, *c*2 are positive constant factors called acceleration factors and *w* is called the inertia weight which represent the value given according to the equation (23):

$$w \leq 1 \quad (23)$$

The cost function of PSO algorithm is the path length and defined as :

$$L = \sum_i ((pf_{path_{x(i+1)}} - pf_{path_{x(i)}}) + (pf_{path_{y(i+1)}} - pf_{path_{y(i)}}))^{0.5} \quad (24)$$

At first, the parameters of PSO and number of via points should be defined, then APF starts in an ordinary way and constructs the path without smoothing. After that, selection of randomly via points is made from the constructed path and employ them as position parameters of PSO equation during the optimization process. The spline equation is applied to connect the via points and generate corresponding smoothed path. The path length which is the cost function of PSO is calculated, then the initial velocity will be set. The next step is going to update the local and global parameters. The process is repeated till the specified points that are chosen from the path are completed. Updating the velocity and position equations is the next step, to converge the via points to the best cost. Again the local and global parameters are updated. Repeat this process for specific number of populations and repeat the whole process for specific number of iterations. Assign the global value which constructs the shortest path.

7. SIMULATION RESULTS

The two modifications will be applied on the two cases (Mass Point and two-link robot arm). The simulation results will be shown in section 7.1 and 7.2.

7.1 In Case of Mass Point

➢ APF Modification

Environment 1, The APF paths of this environment is shown in figure (9), as shown in this figure the path passes in between five obstacles to reach the goal but although it is accepted path but still not an optimal one.

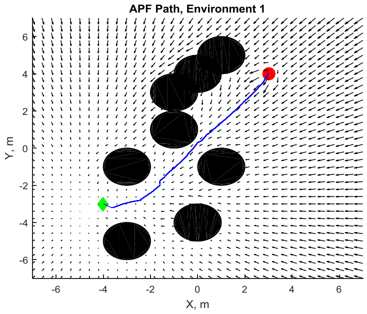


Figure (9) Environment 1

Environment 2: the specifications of this environment is that the path does a hard task and passes through many local minima. Although it is not optimal and not smoothed enough but in many times it escapes away from local minima. Figure (10) show the APF path.

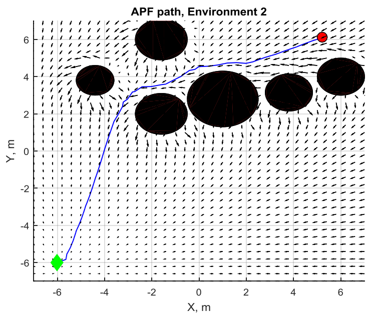
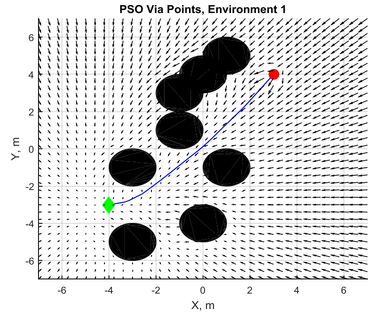


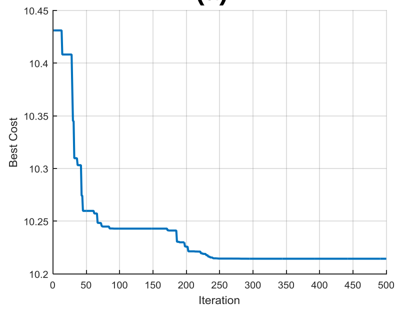
Figure (10) Environment 2

➢ APF based on PSO

Environment 1: the result of this environment shows that the path has improved in smoothing and in cost. Figure (15) shows the APF path based on PSO for Via Point in (a) and the PSO iterations in (b).



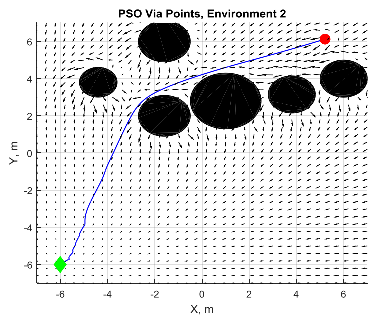
(a)



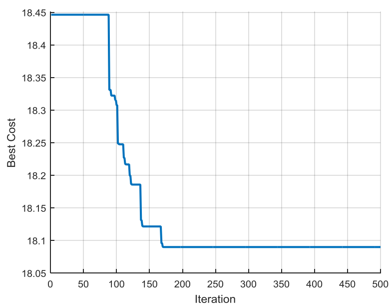
(b)

Figure (15): (a) APF path based on PSO for Via Point (b) PSO iterations

Environment 2: the constructed path get better in smoothing and cost. This approach has good proportional to previous. Figure (16) shows the APF path based on PSO for Via Point in (a) and the PSO iterations in (b).



(a)



(b)

Figure (16): (a) APF path based on PSO for Via Point (b) PSO iterations

Table (2) Mass Point results comparison

Number of environments	1	2
Path length of APF(m)	10.8519	18.7330
Path length of APF based on PSO to optimize via points(m)	10.2144	18.0894

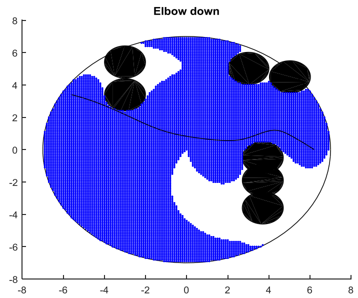
The path generated from APF has many drawbacks even after forces modification. Forces modification ensures solving the simple case of the local minima problem. It is obvious that the path need more development to be more smooth, accurate and optimal or near from the optimality case.

7.2 In Case of Two-link Robot Arm

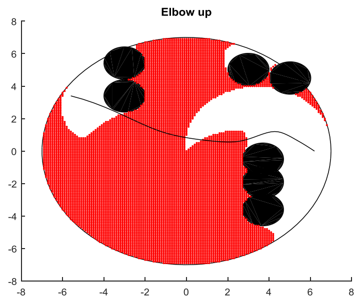
➢ APF Modification

After MP, this approach will be applied on a two-link robot arm. The path will be plotted in each space for all environments, to show the difficulty of the path and the robot arm movement. The arm length equals to the half of the environment, where each link is 3.5 m. The robot has 360 degrees.

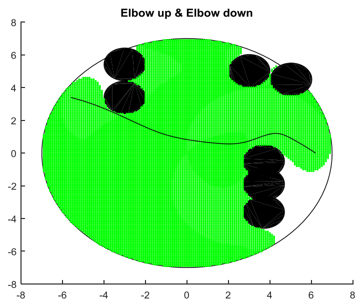
Environment 1: the free space of this environment with APF path is shown in figure (11) elbow down in (a), elbow up in (b) and the two spaces in (c). Figure (12) explains the APF path in (a) and the arm motion in (b).



(a)

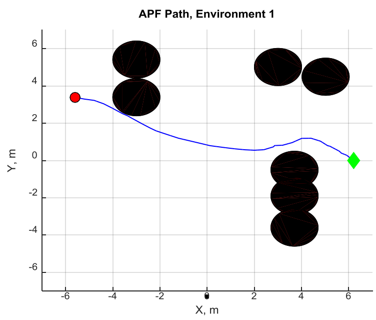


(b)

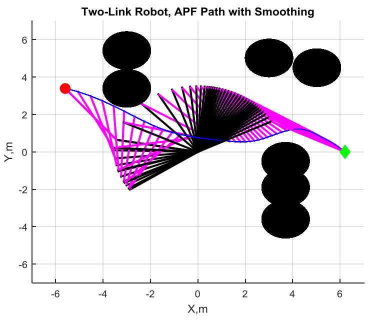


(C)

Figures (11): (a) elbow down (b) elbow up (c) elbow up and down



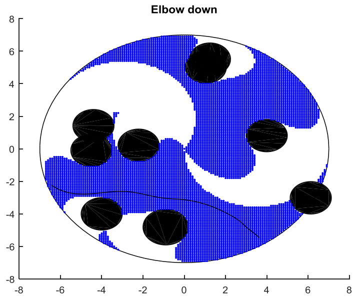
(a)



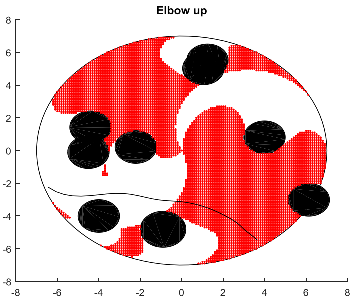
(b)

Figure (12): (a) APF path (b) robot motion

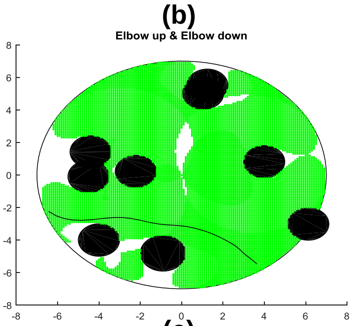
Environment 2: the free space of this environment with APF path is shown in figure (13) elbow down in (a), elbow up in (b) and the two spaces in (c). Figure (14) explains the APF path in (a) and the arm motion (b).



(a)

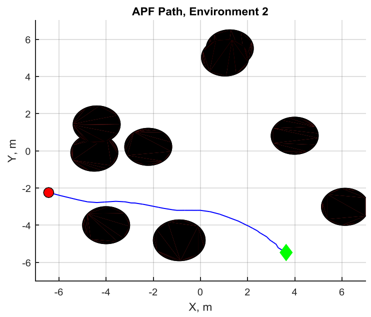


(b)

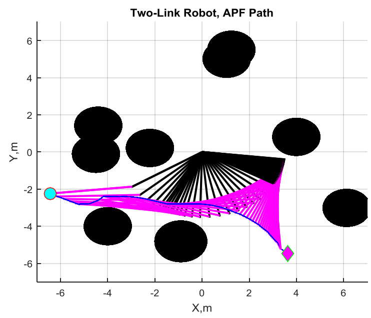


(c)

Figures (13): (a) elbow down (b) elbow up (c) elbow up and down



(a)

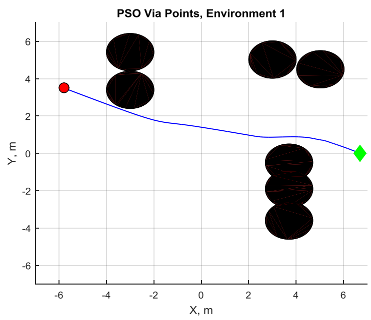


(b)

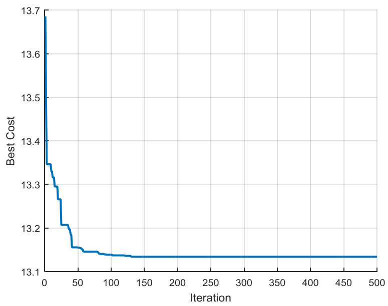
Figure (14): (a) APF Path (b) robot motion on the APF path.

➢ APF based on PSO

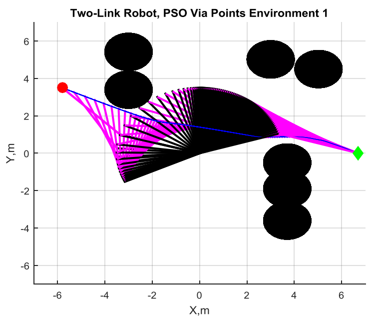
Environment 1: the constructed path has been improved and can get more improvements in cost if there is a new approach. Figure (17) illustrates in (a) APF path based on PSO, in (b) PSO iterations and in (c) the robot motion.



(a)



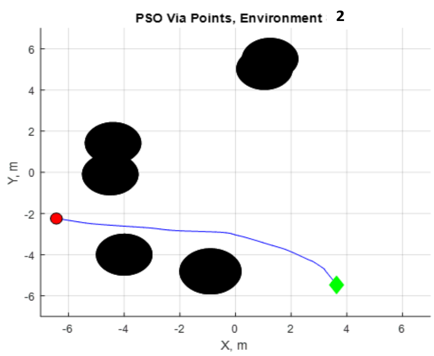
(b)



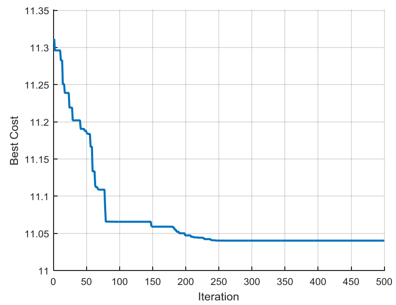
(c)

Figure (17): (a) APF path based on PSO (b) PSO iterations (c) robot motion

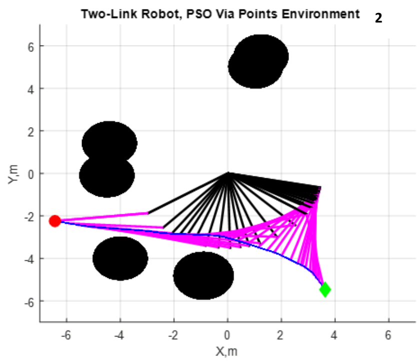
Environment 2: the path constructed in this hard environment is very good compared with the previous modification. Figure (18) illustrates in (a) APF path based on PSO, in (b) PSO iterations and in (c) the robot motion.



(a)



(b)



(c)

Figure (18): (a) APF path based on PSO (b) PSO iterations (c) robot motion

Table (3) Two-link robot arm comparison

Number of environments	1	2
Path length of APF(m)	13.6845	11.5277
Path length of APF based on PSO to optimize via points(m)	13.134	11.0402
Number of populations	100	100
Number of iterations	500	500

The result of the second modification shows the progression in the smoothness and cost of the generated path so clear. The constructed path is optimal compared with the previous modification.

8. Conclusions:

This paper presents an approach of modified path planning using APF and PSO. At the beginning the modified APF was applied then Particle Swarm Optimization method (PSO) applied to find the best position of intermediate via points that chosen from the original Artificial Potential Field path. After each iteration of PSO the APF path will be improved till reaching the best. The results clearly prove the advantages of the mixing approach between APF and PSO for via points optimization. The length of the generated path after optimization processes was minimized in which the traditional APF method could not find it.

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