Influence of Uniaxial Loading on the Strength of Slender Columns

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ABSTRACT

A column is said to be slender if its cross-sectional dimensions are small compared with its length, and its strength is reduced by second-order deformations. Several methods are available for slender column design including codes and researchers. A number (135) of slender column tests is used in this work; from the literature are reanalysis by different methods. Using these tests' results new proposal formulae are made for slender column design. When compared to existing methods the proposed one gives better correlation with test results. The proposed methods were derived using regression analyses for the based on different parameters. Theoretical analyses of columns are sampled to study the major variables affecting the column for each method of analysis. The paper is mainly concerned with making suggest and comparison between several methods analyzing slender columns.

INTRODUCTION

So many investigators have suggested various design and methods to estimate accurately column strengths and deflections. In particular, the factors affecting the behavior of reinforced concrete columns are many. Generally, columns are important structural member, therefore, their behavior especially the slender ones take a large concentration of many researchers and tests. The development of concrete technology and practice has led to increased use of high strength concrete (HSC) especially in compression member. The principal reason for using HSC is that it may offer the most cost-efficient solution for many structural design problems while providing higher strength and improved ductility. One application of HSC has been in the columns of buildings. The main advantage of using HSC in columns is the economic benefits derived from its use in primary structural members as columns [1]. The limit of HSC does not mean that there is a sudden change in material properties at that strength. However, certain differences in mechanical properties and behavior are evident. With the increase in concrete strength, the engineer can design smaller sizes of columns to carry the same loads that a larger member of ordinary strength concrete would carry. Reduced member size increases the amount of rentable space and is especially beneficial when there are architectural restrictions on column size. Producing a more durable material, reduction in cost of forms, is among other advantages [2].

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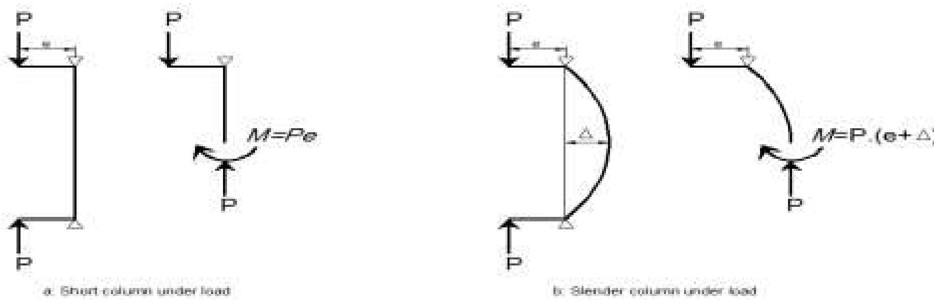
In the few last decades, slender buildings and slender building components have become common, making it necessary to consider stability problems and the deflections produced by lateral loads. Geometric nonlinearity problems have been solved by approximately second-order analysis of frames. Geometrically, two types of displacement lead to slenderness effects. The first is due to displacements of the column relative to the straight chord line joining the ends of the column. This is referred to as the *member stability effect*. The second type of slenderness effect occurs due to lateral displacement of the column relative to its bottom end. This is referred to as the *lateral drift effect* or the *P□□effect.* A rigorous stability analysis of reinforced concrete frames is a rather complicated matter due to the nonlinear load-deflection relationships. The moment magnifier or effective lengths are methods that approximate the *member stability effect* [3]. Show figure (1).



Figure(1): Behavior of columns under loading

Generally the design of a slender reinforced concrete column consists of three stages; (1) the analysis of the structure to determine the forces and moments in each member; (2) the modification of the forces and moments computed in stage (1) to account in some way for the column slenderness; (3) the proportioning of the cross section to resist these forces and moments. Many studies, to carry out the second stage, are made. The analysis and design of reinforced concrete subjected to an axial compressive load is treated by the most familiar codes (ACI [4], CAN [5], NZ [6], and BS [7]) as a combination of the two different actions, and modifications by researches (8 - 16).

Objectives

Methods used for analyzing slender columns are many, ACI Code-14[4], Canadian Code[5], New Zealand Code[6] and British Code[7] and proposed of researches. Each method has many main variables affecting column analysis.

The primary objectives of this research are:

1. Relate available experimental tests of slender column failures with the different methods of analysis.
2. Make a comprehensive comparison between the different analysis methods, and study the behavior of each method separately and compare between them.
3. Study major variables in each method separately [e/h, l/h and *p*].
4. This paper is limited to slender columns under uniaxial loading.

5. Derive simplified equations used in analyzing slender columns, based on available test from the literature that is easy to use and with rational results in comparison with test results.

The assumptions of current design approaches

The current design and proposed methods are based on the following assumption:

1. The tensile strength of the concrete is ignored.
2. The contribution of confinement on strength is ignored.
3. The strains in the distance from the neutral axis (i.e. plane sections prior to loading remain plane after loading).
4. Steel bars behave in an elastic-perfectly plastic manner. Tension and compression steel have identical behavior.
5. It has been assumed that perfect bonding exists between concrete and steel at the interface, and no slip can occur between the two materials.

In addition to the mentioned assumptions, this work considers that all safety factors, strength reduction factors, and stability reduction factors for concrete and steel reinforcement have value of unity for comparison purposes. Also due to the available experimental work used herein, the effect of creep on the strength is neglected (creep factor $\square_d$ becomes zero) because all tests were carried out under short-term loads. In addition, the moments applied at the two ends of the whole specimens were equal and cause a single curvature, thus the equivalent moment factor C_m will always be equal to unity. The British standards specify the cube concrete strength for concrete f_{cu}. The cylinder strength is usually about (70-90) % of the cube strength(3), the cube strength of concrete is assumed equal to *cylinder strength/0.82*(17).

Slender Column Analysis

Reinforced concrete slender columns depend upon the magnitude of the second order moments caused by the lateral deflection of the column and lateral drift of the structure as a whole. The strength of a slender column is affected by many factors such as column length, end restraint conditions, distribution of bending moments, level of axial thrust, creep of concrete, and bracing condition of the column and other factors. Each of the several methods will be discussed separately to study the equations which magnify the second order moment.

Design code requirements

Four codes have been used in this paper for slender column analysis. Some of these codes deal with overall structures [use an overall strength reduction factor take in this paper =1] like ACI Code(4) and New Zealand Code(NZ)(6), while the Canadian Code(CAN)(5) and British Code(BS)(7) deal with the materials [use implicitly strength reduction factors].

Other Methods:

Over the years, many researchers studied various parameters for RC columns to investigate the effects of each variable on the behavior and capacity. Although the designer is told so often what to do, he has never been shown what is likely to happen once he starts making the calculations.

Investigators for HSC have tried to obtain more accurate values of the rectangular stress block factors (α_1) ανδ (β_1).

1. **Junior and Giongo (J&G) [8]:** suggested that the magnified rectangular stress block factor (α_1) is a coefficient calculated as following:

$\alpha_1=\alpha.\alpha_2.\alpha_3$ .....(1)

Where: α = the coefficient, that considers the increase in concrete strength after 28 days. α_2: the coefficient that correlated the compressive strength of plain concrete in a member with those obtained from standard cylinder test. α_3: the coefficient that considers the deleterious effect of long duration loads on the concrete strength. The great majority of the building codes assume α,α_2 and α_3 as 1.2, 0.95 and 0.75, respectively, resulting α_1 equals to 0.85. However, as the concrete strengths were evaluated at the same day of the columns test and the load was applied in an almost static mode in a short period of time. Assuming that α and α_3 are equals to 1.2 and 0.75, respectively. This can be calculated by using equation:

$\alpha_2=-0.136*\ln(f'c)+1.347$ …..(2)

α_1 can be calculated as following: $\alpha_1=1.2*\alpha_2*0.75$, for $f'c$ in MPa …..(3)

2. **Ibrahim and MacGregor (I&M) [9]:** proposed that can calculated factors of rectangular stress block (α_1) and (β_1) as following as:

$\alpha_1= 0.85-0.00125 f'c \geq 0.725$ …..(4)

$\beta_1= 0.95-0.0025 f'c \geq 0.7$ …..(5)

In equations (4) and (5) α_1 and β_1 should not be taken less than that 0.725 and 0.7 respectively.

3. **Shukur Method[10]:** suggested new equations for (α_1) and (β_1). The proposed parameters can be written as following as:

$\alpha_1= 0.895 \quad f'c \leq 58,\ \alpha_1= 0.895-0.0015(f'c -58) \quad f'c \leq 58$ …..(6)

$\beta_1= 0.94 \quad f'c \leq 58,\ \beta_1= 0.94-0.0025\ (f'c -58) \quad f'c \leq 58$ …..(7)

4. **Attard and Stewart Method[11] (A&S):** Proposed new rectangular stress block parameters that based on a probabilistic analysis and stress-strain relationship for HSC. The Proposed parameters can be written as in equations (8) and (9).

$\alpha_1= 1.29(f'c)^{-0.1} \geq 0.71$ ….. (8)

$\beta_1= 1.095(f'c)^{-0.091} \geq 0.67$ …..(9)

For $f'c$ in MPa. In this proposal α_1 and β_1 should not be taken less than 0.71 and 0.67 respectively. They propose to apply equations (8) and (9) to the ACI Code.

5. **AFREM-95 Method[12]:** In this suggestion, equivalent stress block essentially has one variable parameter is proposed, namely the stress block depth parameter (β_1), which varies with concrete strength.

$\beta_1 =1.0 - \left[\frac{0.7}{(4.5-0.025f'c)}\right] for\ f'c\ in\ MPa$ …..(10)

While the width parameter α_1 of the rectangular stress block has a constant value and equal to 0.85 which is similar to the ACI Code.

6. **Proposed Method [1]:** This method represents the modification of the ACI Code rectangular stress block. This method allows a change in α_1 where it is not in ACI Code[4], and allows change in β_1 for f'c beyond 30 MPa. Proposed equations were derived using a statistical study and regression analysis and were applied to 135 test columns.

The ACI Code[4] procures used with proposed that can calculated factors of rectangular stress block (α_1) and (β_1) as following as:

$\alpha_1 = 0.81$ where $f'c \leq 30$ …..(11a)

$\alpha_1 = 0.81-(0.0011*(f'c-30))$ where $f'c > 30$ …..(11b)

$\beta_1 = 0.95$ Where $f'c \leq 30$ …..(12a)

$\beta_1 = 0.95-(0.002*(f'c-30))$ where $f'c > 30$ …..(12b)

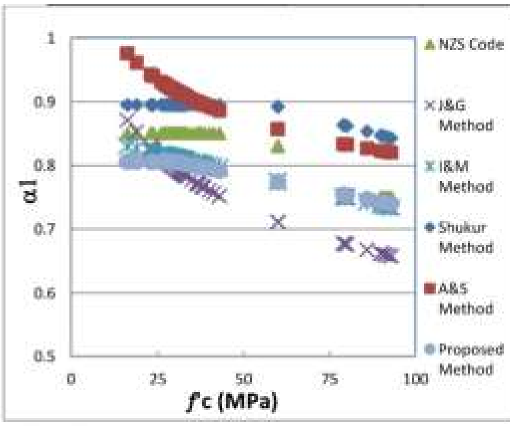
For $f'c$ in MPa. This Proposed suggest that the rectangular stress block parameters (α_1) and (β_1) have constant value for concrete strength less than 30MPa and when concrete strength equal or more than 30MPa, (α_1) and (β_1) have variable values and shall be reduced continuously at a rate of 0.0011 and 0.002 for each one MPa, respectively.

Computer Programs and Statistical results

135 columns were taken from several sources to be available in this paper[18-25] to compared by several existing design methods. Based on these tests statistical and regression analysis are applied leading to a proposed design method for slender columns. All columns are slender columns, rectangular tied, different length and longitudinal reinforcements with eccentricities (uniaxial load). Three tables (1, 2 and 3) of statistical result have been developing in this paper. Each method uses a different procedure for the magnification of moments depending on many variables, which are considered the most important ones. In order to study the efficiency of each method and effects of variable used, a computer program has been developed for each method (Microsoft excel)[27]. This program has been used to compare between these methods.

Table (1): Different values of $P_{test}/P_{calculate}$ for 10 methods.

	MAX.	MIN.	Mean	SD	COV
ACI Code [4]	2.399	0.208	0.965	0.361	37.352
CAN Code[5]	2.309	0.182	0.932	0.365	39.164
NZS Code[6]	2.458	0.205	1.006	0.371	36.829
BS Code[7]	1.739	0.407	0.928	0.326	35.122
J&G Method[8]	2.513	0.344	1.066	0.350	32.846
I&M Method[9]	2.374	0.188	0.961	0.357	37.183
Shukur Method[10]	1.446	0.175	0.842	0.311	36.979
A&S Method[11]	1.475	0.173	0.861	0.313	36.329
AFREM Method[12]	2.399	0.208	0.965	0.361	37.352
Proposed Method[1]	1.797	0.333	1.023	0.269	26.274



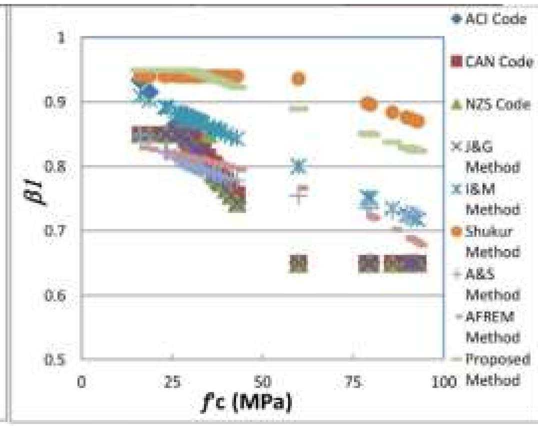


Figure (2a): α_1 expressions versus *f'*c (MPa)   **Figure (2b): β_1 expressions versus *f'*c (MPa)**

Summaries of the developments of α_1 and β_1 are shown in figures.(2a and 2b). In some methods, α_1 and β_1 may have constant values; while in other methods α_1 and β_1 [one or both of them] are variable parameters, see Figure (3).

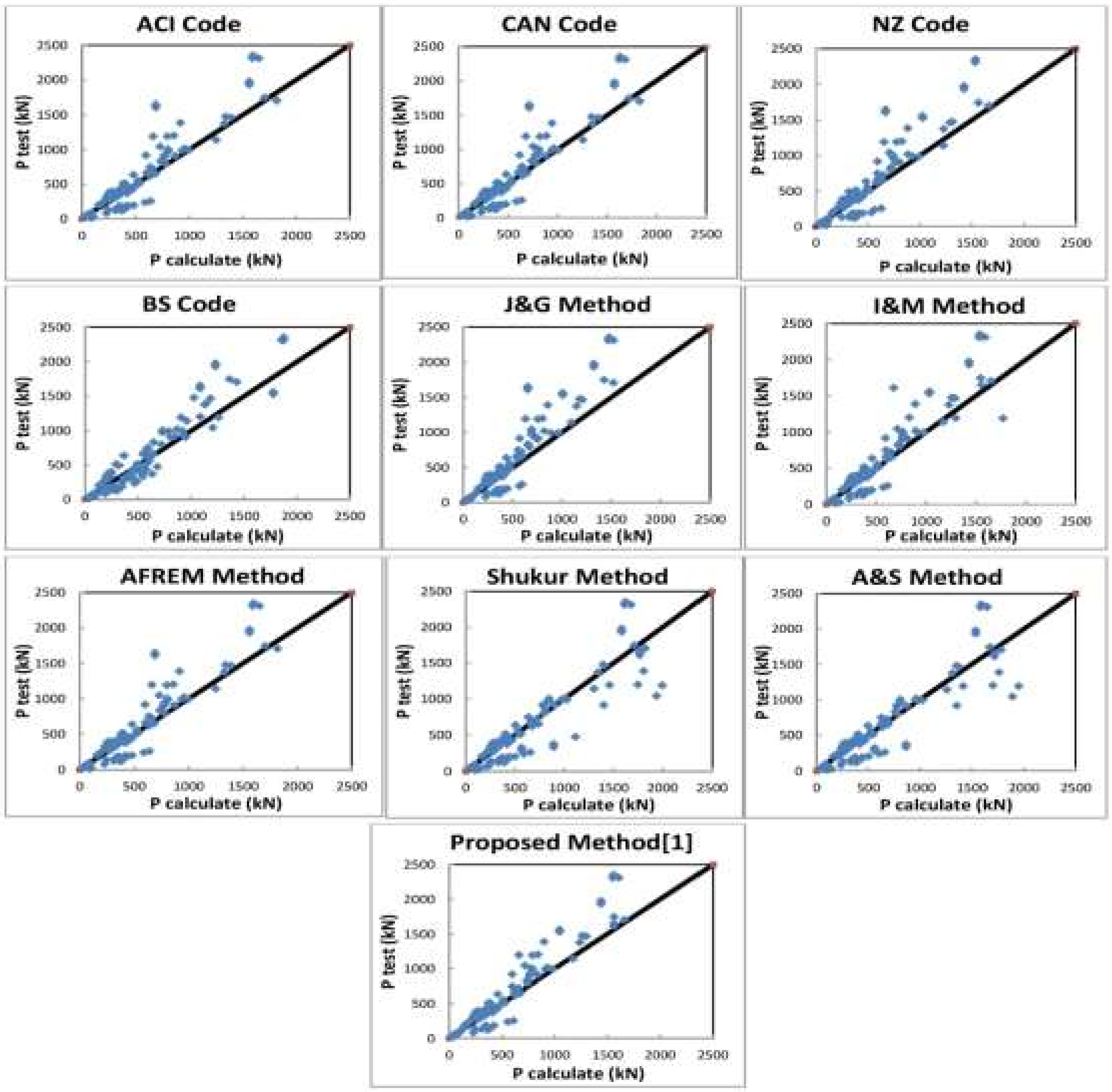


Figure (3): Comparison between Experimental Data and Calculated

Modified-Cranston Method (CMB)[13]: The British standards BS8110-97 states the additional bending approach for the slender RC columns depending on the curvature induced in the RC slender column due to applied loads. Cranston[13] has shown that for a RC column at the ultimate limit state, the curvature ($1/r_m$) at the critical section could be assumed to depend only on the depth of the column section h and the effective height/depth ratio (l_e/h)

$$\frac{1}{r_m} = \frac{1}{175h}\left[1 - 0.0035\frac{l_e}{h}\right] \quad \text{.....(13)}$$

In addition, he suggested that: $e_{add(CMB)} = \frac{l_e^2}{10}\left(\frac{1}{r_m}\right)$ .....(14)

In which ($e_{add(CMB)}$) is the additional eccentricity due to slender column effect [Δ]. Combining equations (13) and (14), relating to current design practice, BS standards. Approximate the resulting equation to Eq.(15). Assuming maximum slenderness ratio (l_e/h=285.7).

$$e_{add(CMB)} = \frac{h}{2000}[\frac{l_e}{b'}]^2 \quad .....(15)$$

Where b'=the smaller dimension of the column section.

7. Al-Bakri [14]: In 1999, Al-bakri[14] proposed equation (18) by modifying Cranston derivation as follows: $\frac{1}{r_m} = \frac{1}{175h}[1 - 0.0035\frac{h}{l_e}]$ .....(16)

He was also proposed to chose the more critical additional eccentricity as: $e_{add(AL-Bakri)} = \frac{l_e^2}{8}(\frac{1}{r_m})$ .....(17)

Combining equations (16) and (17) leads to equation (18), which is applied in a similar manner to BS standards. $e_{add(AL-Bakri)} = \frac{h}{1400}[\frac{l_e}{b'}]^2$ .....(18)

8. Proposed Method[2]: The British standards BS8110-97 states the additional bending approach for the slender RC columns depending on the curvature induced in the RC slender column due to applied loads. In this work assumed the curvature ($1/r_m$) at the critical section depend only on the depth of the column section h and the effective height/depth ratio (l_e/h)

$$\frac{1}{r_m} = \frac{1}{175h}[1 - 0.0035\frac{h}{l_e}] \quad .....(19)$$

This work was also proposed to choose the more critical additional eccentricity as: $e_{add(Proposed)} = \frac{l_e^2}{13}(\frac{1}{r_m})$ .....(20)

Combining equations (19) and (20) leads to equation (21), which is applied in a similar manner to BS standards. $e_{add(Proposed)} = \frac{h}{2235}[\frac{l_e}{b'}]^2$ .....(21)

Table (2): Different values of $e_{test}/e_{calculate}$ for 12 methods.

	MAX.	MIN.	Mean	SD	COV
ACI Code [4]	1.0	0.185	0.598	0.227	37.964
CAN C ode[5]	1.0	0.262	0.622	0.224	36.008
NZS Code[6]	1.0	0.185	0.588	0.224	38.047
BS Code[7]	0.988	0.340	0.780	0.155	19.887
J&G Method[8]	1.0	0.186	0.567	0.212	37.352
I&M Method[9]	1.0	0.219	0.627	0.226	36.005
Shukur Method[10]	1.0	0.252	0.678	0.232	34.204
A&S Method[11]	1.0	0.260	0.674	0.228	33.792
AFREM Method[12]	1.0	0.185	0.598	0.227	37.965
CMB Method[13]	1.599	0.999	1.065	0.116	10.878
Al-Bakri Method[14]	3.280	1.007	1.346	0.412	30.623
Proposed Method[2]	1.434	0.831	1.016	0.103	10.102

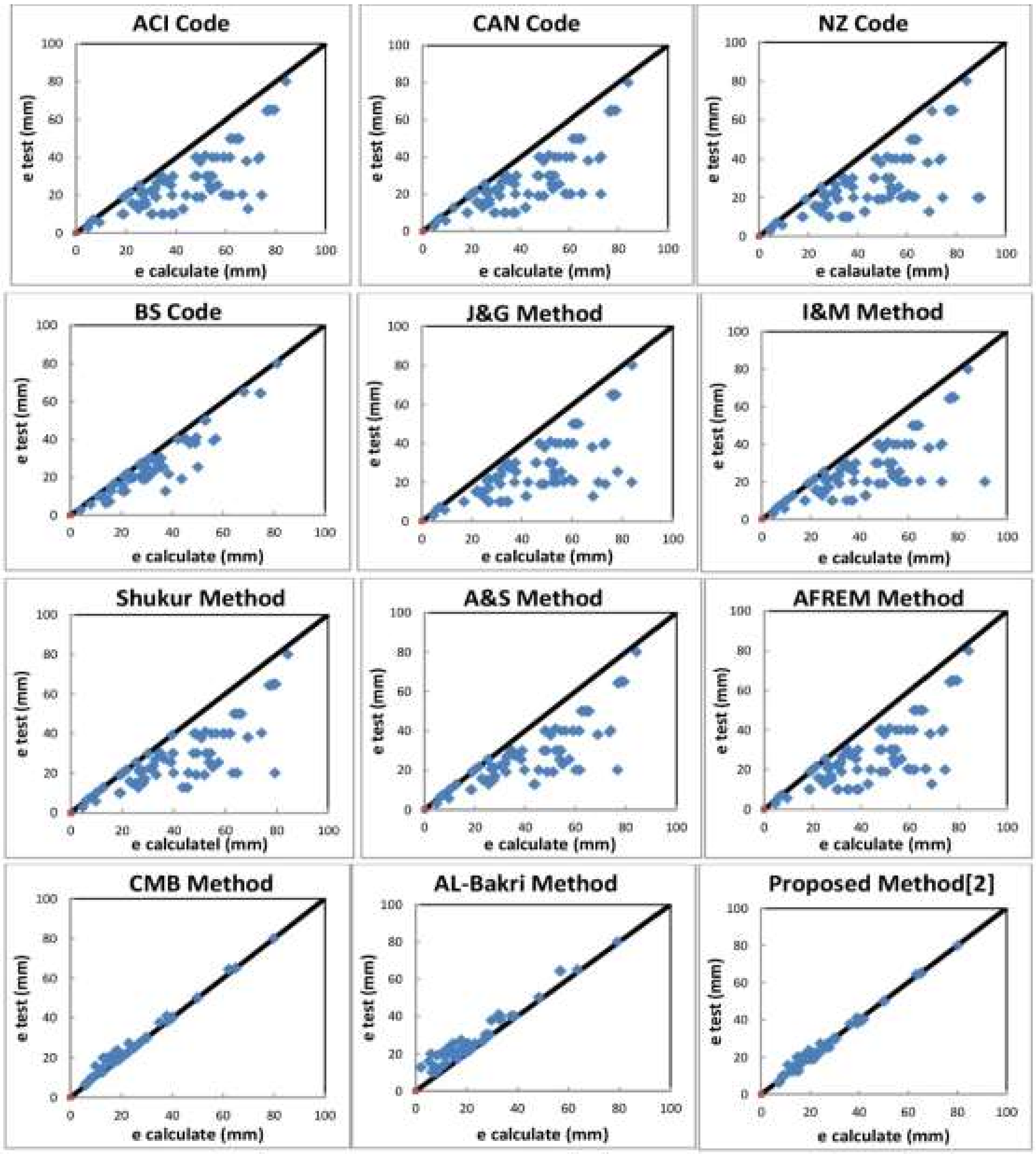


Figure (4): Comparison between Experimental Data and Calculated

Rangan Method[15]: This refers to a proposed method by Rangan in 1990 [15].It is a method for calculating the strength of reinforced concrete slender columns in braced frames. It is based on the stability analysis of slender columns. The creep deflection, which is included in the analysis as an additional eccentricity, is included by equations (22),(25), and (27).

$\Delta_{\tau o \tau} = \varepsilon / [(\Pi_{\chi} / \phi\, \Pi) - 1]$ …..(22)

$P_c = \pi^2 EI / L^2$ …..(23)

$EI = \lambda\, E_c\, I_g / (1+0.8\, \phi_{cc})$ …..(24)

$\lambda = [0.6+(e_b /8e)] \leq 1.0$ …..(25)

$\Delta_o = e / [(P_{co}/ P_n)-1]$ …..(26)

$P_{co} = \lambda\, \pi^2\, E_c\, I_g / L^2$ …..(27)

$\Delta_{cp} = \Delta_{tot} - \Delta_o$ …..(28)

Where (ϕ_{cc}) is the creep factor, taken as the average value between 1.8 and 3.8 equal to 2.5.
To calculate the eccentricity e for the column: e = M / P. Δ_{yo} is given by:
$\Delta_{yo} = 1.6\, \varepsilon_y\, L^2 / \pi^2\, d$ …..(29)

For $P_u > \phi P_b$: $\Delta_y = \Delta_{yb}(\phi P_o - P_u)/(\phi P_o - \phi P_b)$ …..(30)

For $P_u \leq \phi P_b$: $\Delta_y = \Delta_{yo} + (\Delta_{yb} - \Delta_{yo})/(P_u/\phi P_b)$ …..(31)

And for design proposes:- $M_c = P_c\, (e+\Delta_{cp}+\Delta_y)$ …..(32)

9. Abbas Method(16): Abbas Method(16) proposed equations (33,34) by modifying Rangan Method(15), study on the methods [ACI,CAN,BS and RAN]of analysis separately and make a comparison between them, 56 columns test used in order to make the convenient for all methods.
The Abbas equations are: For $P_u \geq \phi P_b$: $\Delta_y = Cf.\, \Delta_{yb}(\phi P_o - P_u)/(\phi P_o - \phi P_b)$ …..(33)

For $P_u \leq \phi P_b$: $\Delta_y = Tf.\Delta_{yo} + Tf.(\Delta_{yb} - \Delta_{yo})/(P_u/\phi P_b)$ …..(34)

Where: Cf= reduction strength factor in compression control = 1.5.
Tf= reduction strength factor in tension control = 1.1.
10. Proposed Method[3]: This proposed method is based on the best fit of the previously mentioned method Rangan method(15). It has the same concepts used in magnification of slender column moments with factored equations to get more accurate and conservative results. Rangan method(15) depends on two separate equations: one is for the tension control region ($P_u < P_b$) and second is for the compression control one. In the proposed method both of these main magnification equations were modified separately depending on a statistical study of the test and analytical results, show figure (5).The Proposed equations are:
For $P_u \geq \phi P_b$: $\Delta_y = Cf.\, \Delta_{yb}(\phi P_o - P_u)/(\phi P_o - \phi P_b)$ …..(35)

For $P_u \leq \phi P_b$: $\Delta_y = Tf.\Delta_{yo} + Tf.(\Delta_{yb} - \Delta_{yo})/(P_u/\phi P_b)$ …..(36)

Where: Cf= factor in compression control =0.5 and Tf= factor in tension control = 0.55.

Table (3): Different values of $P_{test}/P_{calculate}$ for 12 methods.

	MAX.	MIN.	Mean	SD	COV
Rangan Method(15)	1.991	0.405	1.278	0.333	26.043
Abbas Method(16)	2.399	0.425	1.367	0.403	29.511
Proposed Method[3]	1.756	0.401	1.127	0.252	22.397

*see table (1)

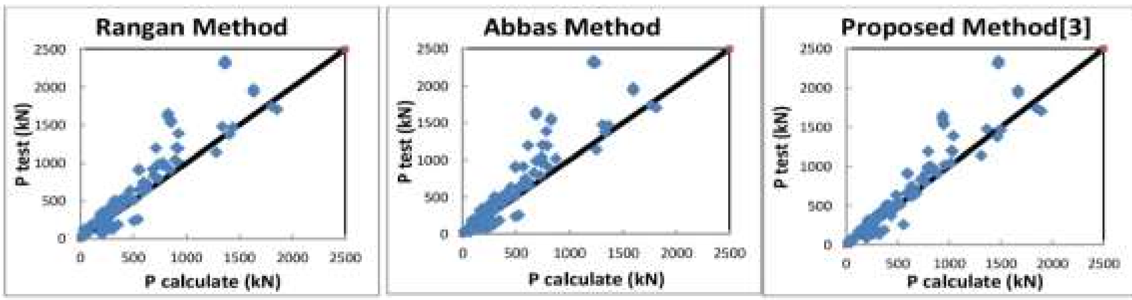


Figure (5): Comparison between Experimental Data and Calculated

Major Factors Affecting Strength of Column

The same 135 column test results (18- 26) were used to investigate the representation of analysis methods for the column strength, when comparisons are made between method using the main factors affecting column strength (e/h, l/h and p) on the value of relate axial strength value ($P_{test}/P_{calculate}$).

1. Effect of l/h: Figure (6) show the effect of increasing l/h ratio on uniaxial load. We can notice that ACI(4), CAN(5), NZ(6) and BS(7) Codes give significantly lower values of $P_{test}/P_{calculate}$ with increasing of l/h ratio. I&M(9), Shukur(10), A&S(11) and AFREM(12) methods give lowest values $P_{test}/P_{calculate}$ with increasing of l/h ratio. This leads to a drop in the safety with rising l/h ratio. J&G(8), Proposed [1], CMB(13), AL-Bakri(14), Proposed[2], Rangan(15), Abbas(16) and Proposed[3] method give higher values $P_{test}/P_{calculate}$ with increasing, of l/h ratio. This leads safety with rising l/h ratio.

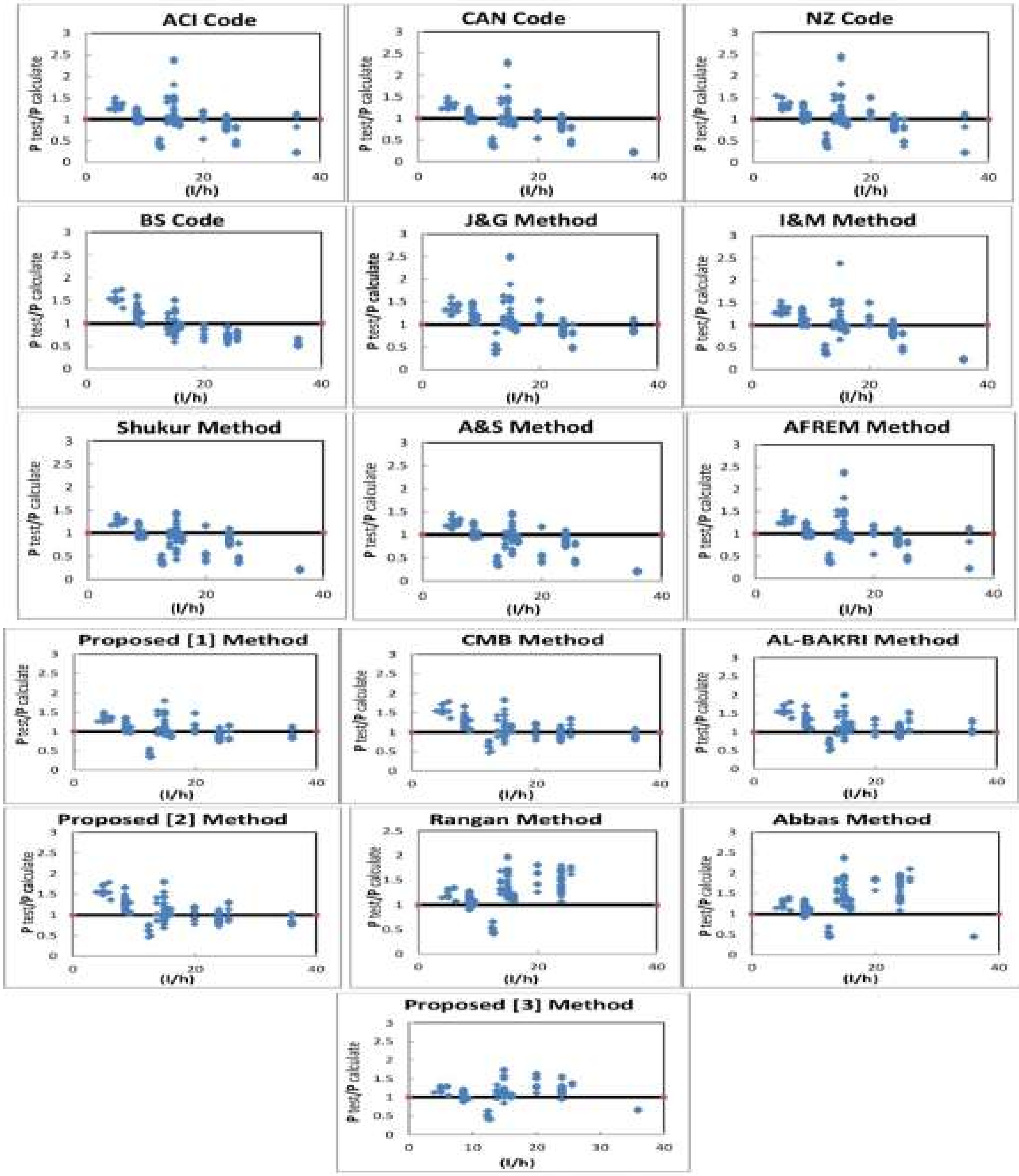


Figure (6): Effect of (*l*/h) on the strength ratio of overall reinforced concrete columns

2. Effect of e/h: Figure (7) show the effect of increasing e/h (eccentricity ratio) on the relative uniaxial strength value which represents the ratio of test to theoretical load. From the Figure (8) we can notice that the value of relative axial strength value decrease

3. with increasing e/h in ACI[4], CAN[5], NZ[6] and BS[7] Codes but in other methods still in almost the same level with increasing e/h.

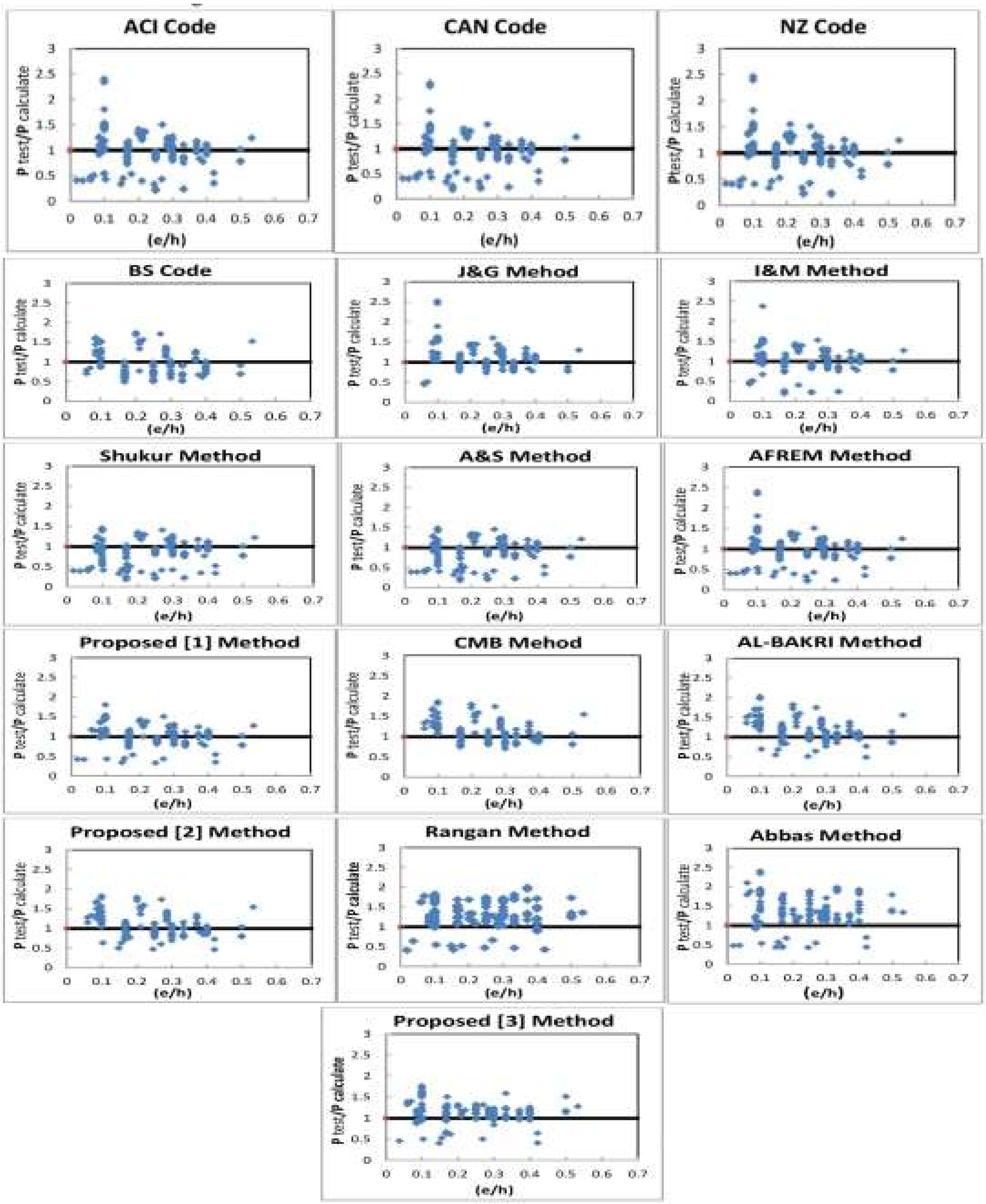


Figure (7): Effect of e/h (eccentricity ratio) on the strength ratio of overall reinforced concrete columns

4. Effect of ρ%: Figure (8) show the effect of increasing reinforcement ratio on uniaxial load. This figure show that some resistance predictions of test capacity are safe for the specimens columns in ACI[4], CAN[5], NZ[6] and BS[7] Codes, J&G[8], I&M[9], Shukur[10], A&S[11] and AFREM[12] Methods. As well as more note resistance prediction of test capacity are safe for the specimens columns in others methods.

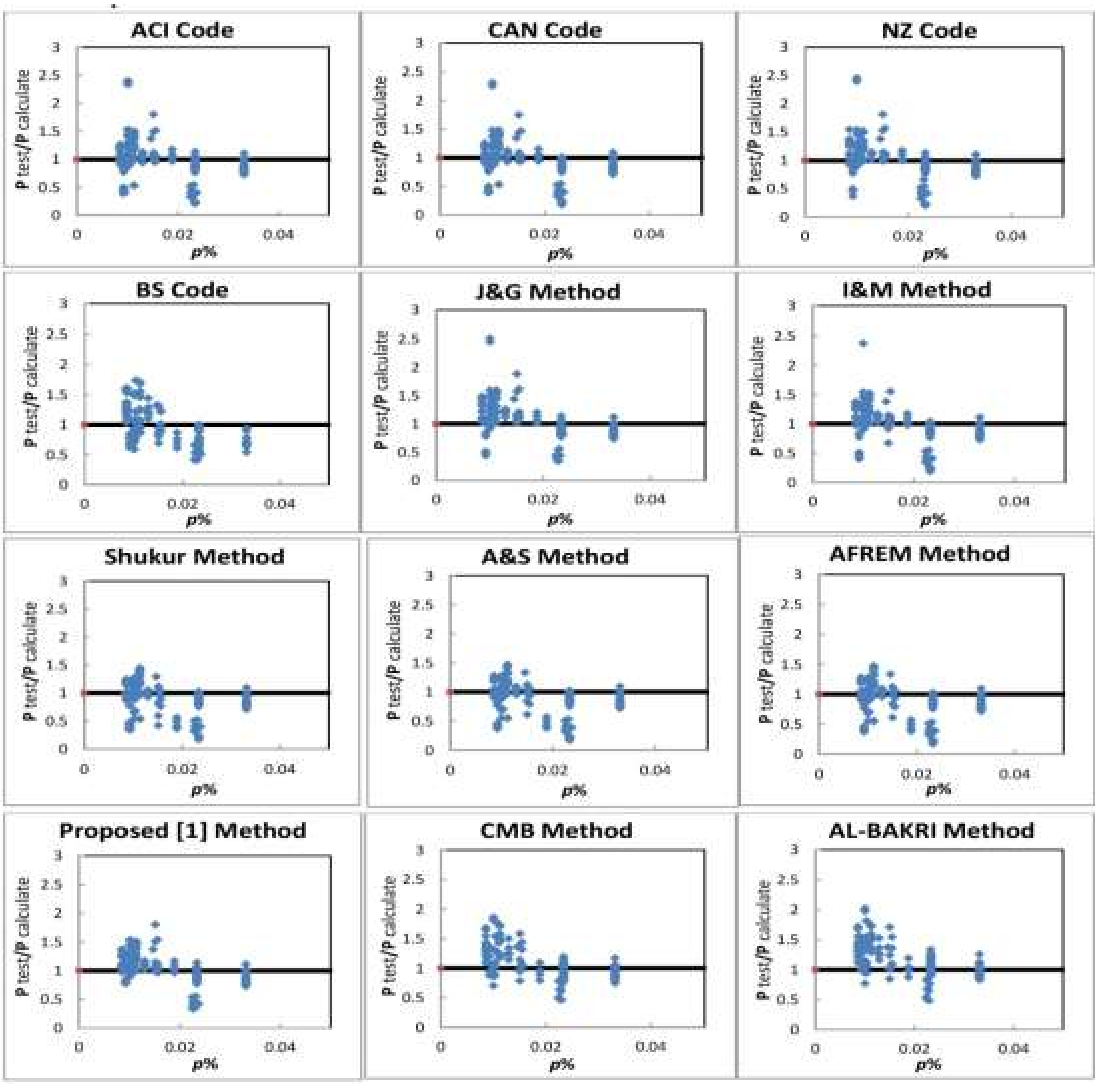


Figure (8): Continue

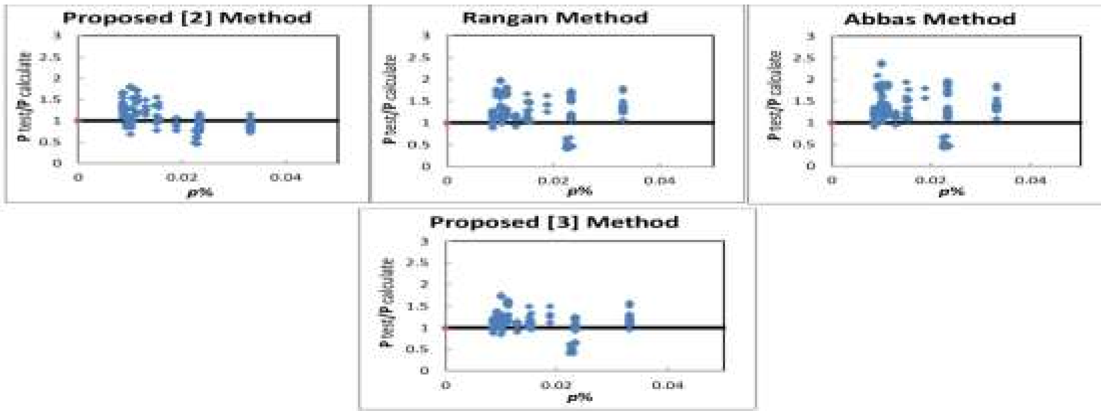


Figure (8): Effect of longitudinal steel ratio (ρ) on the strength ratio of overall reinforced concrete columns

CONCLUSIONS

The present study was prepared to investigate methods of slender column analysis and design. Comparisons were made between these methods to study the concrete stress distribution [rectangular stress block] for each method and study the safety factor for each one, differences between them and the major factors affecting these methods. The proposed equations of the proposed methods one of the best applicable equations for rectangular stress block for slender columns design and analysis. This method is more suitable for HSC columns.

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