Using Exponential smoothing Models in Forecasting about The Consumption of Gasoline in Iraq

Dr. Hazim Mansoor Gorgess   Luma Saad Abdul Zahra
Dept. of Mathematics/College of Education for Pure Science
(Ibn AL-Haitham)/University of Baghdad

Abstract

In this paper, the exponential smoothing approach is used in order to forecast about the consumption of Gasoline in Iraq for the years from 2014 to 2024 making use of the data obtained from the Oil Products Distribution Company. In this situation, three methods were applied, namely, Single, Double, and Winters' exponential smoothing. We employ; the mean absolute percentage error (MAPE), the mean absolute deviation (MAD) and mean squared deviation (MSD) as a criterion for compare between these methods. And we conclude that the winters' method is the best according to MAPE and MAD criterion.

Key Words: Forecasting , exponential smoothing , Single exponential smoothing , Double exponential smoothing , Winters' exponential smoothing , MAPE(mean absolute percentage error), MAD(mean absolute deviation), MSD(mean squared deviation).

Introduction

The formulation of exponential smoothing forecasting methods arose in the 1950's from the original work of Brown (1959,1962) and Holt (1960) who were working on creating forecasting models for inventory control systems. One of the basic ideas of smoothing models is to construct forecasts of future values as weighted averages of past observations with the more recent observation carrying more weight in determining forecasts than observations in the more distant past. By forming forecasts based on weighted averages we are using a "smoothing" method. The adjective "exponential" derives from the fact that some of the exponential smoothing models not only have weights that diminish with time but they do so in an exponential way, as in $\lambda_j = \lambda^j$ where -1< λ <1 and j=1,2,….represents the specific period in the past.

In this paper a detailed description of the three (ES) methods are presented, namely Single, Double and Winters' exponential smoothing.

1- Single Exponential Smoothing (SES)[3]

This model should be used when the time series data has no trend and no seasonality.

The specific formula for single exponential smoothing is :

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_t \qquad \ldots(1)$$

Where

y_t is the actual value for time period t ,

$\hat{y}_t$ is the forecast value of the variable y for time period t ,

$\hat{y}_{t+1}$ is the forecast value for time t + 1 and

α is smoothing constant ($0<\alpha<1$)

The forecast $\hat{y}_{t+1}$ is based on weighting the most recent observation with y_t a weight α and weighting the most recent forecast $\hat{y}_t$ with a weight of $(1 - \alpha)$.

To state the algorithm, we need an initial forecast, an actual value and a smoothing constant since $\hat{y}_t$ is not known, we can:

(i) Set the first estimate equal to the first observation. Thus we can use $\hat{y}_t = y_t$

(ii) Use the average of the first five or six observations for the initial smoothed value.

Smoothing constant α is a selected number between zero and one, $0 < \alpha < 1$. When $\alpha = 1$, the original and smoothed version of the series are identical. At the other extreme, when $\alpha = 0$, the series is smoothed flat. Rewriting the model (1) to see one of the neat things about the (SES) model:

$$\hat{y}_{t+1} - \hat{y}_t = \alpha(y_t - \hat{y}_t) \qquad \ldots(2)$$

That is , the new one-step ahead forecast is the previous forecast , partially adjusted by the amount that forecast was in error. Since this expression considers only the one-step-ahead forecasts, it may also be written as :

$$\hat{y}_{t-1} = \hat{y}_t + \alpha e_t \qquad \ldots(3)$$

Where residual $e_t = y_t - \hat{y}_t$ is forecast error for time period t.

So, the exponential smoothing forecast is the old forecast plus an adjustment for the error that occurred in the last forecast, [4]. By continuing to substitute previous forecasting value back to the stating point of the data we get:[3]

$$\hat{y}_{t+1} = \alpha y_t + \left(1-\alpha\right)\hat{y}_t$$

Thus we can write:

$$\hat{y}_t = \alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1}$$

Hence,

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1}\right]$$

$$= \alpha y_t + \alpha(1-\alpha) y_{t-1} + (1-\alpha)^2 \hat{y}_{t-1}$$

Resubstituting the value of $\hat{y}_{t-1}$ we get:

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \alpha(1-\alpha)^3 \hat{y}_{t-3}$$

The last formula can be extended as following:

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + ... + \alpha(1-\alpha)^{t-2} y_t + \alpha(1-\alpha)^{t-1} y_1$$

Or

$$\hat{y}_{t+1} = \alpha \sum_{k=0}^{t-1} (1-\alpha)^k y_{t-k} \qquad \ldots(4)$$

Where $\hat{y}_{t+1}$ is the weighted moving average of all past observations. The series of weights used in producing the forecast $\hat{y}_{t+1}$ is α, $\alpha(1-\alpha)$, $\alpha(1-\alpha)^2$, …. . These weights decline toward zero in an exponential fashion; thus as we go back in the series, each value has a smaller weight in terms of its effect on the forecast.

2- Double Exponential Smoothing

Holt (1957) extended single exponential smoothing to linear exponential smoothing to allow forecasting of data with trends. Exponential smoothing with a trend works much like from single smoothing except that the two components must be updated each period, namely, level and trend. The forecast for Double exponential smoothing is found by using two smoothing constants, α and β (with values between 0 and 1). The level is a smoothed estimate of the value of the data at the end of each period. The trend is a smoothed estimate of average growth at the end of each period.

The specific formula for this method is:[1]

Level $s_t = \alpha y_t + (1-\alpha)[s_{t-1} + b_{t-1}]$, $0 < \alpha < 1$ …(5)

Trend $b_t = \beta(s_t - s_{t-1}) + (1-\beta)\, b_{t-1}$, $0 < \beta < 1$ …(6)

Forecasts are made with the expression:

$H_{t+m} = s_t + mb_t$ …(7)

where

s_t is forecast value for period t + 1, which can be denoted as $\hat{y}_{t+1}$, y_t is actual value in period t,
α smoothing constant for the data $0 < \alpha < 1$,
s_{t-1} forecast value for period t,
b_{t-1} is trend,
β is smoothing constant for the trend estimate $0 < \beta < 1$,
m is the number of periods ahead to be forecast, and
H_{t+m} is Double forecast value of period t + m.
Double exponential smoothing is sometimes called "Holt's exponential smoothing" (HES).

3- Winters' Exponential Smoothing

The Winters' model contains three parameters, one for actual data and the other two are for trend and seasonal factors. Four equations can be formulated:[2]

$$F_t=\alpha(Y_t/S_{t-p})+(1-\alpha)(F_{t-1} + T_{t-1}) \quad ...(8)$$

$$S_t=\gamma(Y_t/S_t)+(1-\gamma)(S_{t-p}) \quad ...(9)$$

$$T_t = \beta\left(F_t - F_{t-1}\right) + \left(1-\beta\right)T_{t-1} \quad ...(10)$$

$$W_{t+m}=(F_t+mT_t)S_{t+m-p} \quad ...(11)$$

where
F_t is smoothed value of the level for period t,
F_{t-1} is smoothed value for period t – 1,
Y_t actual value in period t,
T_t trend estimate,
S_t seasonality estimate,
α smoothing constant for the data ($0 < \alpha < 1$),
β smoothing constant for trend estimate ($0 < \beta < 1$),
γ smoothing constant for seasonality estimate ($0 < \gamma < 1$),
P number of periods in seasonal cycle,
m number of periods ahead to be forecast,
w_{t+m} winters' forecast for m periods into the future,
There are two main winter's exponential models, depending on the type of seasonality:

(1) **Additive winters' model** a time series with a local linear trend and an additive seasonality can be represented by a model of the form:

$$y_t = \beta_0 + \beta_1 t + S_t + \in_t \quad ...(12)$$

Where S_t is the seasonal factor at time t.

$\in_t$ is the denotes a random error term.

(2) **Multiplicative winters' model** a time series with a local linear trend and a multiplicative seasonality can be represented by a model of the form:

$$y_t = (\beta_0 + \beta_1 t) * S_t + \in_t \qquad \ldots(13)$$

4- Description of Various Forecast Performance Measures

In this section we discuss about the commonly used performance measures and their important properties. Let us assume that y_t t=1,2,…,n is the actual value, f_t is the forecasted value, $e_t = y_t - f_t$ is the forecast error.

4.1 The Mean Absolute Percentage Error (MAPE)[5]

This measure is given by MAPE = $\frac{1}{n}\sum_{t=1}^{n}\left|\frac{e_t}{y_t}\right| * 100$

Its important features are:

- This measure represents the percentage of average absolute error occurred.
- It is independent of the scale of measurement, but affected by data transformation.
- It does not show the direction of error.
- MAPE does not panelize extreme deviations.
- In this measure, opposite signed errors do not offset each other.

4.2 The Mean Squared Deviation (MSD)[8]

One of commonly used measure of accuracy of fitted time series values. Is known as mean squared deviation (MSD) which is defined as:

MSD = $\frac{\sum_{t=1}^{n}\left|y_t - \hat{y}_t\right|^2}{n}$ where y_t equals the actual value , $\hat{y}_t$ equals the forecast value ,and n equals the number of forecasts.

4.3 The Mean Absolute Deviation (MAD)[9]

An alternative measure of accuracy of fitted time series values is known as mean absolute deviation (MAD) which is given by the formula

MAD= $\frac{\sum_{t=1}^{n}\left|y_t - \hat{y}_t\right|}{n}$. This measurement is less affected by the outliers than the MSD.

5- Numerical Example

In this section we use the exponential smoothing models disculled earlier to analyze the time series for the Gasoline consumption in Iraq from the year 2004 to 2013 then try to forecast the values of consumption from 2014 to 2024.

<table>
<tr><th rowspan="2">MAPE</th><th colspan="3">Parameters</th><th rowspan="2" colspan="2">Smoothing Method</th></tr>
<tr><th>β</th><th>α</th><th>γ</th></tr>
<tr><td>0.488091</td><td>-</td><td>0.939612</td><td>-</td><td colspan="2">Single expon. smooth</td></tr>
<tr><td>0.497486</td><td>-</td><td>1.03169</td><td>0.00999</td><td colspan="2">Double expon. smooth</td></tr>
<tr><td>0.485637</td><td>0.2</td><td>0.2</td><td>0.2</td><td>Multiplicative winters'</td><td rowspan="2">winters' expon. smooth</td></tr>
<tr><td>0.483318</td><td>0.2</td><td>0.2</td><td>0.2</td><td>Additive winters'</td></tr>
</table>

Table (1) represents smooth methods which applying use MAPE criterion to choose the best parameters for Gasoline series.

<table>
<tr><th rowspan="2">MAD</th><th colspan="3">Parameters</th><th rowspan="2" colspan="2">Smoothing Method</th></tr>
<tr><th>β</th><th>α</th><th>γ</th></tr>
<tr><td>0.064294</td><td>-</td><td>0.939612</td><td>-</td><td colspan="2">Single expon. smooth</td></tr>
<tr><td>0.065531</td><td>-</td><td>1.03169</td><td>0.00999</td><td colspan="2">Double expon. smooth</td></tr>
<tr><td>. 0063859</td><td>0.2</td><td>0.2</td><td>0.2</td><td>Multiplicative winters'</td><td rowspan="2">winters' expon. smooth</td></tr>
<tr><td>0.063541</td><td>0.2</td><td>0.2</td><td>0.2</td><td>Additive winters'</td></tr>
</table>

Table (2) represents smooth methods which applying use MAD criterion to choose the best parameters for Gasoline series.

<table>
<tr><th rowspan="2">MSD</th><th colspan="3">Parameters</th><th rowspan="2" colspan="2">Smoothing Method</th></tr>
<tr><th>β</th><th>α</th><th>γ</th></tr>
<tr><td>0.006286</td><td>-</td><td>0.939612</td><td>-</td><td colspan="2">Single expon. smooth</td></tr>
<tr><td>0.006545</td><td>-</td><td>1.03169</td><td>0.00999</td><td colspan="2">Double expon. smooth</td></tr>
<tr><td>0.006776</td><td>0.2</td><td>0.2</td><td>0.2</td><td>Multiplicative winters'</td><td rowspan="2">winters' expon. smooth</td></tr>
<tr><td>0.006856</td><td>0.2</td><td>0.2</td><td>0.2</td><td>Additive winters'</td></tr>
</table>

Table (3) represents smooth methods which applying use MSD criterion to choose the best parameters for Gasoline series.

Period	Forecast Value	Forecast Period , α = 0.5	
		Lower	Upper
121	13.3038	13.1463	13.4613
122	13.3038	13.1463	13.4613
123	13.3038	13.1463	13.4613
124	13.3038	13.1463	13.4613
125	13.3038	13.1463	13.4613
126	13.3038	13.1463	13.4613
127	13.3038	13.1463	13.4613
128	13.3038	13.1463	13.4613
129	13.3038	13.1463	13.4613
130	13.3038	13.1463	13.4613

Table(4) represents forecast value for Gasoline series consumption by using single exponential method .

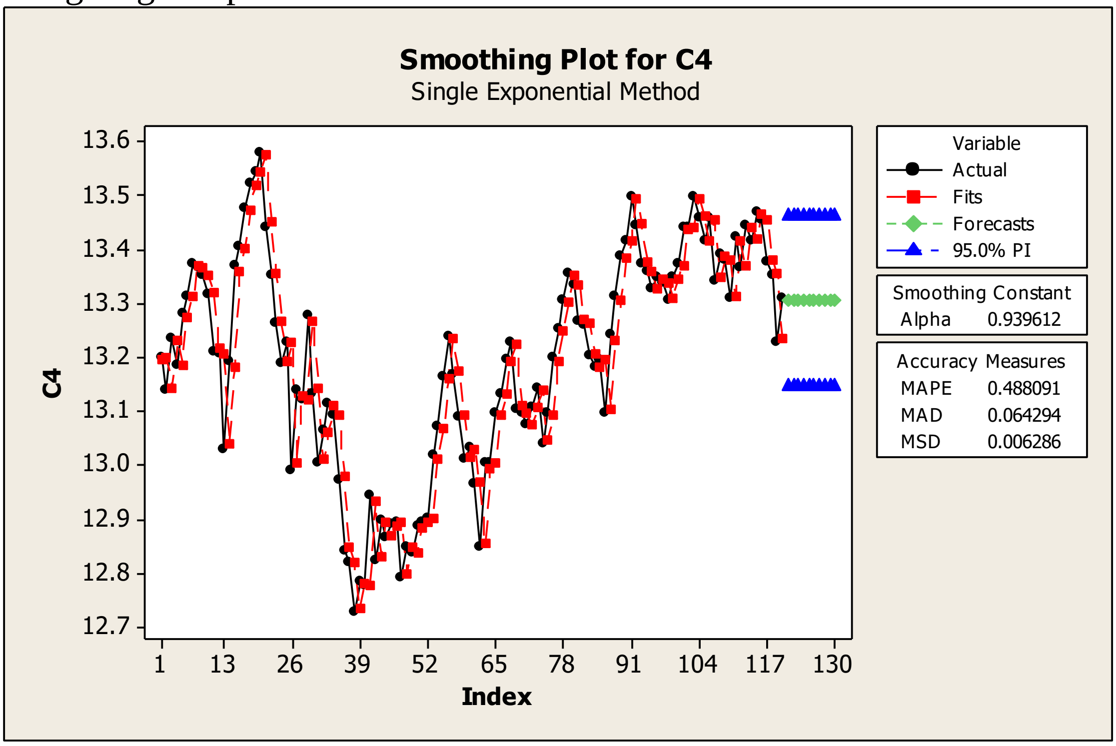

Figure (1) refers to single exponential for Gasoline series with α=0.939612

Period	Forecast Value	Forecast Period , α = 0.5	
		Lower	Upper
121	13.3099	13.1493	13.4704
122	13.3088	13.0583	13.5593
123	13.3076	12.9630	13.6523
124	13.3065	12.8662	13.7469
125	13.3054	12.7687	13.8421
126	13.3043	12.6708	13.9378
127	13.3032	12.5726	14.0337
128	13.3020	12.4744	14.1227
129	13.3009	12.3760	14.2259
130	13.2998	12.2775	14.3221

Table(5) represents forecast value for Gasoline series consumption by using Double exponential method .

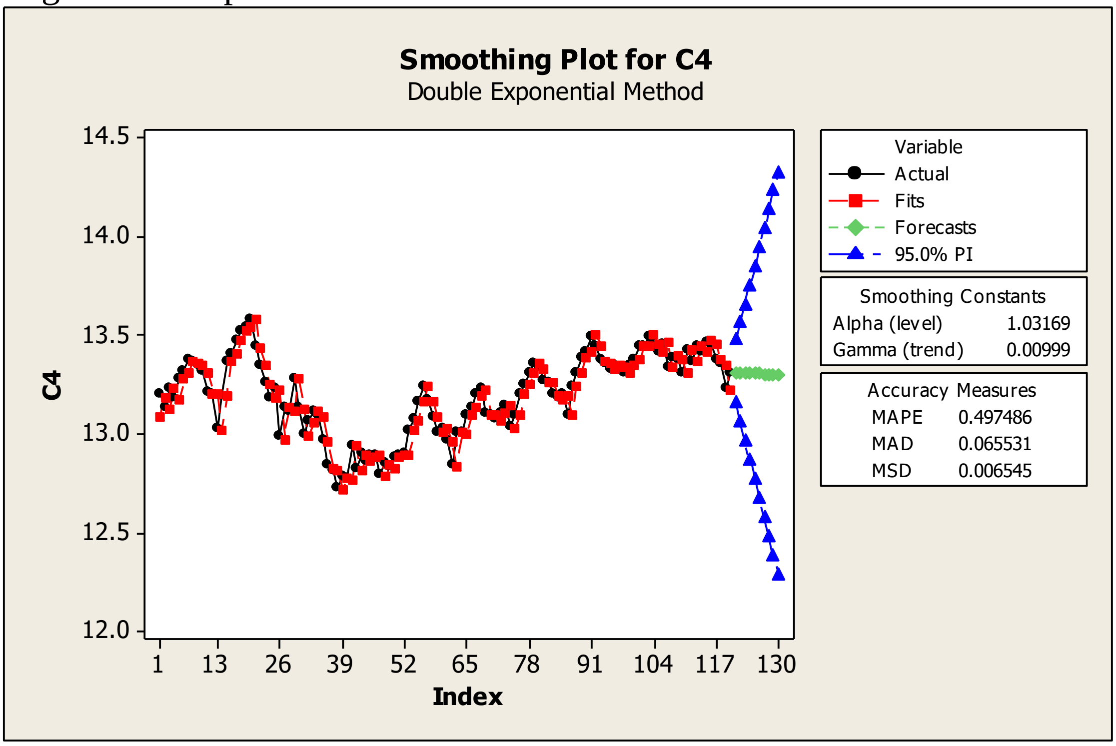

Figure (2) refers to Double exponential for Gasoline series with **α=1.03169** and γ=**0.00999**

Period	Forecast Value	Forecast Period , α = 0.5	
		Lower	Upper
121	13.2515	13.0951	13.4080
122	13.1676	13.0087	13.3265
123	13.2474	13.0858	13.4091
124	13.2461	13.0815	13.4107
125	13.3241	13.1562	13.4920
126	13.3121	13.1407	13.4835
127	13.3505	13.1754	13.5256
128	13.3338	13.1547	13.5128
129	13.2653	13.0822	13.4485
130	13.2426	13.0552	13.4301

Table(6) represents forecast value for Gasoline series consumption by using Winters' exponential method , (Multiplicative Method).

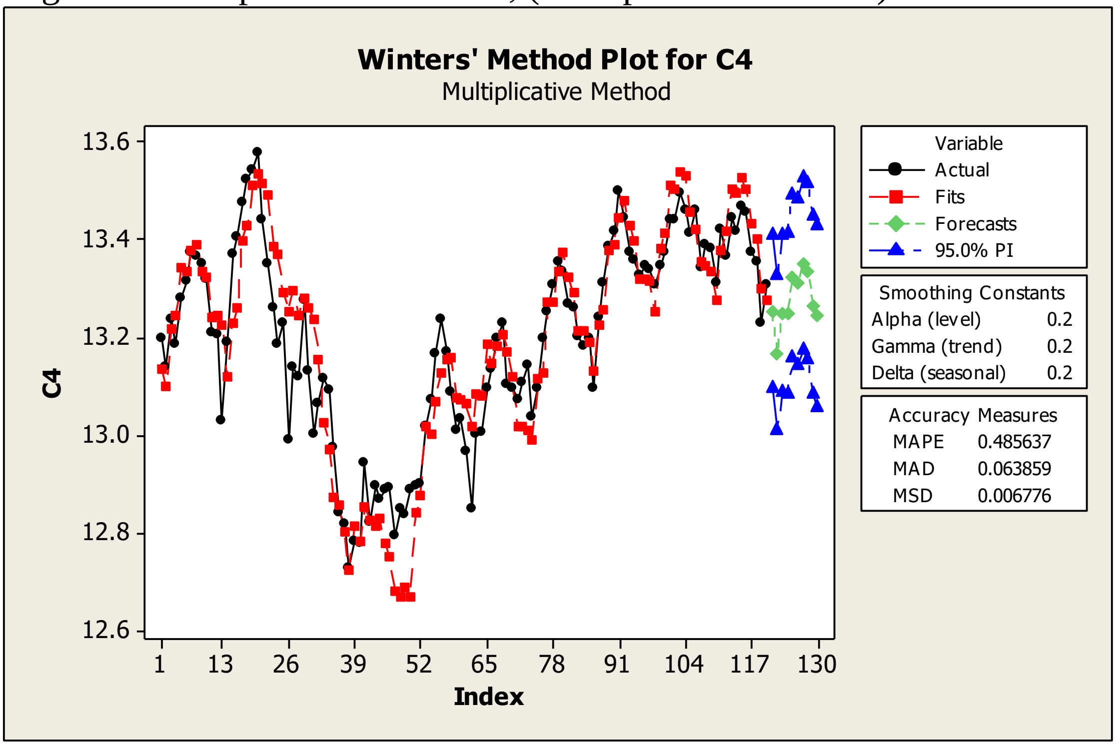

Figure (3) refers to winters method (Multiplicative Method) for Gasoline series with **α=0.2 , β=0.2 and γ=0.2**

Period	Forecast Value	Forecast Period , α = 0.5	
		Lower	Upper
121	13.2530	13.0973	13.4086
122	13.1698	13.0117	13.3279
123	13.2497	13.0889	13.4105
124	13.2486	13.0848	13.4124
125	13.3264	13.1593	13.4934
126	13.3147	13.1442	13.4852
127	13.3533	13.1791	13.5275
128	13.3364	13.1582	13.5145
129	13.2675	13.0853	13.4498
130	13.2446	13.0581	13.4311

Table(7) represents forecast value for Gasoline series consumption by using Winters' exponential method , (Additive Method).

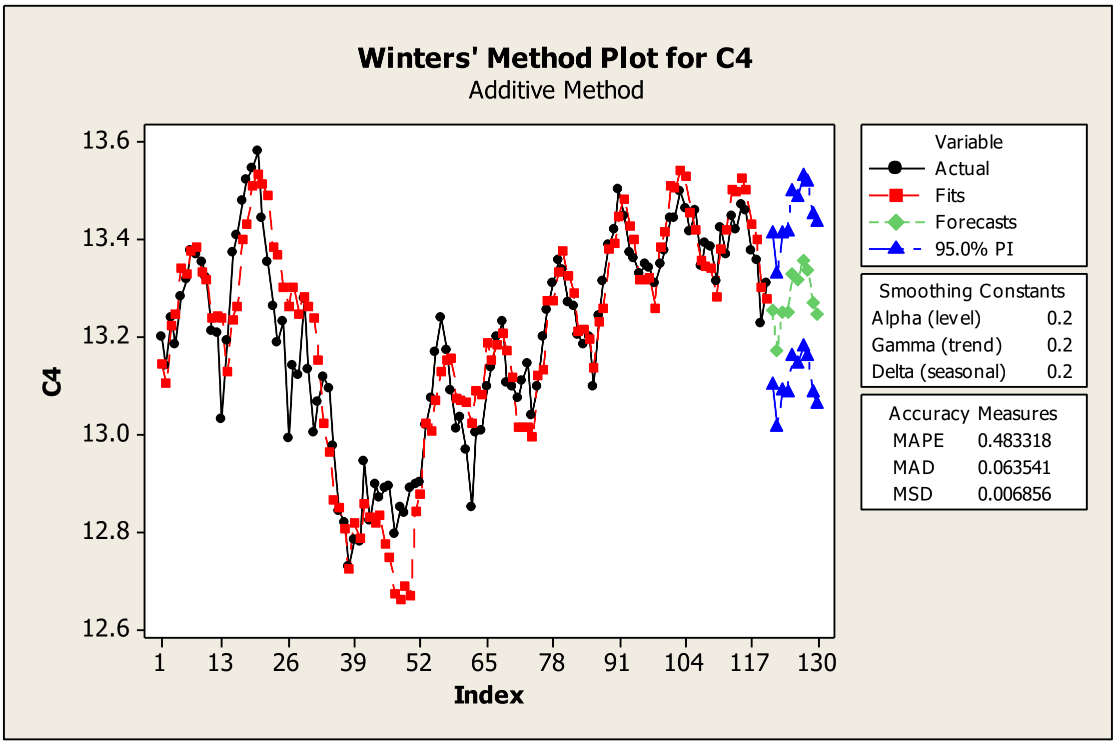

Figure (4) refers to winters method (Additive Method) for Gasoline series with **α=0.2 , β=0.2 and γ=0.2**

6- Conclusion

By applying MAPE, MAD as measures of performance we conclude that the additive Winters' Method is the best while the Single exponential smoothing is the best when the measurement MSD was applied.

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أستخدام نماذج التمهيد الأسي للتنبؤ باستهلاك مادة البنزين في العراق

د. حازم منصور كوركيس **لمى سعد عبد الزهرة**

قسم علوم الرياضيات/ كلية التربية للعلوم الصرفة(ابن الهيثم)/ جامعة بغداد

الخلاصة

في هذا البحث , تم أستخدام أسلوب التمهيد الأسي للتنبؤ بأستهلاك مادة البنزين في العراق للسنوات من 2014–2024 بأستخدام بيانات تم الحصول عليها من شركة توزيع المنتجات النفطية. وفي هذا الصدد تم أستخدام ثلاثة طرق هي التمهيد الأسي البسيط , التمهيد الأسي المزدوج وطريقة ونترز للمتسلسلات الموسمية وقد أعتمدت المعايير MAPE, MAD, MSD لغرض المقارنة بين هذه الطرق وقد أستنتجنا أن طريقة ونترز هي الأفضل بأستخدام المعيار MAD, MAPE .

الكلمات المفتاحية: التنبؤ، التمهيد اللأسي ، التمهيد الأسي البسيط، التمهيد الأسي المزدوج، التمهيد الأسي بواسطة ونترز للمتسلسلات الموسمية MAPE(متوسط النسبة المطلقة للخطأ)، MAD(متوسط القيمة المطلقة للخطأ)، MSD(متوسط مربعات الانحرافات).