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ULTIMATE STRENGTH OF CLAMPED R.C SQUARE SLABS WITH CENTRAL SQUARE OPENING UNDER COMBIEND BENDING AND MEMBRANE ACTION

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المقاومة القصوى في بلاطة خرسانية مربعة الشكل ذات فتحة مركزية مربعة الشكل ومقيدة من جميع الإتجاهات تحت التأثير المشترك للإنحاء والقوة الغشائية

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الخلاصة

:بالاعتماد على نظرية سيولة الاجسام الجاسئة اللدنة تم استحداث طريقة جديدة لاحتساب الحمولة القصوى وايجاد التصرف اللدن لبلاطات خرسانية مسلحة . مربعة الشكل . ومقيدة الجوانب ضد الدوران والحركة المحورية . وتحتوي على فتحة مربعة الشكل في منتصفها . ومحملة حملا منتظما. تأخذ الطريقة بنظر الاعتبار التأثيرات المهمة للقوى الغشائية المتولدة في مستوى البلاطة على طول خطوط الخضوع اثناء هطول البلاطة. استخدمت قوانين الاتزان والتوافق لعناصر البلاطة المشوهة لا شتقاق علاقات الحمل بالهطول بدءاً بالضغط الغشائي المبكر ولحين حصول تشققات نافذة في البلاطة وشد غشائي مركزي. يشير التحليل الى الزيادة الواضحة في تحمل البلاطات المسندة محوريا يفوق تخمينات نظرية خط الخضوع ، وان الزيادة في نسبة الحمولة هذة تتضاعف كلما قل في البلاطة سمكها وحجم فتحتها وخف التسليح فيها . عند قيمة معينة للهطول تم احتساب الاحمال بطريقة النظرية الجديدة فوجدت مقاربة لنتائج الفحوصات السابقة. اجريت دراسة نظرية على قيمة الحمولة القصوى فاتضح انه بالامكان تقليل حديد التسليح باستخدام خرسانة اقوى. تمت التوصية على امكانية توفير بحدود ٤٨% من حديد التسليح المصمم بطريقة خط الخضوع لبلاطة وسطى وتحتوي على فتحة مصعد مربعة الشكل في نظام سقفي لاحمال خفيفة و ٢٥% عندما تكون الاحمال معتدلة .

Abstract

Based on the flow theory of rigid-plastic bodies, a method is developed for assessing the ultimate strength as well as determining the post yield behavior of uniformly loaded R.C square slabs with central square opening which have all edges restrained against rotation and lateral movement . The method takes into account the significant effects of membrane forces which are usually induced in the plane of the slab along sagging and hogging yield lines as the slab deflects. Considering equilibrium and compatibility of the deformed slab element, load-deflections relations are derived starting from the initial compressive membrane action up to tensile membrane and full-depth cracking at large deflection.

The solution shows that axially restrained slabs can sustain loads far beyond those predicated by Johansen's yield line theory and the enhancement in load is greatest for thinner slabs having lighter reinforcement and smaller opening. On the basis of an estimated deflection, theoretical ultimate loads are found comparable with those of existing experimental tests. A theoretical study of the maximum yield load reveals a promising saving in reinforcement for using stronger concrete. For an interior square panel with central square opening, such saving in reinforcement could be as high as 48% if the panel is lightly loaded, and 25% for a panel under moderate loads.

1. Introduction

In R.C slabs, bending is usually accompanied by lateral displacement at the edges of the slab. In axially restrained slabs, such displacements are prevented by the support restraints and, therefore, in-plane compressive membrane forces appear. These forces generate higher moment capacities for the slab sections at yield lines and consequently enable the slab to carry ultimate loads that are far in excess than those indicated by yield line theory [1], see Fig.(1). Even in the case of unrestrained slabs, a self balanced in-plane membrane forces have been found [2] to take place in the slab which result in moderate increases in the yield load with continuing deflection.

Previous studies of membrane action in axially restrained slabs have rendered several rigorous solutions. These are in particular, the rigid-plastic and elastic–plastic solutions of slab strips [3, 4, 5, 6].

More studies on the problem were experimental in natural [7, 8, 9] and empirical correlations were used to explain the high peak loads obtained in tests. In this research, a rigorous method is presented for the analysis of membrane action in clamped R.C square slabs with central square opening covering a wider range of slab deflections and taking into consideration the crucial effects of the in-plane axial forces.

2. Basic Assumptions

To determine the effect of membrane action in clamped R.C square slabs with central square opening, it is necessary to make several assumptions. These are as follows:

1. The slab is carrying a uniformly distributed load.
2. The materials are rigid-perfectly plastic.

3. The slab is considered, isotropically reinforced in the bottom face only at middle and column strips and with same amount of reinforcement in the top face only at support.
4. The slab yields under the simultaneous action of bending moment M and a compressive axial force N acting at the slab mid-depth.

3. Yield Criterion

For the stress distribution on the slab section at yield shown in Fig. (2), the equilibrium equations of axial forces and bending moment are:

$$N = C - T_{\circ} \qquad (1)$$

$$M = C\left(\frac{h}{2} - k_2 a\right) + T_{\circ}\left(d - \frac{h}{2}\right) \qquad (2)$$

Where $C = k_1 k_3 \; f_c' \; a$, $T_{\circ} = A_s \; f_y = \rho \; d \; f_y$.

And the concrete compressive stress block parameter k_1k_3 and k_2 are Hognestad [10] parameters; $k_1 k_3 = \dfrac{27 + 0.35 f_c'}{22 + f_c'}$, $k_2 = 0.5 - \dfrac{f_c'}{550}$

A combination of equations (1) and (2) leads to the following non-dimensional yield criterion:

$$\frac{M}{M_{\circ}} = 1 + \alpha\left(\frac{N}{T_{\circ}}\right) - \beta\left(\frac{N}{T_{\circ}}\right)^2 \qquad (3)$$

Where M_o is the yield moment corresponding to N=0;

$M_{\circ} = \rho \; f_y \, d^2 \; (1 - \dfrac{k_2}{k_1 k_3}\rho\dfrac{f_y}{f_c'})$ and $\alpha = \dfrac{\left(\dfrac{1}{2}\dfrac{h}{d} - \dfrac{2k_2}{k_1k_3}\rho\;\dfrac{f_y}{f_c'}\right)}{\left(1 - \dfrac{k_2}{k_1k_3}\rho\;\dfrac{f_y}{f_c'}\right)}$, $\beta = \dfrac{\dfrac{k_2}{k_1k_3}\rho\;\dfrac{f_y}{f_c'}}{\left(1 - \dfrac{k_2}{k_1k_3}\rho\dfrac{f_y}{f_c'}\right)}$ are constants for a particular slab section.

Where A_s is Area of tensile reinforcement per unit-width of slab, a is depth of the equivalent rectangular compression block of concrete, C is compressive force on concrete unit-width of slab, $T_{\circ}$ is Yield force in tensile reinforcement per unit-width of slab , d is effective depth of slab , f_c' is concrete cylinder strength, f_y is yield stress of steel reinforcement, h is Overall depth of slab , ρ is Ratio of steel area to effective area of concrete $= A_s / d$.

If this yield criterion is denoted by a function *f*, the ratio of the plastic axial rate $d\varepsilon$ to the plastic curvature rate dk according to the plastic potential flow rule must be ; $\frac{d\varepsilon}{dk} = \mu = \frac{\partial f / \partial N}{\partial f / \partial M} = \frac{\frac{\alpha}{T_\circ} - 2\beta \frac{N}{T_\circ^2}}{\frac{1}{M_\circ}}$

$$\frac{N}{T_\circ} = \frac{\alpha}{2\beta} - \frac{\mu}{2\beta}\frac{T_\circ}{M_\circ} \qquad (4)$$

With deflection of the slab, the neutral axis (μ) moves towards the compressed face of the section .A case will then be reached when the crack penetrates throughout the whole thickness of the slab. Thus, for two possible ranges of the neutral axis depth μ, the form of the yield criterion will be (as in Table -1).

4. Yield Mechanism and Compatibility Equations

An initial collapse mode of the type shown in Fig (3) is considered and assumed at large deflection. If the vertical deflection at the edges of the opening is Δ, the corresponding rotation of the trapezoidal middle surface element (Fig.4) relative to the supporting edges is θ. This plastic rotation is related to the plastic axial elongations e_1 at hogging yield lines and e_2, e_o at sagging yield lines according to the following compatibility equations:

<u>Sec. 1-1</u>: $2e_1 + 2\frac{L}{2}(1-R)\cos\theta + 2e_\circ \sin 45 + RL = L$.

For small angle of θ, $\cos\theta = 1 - \frac{\theta^2}{2}$

$$e_1 + e_\circ \sin 45 = \frac{L}{4}(1-R)\theta^2 \qquad (7)$$

<u>Sec. 2-2</u>: $e_1 + e_2 \sin 45 = \frac{\theta^2}{2} x$ (8)

A combination of equations (7) and (8) gives:

$$e_2 \sin 45 - e_\circ \sin 45 = \frac{\theta^2}{2}\left[x - \frac{L}{2}(1-R)\right] \qquad (9)$$

By differentiating Eq. (7) and (9) with respect to θ and noting that (see Fig.5)

$\mu_1 = \frac{de_1}{d\theta}$, $\mu_\circ = 0.707\frac{de_\circ}{d\theta}$, $\mu_2 = 0.707\frac{de_2}{d\theta}$ and $\theta = \frac{2\Delta}{L(1-R)}$ (see Fig.4) give:

$$\mu_1 = \Delta - \mu_\circ \qquad (10)$$

$$\mu_2 = \mu_\circ - \Delta\left(1 - \frac{2x}{L(1-R)}\right) \qquad (11)$$

Where L is span of square slab , R is ratio of opening span to slab span.
Eq. (10) defines the neutral axis depth along hogging yield lines whereas Eq.(11) defines the depth of the neutral axis at the four inclined yield lines.

5. Horizontal Equilibrium Equations

The only unknown in Eqs.(10) and (11), for a given deflection (Δ),is ($\mu_\circ$) and this may be determined by considering the horizontal equilibrium of trapezoidal element in Fig.(6).

For $\mu_\circ \le h/2$: Resolving forces perpendicular to the fixed edge of the trapezoidal element gives; $2\int_0^{\frac{L}{2}(1-R)\csc 45} N_2 ds \ \cos 45 - N_1 L = 0$ Where $ds = dx \csc 45$

The values of the membrane forces N_1, N_2 acting perpendicular to the yield lines, for this case of $\mu_\circ \le h/2$, are obtained by substituting the values of μ_1, μ_2 defined by Eqs.(10)and (11) each at a time into Eq.(5a), which give;
$\frac{N_1}{T_\circ} = \frac{\alpha}{2\beta} + \frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ} - \frac{\Delta}{2\beta}\frac{T_\circ}{M_\circ}$, $\frac{N_2}{T_\circ} = \frac{\alpha}{2\beta} - \frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ} + \frac{\Delta}{2\beta}(1 - \frac{2x}{L(1-R)})\frac{T_\circ}{M_\circ}$

When these values of the membrane forces are used, the horizontal equilibrium equation for trapezoidal element becomes:

$$2T_\circ \int_0^{\frac{L}{2}(1-R)} \left[\frac{\alpha}{2\beta} - \frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ} + \frac{\Delta}{2\beta}(1 - \frac{2x}{L(1-R)})\frac{T_\circ}{M_\circ}\right] dx \ - T_\circ \left[\frac{\alpha}{2\beta} + \frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ} - \frac{\Delta}{2\beta}\frac{T_\circ}{M_\circ}\right] L = 0 \qquad (12)$$

Solving.(12) gives: $$\mu_\circ = \frac{1}{2-R}\left[\Delta(1.5 - 0.5R) - \frac{\alpha R \frac{h}{2}}{\alpha + 2\beta}\right] \qquad (13)$$

The limiting deflection Δ' for which Eq.(13) is valid is obtained when $\mu_\circ = h/2$ (the slab is cracked throughout the depth at end of sagging yield lines);

$$\frac{\Delta'}{h} = \frac{1}{3-R}\left[2 - R + \frac{\alpha R}{\alpha + 2\beta}\right] \qquad (14)$$

For $\mu_\circ \ge h/2$: Once the deflection at edge of the opening exceeds the value (Δ'), the tensile membrane action (N=-T_0) extends from($x = \frac{L}{2}(1-R)$) to a position ($x = \frac{L'}{2}(1-R)$) as in Fig.(6b) , there will be a discontinuity in the distribution of (N) along the inclined yield lines at ($x=\frac{L'}{2}(1-R)$) and the horizontal equilibrium equation becomes;

$$2T_\circ \int_0^{\frac{L'}{2}(1-R)} \left[\frac{\alpha}{2\beta}-\frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ}+\frac{\Delta}{2\beta}(1-\frac{2x}{L(1-R)})\frac{T_\circ}{M_\circ}\right] dx+2T_\circ \int_{\frac{L'}{2}(1-R)}^{\frac{L}{2}(1-R)} (-1)dx-T_\circ\left[\frac{\alpha}{2\beta}+\frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ}-\frac{\Delta}{2\beta}\frac{T_\circ}{M_\circ}\right]L=0 \qquad (15)$$

Solving Eq. (15) gives:

$$\mu_\circ=\frac{h}{2}\frac{[\frac{\alpha}{2\beta}(\frac{L'}{L})-\frac{\alpha}{2\beta}\frac{1}{1-R}+(\frac{L'}{L}-1)]}{[(\frac{\alpha}{2\beta}+1)(\frac{L'}{L})+(\frac{\alpha}{2\beta}+1)\frac{1}{1-R}]}+\Delta\left(1-\frac{\frac{1}{2}(\frac{L'}{L})^2}{[(\frac{L'}{L})+\frac{1}{1-R}]}\right) \qquad (16)$$

But at $x=\frac{L'}{2}(1-R)$; $\frac{N_2}{T_\circ}=\frac{\alpha}{2\beta}-\frac{\mu_\circ}{2\beta}\frac{T_\circ}{M_\circ}+\frac{\Delta}{2\beta}(1-\frac{L'}{L})\frac{T_\circ}{M_\circ}=-1$ Solving for $\mu_\circ$;

$$\mu_\circ=\frac{h}{2}+\Delta(1-\frac{L'}{L}) \qquad (17)$$

Solving Eq.(16)and (17) gives:

$$\mu_\circ=\Delta\left[1-\frac{\left(1+\frac{\alpha}{2\beta}\frac{1}{1-R}\right)(\frac{L'}{L})+\frac{1}{2}(\frac{\alpha}{2\beta}+1)(\frac{L'}{L})^2}{1+(\frac{\alpha}{\beta}+1)\frac{1}{1-R}}\right] \qquad (18)$$

From Eqs.(17)and (18), the value of ($\frac{L'}{L}$) can be obtained to be;

$$\frac{L'}{L}=\frac{1}{1-R}\left[\sqrt{1+2(1-R)\left(\frac{h}{\Delta}\right)\left(1-(\frac{\beta}{\alpha+2\beta})R\right)}-1\right] \qquad (19)$$

6. Yield Moment and Axial Forces

For $\Delta\le\Delta'$:The axial force can be determined by introducing the value of μ_o from Eq. (13) into Eq. (5a) and with use of Eqs. (10) and (11) the results will be:

$$\frac{N_1}{T_\circ} = \frac{\alpha}{2\beta} + (\frac{\alpha}{2\beta} + 1)\left(\frac{R\alpha}{(R-2)(\alpha + 2\beta)} - \frac{1-R}{2-R}\frac{\Delta}{h} \right) \quad (20)$$

$$\frac{N_2}{T_\circ} = \frac{\alpha}{2\beta} - (\frac{\alpha}{2\beta} + 1)\left(\frac{R\alpha}{(R-2)(\alpha + 2\beta)} - (\frac{1-R}{2-R} - \frac{4x}{L(1-R)})\frac{\Delta}{h} \right) \quad (21)$$

Substituting these expressions of N/T_o into Eq. (5b) gives the corresponding equations for the yield moments

$$\frac{M_1}{M_\circ} = 1 + \frac{\alpha^2}{4\beta} - \beta(\frac{\alpha}{2\beta} + 1)^2 \left(\frac{R\alpha}{(R-2)(\alpha + 2\beta)} - \frac{1-R}{2-R}\frac{\Delta}{h} \right)^2 \quad (23)$$

$$\frac{M_2}{M_\circ} = 1 + \frac{\alpha^2}{4\beta} - \beta(\frac{\alpha}{2\beta} + 1)^2 \left(\frac{R\alpha}{(R-2)(\alpha + 2\beta)} - (\frac{1-R}{2-R} - \frac{4x}{L(1-R)})\frac{\Delta}{h} \right)^2 \quad (24)$$

<u>$For\ \Delta \geq \Delta'$</u> : The yield axial force and bending moment along part of the four inclined yield lines within region ($\frac{L'}{2}(1-R) \leq x \leq \frac{L}{2}(1-R)$) are given directly by Eqs.(6);

$$\frac{N_2}{T_\circ} = -1 \quad (6a)$$

$$\frac{M_2}{M_\circ} = 1 - \alpha - \beta \quad (6b)$$

For values of ($0 \leq x \leq \frac{L'}{2}(1-R)$), the axial force N_2 is found by substituting ($\mu_\circ$) from Eq.(18) into Eq.(5a) and making use of Eq.(11).

$$\frac{N_2}{T_\circ} = \frac{\alpha}{2\beta} + 2(\frac{\alpha}{2\beta} + 1)\left(\frac{\left(1 + \frac{\alpha}{2\beta}\frac{1}{1-R}\right)(\frac{L'}{L}) - \frac{1}{2}(\frac{\alpha}{2\beta} + 1)(\frac{L'}{L})^2}{1 + (\frac{\alpha}{\beta} + 1)\frac{1}{1-R}} - \frac{2x}{L(1-R)} \right)(\frac{\Delta}{h}) \quad (25)$$

And the corresponding yield moment equation is;

$$\frac{M_2}{M_\circ} = 1 + \frac{\alpha^2}{4\beta} - 4\beta\ (\frac{\alpha}{2\beta} + 1)^2 \left(\frac{\left(1 + \frac{\alpha}{2\beta}\frac{1}{1-R}\right)(\frac{L'}{L}) - \frac{1}{2}(\frac{\alpha}{2\beta} + 1)(\frac{L'}{L})^2}{1 + (\frac{\alpha}{\beta} + 1)\frac{1}{1-R}} - \frac{2x}{L(1-R)} \right)^2 (\frac{\Delta}{h})^2 \quad (26)$$

At the slab edges, the axial force is found by a combination of Eqs. (18), (10) and (5a), the result will be;

$$\frac{N_1}{T_\circ} = \frac{\alpha}{2\beta} - 2(\frac{\alpha}{2\beta}+1)\left(\frac{\left(1+\frac{\alpha}{2\beta}\frac{1}{1-R}\right)(\frac{L'}{L}) - \frac{1}{2}(\frac{\alpha}{2\beta}+1)(\frac{L'}{L})^2}{1+(\frac{\alpha}{\beta}+1)\frac{1}{1-R}}\right)(\frac{\Delta}{h}) \qquad (27)$$

And the corresponding yield moment equation is;

$$\frac{M_1}{M_\circ} = 1+\frac{\alpha^2}{4\beta} - 4\beta(\frac{\alpha}{2\beta}+1)^2\left(\frac{\left(1+\frac{\alpha}{2\beta}\frac{1}{1-R}\right)(\frac{L'}{L}) - \frac{1}{2}(\frac{\alpha}{2\beta}+1)(\frac{L'}{L})^2}{1+(\frac{\alpha}{\beta}+1)\frac{1}{1-R}}\right)^2(\frac{\Delta}{h})^2 \qquad (28)$$

7. Yield Loads

Having expressed the values of the axial forces $N/T_\circ$ and yield moments $M/M_\circ$ in terms of deflection Δ, the yield loads w corresponding to any given deflection can now be found by considering the equilibrium of the slab trapezoidal element.

Referring to Fig.(7), by taking moments about the mid-depth of the slab fixed edge, the following equilibrium equation is obtained;

$$w\left(\frac{RL^3(1-R)^2}{8}+\frac{L^3(1-R)^3}{24}\right)+2\int_0^{\frac{L}{2}(1-R)\csc 45} N_2\,ds\,\cos 45\frac{2\Delta.y}{L(1-R)} - M_1\,L - 2\int_0^{\frac{L}{2}(1-R)\csc 45} M_2\,ds\cos 45 = 0 \qquad (29)$$

Noting that the yield load predicated by Johansen's yield line theory for this type of slabs is;

$$w_J = 24\frac{M_\circ}{L^2}\left(\frac{2-R}{1-3R^2+2R^3}\right) \qquad (30)$$

And $ds = dy\csc 45$, $x = y$ therefore Eq. (29) can be written in the following form

$$\frac{w}{w_J} = \frac{1-R}{4-2R}\left[\frac{4}{L(1-R)}\int_0^{\frac{L}{2}(1-R)}\frac{M_2}{M_\circ}dy + \frac{2}{1-R}\frac{M1}{M_\circ} - 8\beta\,(\frac{\alpha}{2\beta}+1)(\frac{\Delta}{h})(\frac{2}{L(1-R)})^2\int_0^{\frac{L}{2}(1-R)}\frac{N_{21}}{T_\circ}y\,dy\right] \qquad (31)$$

$\Delta \le \Delta'$:For deflections less than the critical value Δ' , the appropriate expressions for $N_{1\to 2}/T_\circ$ and $M_{1\to 2}/M_\circ$ are given by Eqs.(20) to (24) . Substituting these expressions into Eq.(31) and reducing;

$$\frac{w}{w_J}=\frac{1-R}{4-2R}\left\{ (\frac{\alpha^2}{4\beta}+1)\ \frac{4-2R}{1-R}\ -\frac{\Delta}{h}2\alpha(\frac{\alpha}{2\beta}+1) + \beta(\frac{\alpha}{2\beta}+1)^2\left((\frac{\Delta}{h})^2\frac{8}{3}-(\frac{R\alpha}{(R-2)(\alpha+2\beta)}-\frac{1-R}{2-R}\frac{\Delta}{h})^2\frac{4-2R}{1-R}\right)\right\} \tag{32}$$

$\Delta \geq \Delta'$: For deflections greater than the critical value Δ' , the appropriate expressions for $N_2/T_\circ$ and $M_2/M_\circ$ for the central region of the slab where ($\frac{L'}{2}(1-R) \leq y \leq \frac{L}{2}(1-R)$) are given by Eqs.(6a) and (6b), but for the corner regions of the slab , where ($0 \leq y \leq \frac{L'}{2}(1-R)$), Eqs.(25) and (26) hold instead. Along the fixed edges of the slab, the values of $N_1/T_\circ$ and $M_1/M_\circ$ are given by Eqs. (27) and (28) .Substituting these expressions into Eq. (31) and reducing;

$$\frac{w}{w_J}=\frac{1-R}{4-2R}\left\{2(\frac{\alpha^2}{4\beta}+1)\left(\frac{L'}{L}+\frac{1}{1-R}\right)+2(1-\alpha-\beta)(1-\frac{L'}{L})+(\frac{\Delta}{h})4\beta(\frac{\alpha}{2\beta}+1)(1-(\frac{\alpha}{2\beta}+1)(\frac{L'}{L})^2) -(\frac{\Delta}{h}\frac{L'}{L})^2 8\beta(\frac{\alpha}{2\beta}+1)^2\left((\frac{\left(1+\frac{\alpha}{2\beta}\frac{1}{1-R}\right)-\frac{1}{2}(\frac{\alpha}{2\beta}+1)(\frac{L'}{L})}{1+(\frac{\alpha}{\beta}+1)\frac{1}{1-R}})^2(\frac{L'}{L}+\frac{1}{1-R})-\frac{1}{3}\frac{L'}{L}\right)\right\} \tag{33}$$

Graphical representations of Eqs.(32) and (33) are given in Figs.(8),(9) and (10).The first two of these figures show respectively the possible enhancement in the load carrying capacity of clamped square slabs with central square opening above Johansen's load due to effects of variation in the percentage of reinforcement (ρ),and the ratio of opening(R),whereas Fig.(10) shows the amount of percentage increase in the yield load with variation in the parameter ($\rho\, f_y / f'_c$) and the slab deflection.

8. Study of the Maximum Yield Load

It is apparent from the present analysis that due to compressive membrane action, fully restrained R.C square slabs with central square opening can carry load far beyond those of Johansen's simple yield line theory .The reserves in strength are found to be more pronounced in lightly reinforced slabs made of concrete with high strength. The assumption of rigid-perfectly plastic behavior of R.C slabs with edges fully restrained against rotation and horizontal translation implies that the maximum yield load is attained at zero deflection. Therefore, from Eq.(32):

$$\frac{w}{w_J}=\left((\frac{\alpha^2}{4\beta}+1)-\left(\frac{R\alpha}{(R-2)(\alpha+2\beta)}\right)^2\beta(\frac{\alpha}{2\beta}+1)^2\right)$$

If the expression of Johansen's load (w_J) given by Eq. (30) is introduced, the maximum yield uniform load will have the following explicit dimensional form;

$$\frac{w_{max}}{(h/L)^2}=\frac{24(2-R)}{(1-3R^2+2R^3)}\frac{\rho f_y}{(h/d)^2}(1-\frac{k_2}{k_1k_3}\rho\frac{f_y}{f_c'})\left((\frac{\alpha^2}{4\beta}+1)-\left(\frac{R\alpha}{(R-2)(\alpha+2\beta)}\right)^2\beta(\frac{\alpha}{2\beta}+1)^2\right) \qquad (34)$$

For a certain slab with a specified ratio of opening (R) ,total depth to effective depth ratio (h/d) and steel yield stress, the maximum yield uniform load increase linearly with the percentage of reinforcement (ρ) and non-linear with the concrete strength f_c' , the latter being the more dominant factor . This is clearly shown in Fig.(11) where Eq.(34) is plotted for a typical slab with R=0.25, h/d=1.2 and f_y=300MPa.as for example , the same maximum yield load ,say $172\,(h/L)^2$ in unit of N/mm^2 , can be obtained by using a typical slab with $f_c'=25MPa$ and percentage of reinforcement ρ=1% or with minimum ρ of 0.2% but $f_c'=33MPa$.Thus, to obtain equal maximum yield loads the amount of reinforcement can be reduced by using a higher strength concrete.

9. Comparison with Available Experimental Tests

Previous experimental investigations [7,8,9] have show that the actual load-deflection relationship of axially restrained slabs consists of an initial ascending part represent the elastic deformation (see Fig.1) followed by an elastic-plastic stage until peak load is reached at a critical slab deflection and thereafter declines rapidly . Obviously, a rigid-plastic solution does not predict such initial rising part of the load-deflection relation but shows instead a rather continuous descending curve starting from a maximum value of yield load at zero deflection. Therefore to make use of the rigid-plastic solution in estimating actual ultimate load, the value of the deflection corresponding to the ultimate load has to be specified and use directly. The previous tests on R.C slabs have indicated different values for such deflection depending on the flexural rigidity ratio of the surrounding elements of the slab. This can be readily seen in Table (2).Which is basically constructed to show a full detailed comparison between the results of the present theoretical method and the finding of previous experimental study [7].

The table clearly indicates variation in the value of the ratio (*Δ/h*) corresponding to ultimate load, ranging between the case of slabs with stiff bounders to slabs surrounded by weaker R.C beams. The table also shows that ultimate loads found by tests are comparable with the corresponding rigid-plastic theoretical load, with an average value of the ratio (w test/w theory) of 0.9. The 10% deviation between the rigid-plastic solution and experimental peak load is, therefore, a measure of the effect of neglecting elastic deformations in the theoretical method of analysis (see Fig.1).

Accordingly , if $\Delta/h=0.7$ (considered quite conservative for practical R.C slab –beam panels) is inserted into the appropriate equation (32) or (33) and 90% of the resulting yield load is taken, the theoretical estimation of the ultimate load will be close enough to the corresponding test value. Such a procedure is followed to construct Fig.(12) &(13).Fig(12) indicates remarkable enhancements in the ultimate loads above those suggested by yield line theory . Especially for slabs having smaller values of both the opening ratio (R) and the parameter ($t=\rho\ f_y/f_c'$) . A practical significance of results is show in Fig.(13) where certain percentage saving reinforcement is seen to be possibly made in the slab due to the inclusion of membrane action in the method of slab design ;

$$\%\,\text{reduction in } A_s = 100 - \frac{100}{w/w_J}$$

Thus for a lightly loaded interior panel in slab-beam system where only light steel reinforcement is usually used, say t=0.04 , 48% of the reinforcement design by the simple yield line theory can theoretically be saved and that if the panel sustains moderate load (say t=0.08), saving in the steel reinforcement of the order 25% seems quite possible.

10. Concluding Remarks

The main conclusions to be drawn from the present study are:

1. The analysis shows that reinforced concrete square slabs with central square opening and with clamped edges can sustain loads more than those predicted by Johansen's yield line theory by a big margin.
2. The enhancement in ultimate load above Johansen's load is more pronounced in slabs with low value of (ρ) and (f_y) and high value of (f'_c).
3. The predicated enhancement in the load carrying capacity decreases with increasing slab opening.
4. In R.C slab-beam panel, the degree of restraint provided by the surrounding beams has a major influence on the amount of enhancement in the ultimate load of the slab.

5. Practically, the predicated increases in the ultimate load can be made use of in design if corresponding deflection are proved to satisfy serviceability requirement.
6. Membrane action in restrained slab is a self-prestressing action where the compressive strength of concrete is the dominate parameter. Thus, some reinforcement can be saved in a slab by using stronger concrete. For square panels of slab with central square opening that have edges either continuous or highly restrained against rotation and translation, such saving in reinforcement could be as high as 48% if the panel is lightly loaded, and 25% for a panel under moderate load.

Table (1) Cases of Neutral Axis Depth

Neutral Axis Depth	Description	Yield Criterion
$\mu \le h/2$	The slab is not cracked throughout its depth and there are some compressive stresses on the concrete	$\frac{N}{T_o} = \frac{\alpha}{2\beta} - \frac{\mu}{2\beta}\frac{T_o}{M_o}$ (5a) $\frac{M}{M_o} = 1 + \alpha\left(\frac{N}{T_o}\right) - \beta\left(\frac{N}{T_o}\right)^2$ (5b)
$\mu \ge h/2$	The slab is cracked throughout its depth and there are no compressive stresses on the concrete (i.e C=0)	$\frac{N}{T_o} = -1$ (6a) $\frac{M}{M_o} = 1 - \alpha - \beta$ (6b)

Table (2) Comparison between Experiments and Theory

Authority	Description of test	Slab Mark	ρ	Ratio of flexural rigidity of the surrounding beams to flexural rigidity of the slab	Δ/h	w_{test}/w_J	w_{theory}/w_J	w_{test}/w_{Theory}
Datta and Ramesh (Ref.7)	Isotropically reinforced square concrete panels surrounded by R.C beams of varying widths were uniformly loaded to failure(R=0 , L=1220mm , h=38mm , $f_y/f_c' = 10$)	BN1	0.283%	9.5	0.8	2.13	2.99	0.71
		BN2	0.283%	12	0.7	2.62	3.15	0.83
		BN3	0.283%	13.5	0.65	2.95	3.24	0.91
		CN1	0.526%	12.5	0.8	1.51	1.86	0.81
		CN2	0.526%	13.5	0.75	1.66	1.89	0.88
		CN3	0.526%	14.5	0.7	1.77	1.93	0.92
		DN1	1.2%	23	0.8	1.3	1.28	1.02
		DN1	1.2%	22	0.75	1.26	1.29	0.98
		DN1	1.2%	24	0.7	1.37	1.30	1.05
Average								0.9

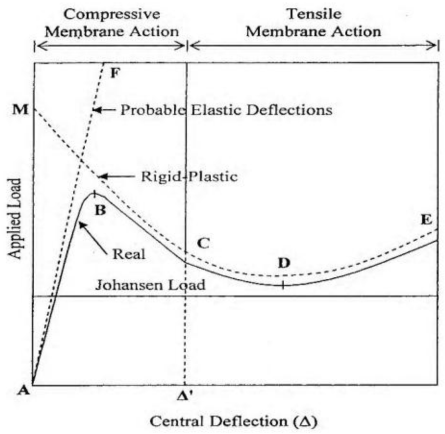


Fig. (1) Load –Deflection Relations for Laterally Restrained R.C Slabs (Reference 3)

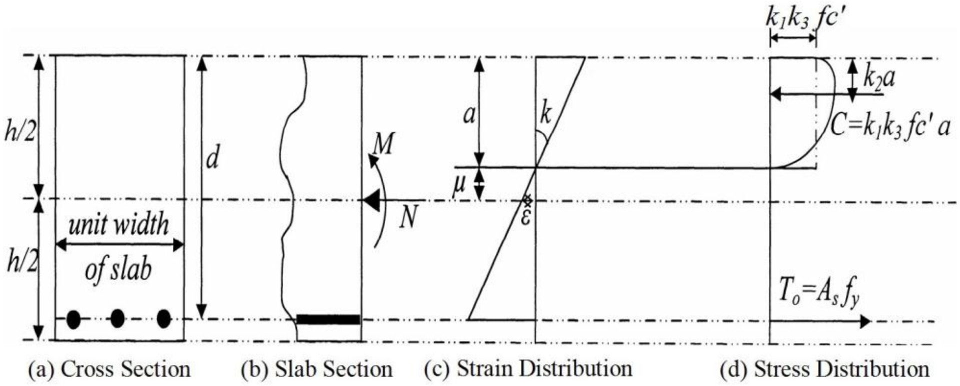


Fig.(2) Stress Distribution on a Slab Section at Yield

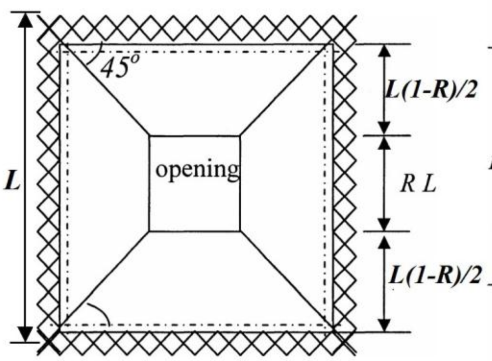


Fig (3). Yield Line Pattern

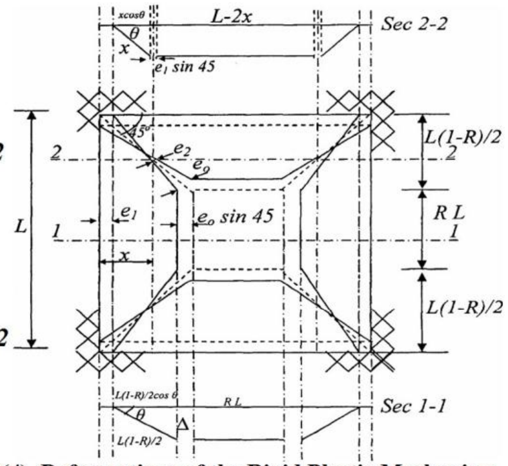
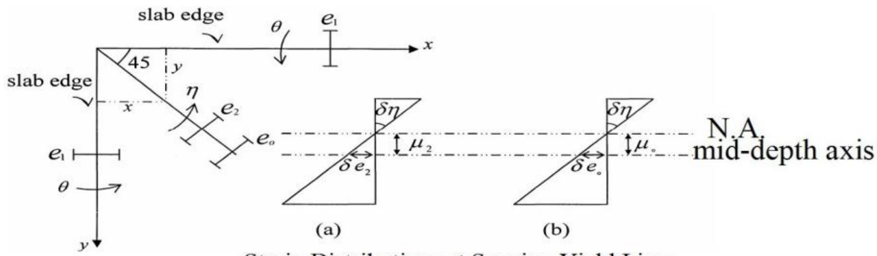
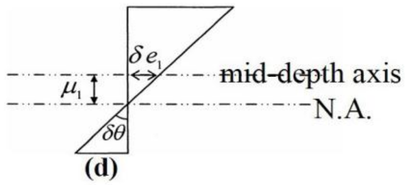


Fig (4). Deformations of the Rigid Plastic Mechanism



Strain Distributions at Sagging Yield Lines

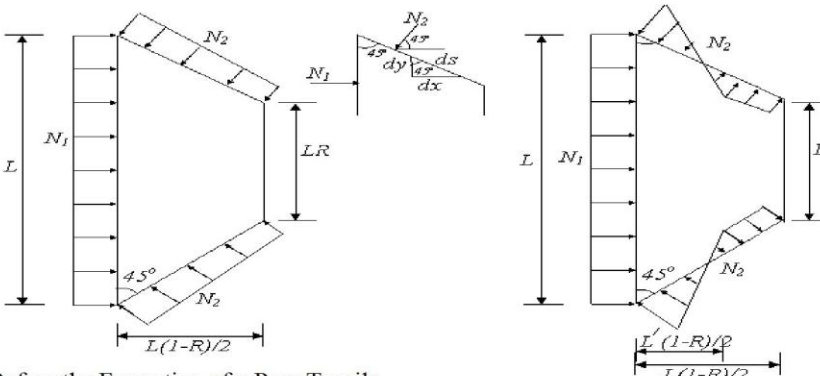
$$\delta\eta = \delta\theta\cos 45 + \delta\theta\sin 45$$
$$= 1.414\delta\theta$$
$$\mu_o = \frac{\delta e_o}{\delta\eta} = 0.707\frac{de_o}{d\theta}$$
$$\mu_1 = \frac{\delta e_1}{\delta\theta} = \frac{de_1}{d\theta}$$



(d)

Strain Distributions at Hogging Yield Lines

Fig (5). Strain Distributions at Sagging and Hogging Yield Lines



(a) Before the Formation of a Pure Tensile Membrane

(b) After the Formation of a Pure Tensile Membrane

Fig.(6) Horizontal Forces on Slab Element

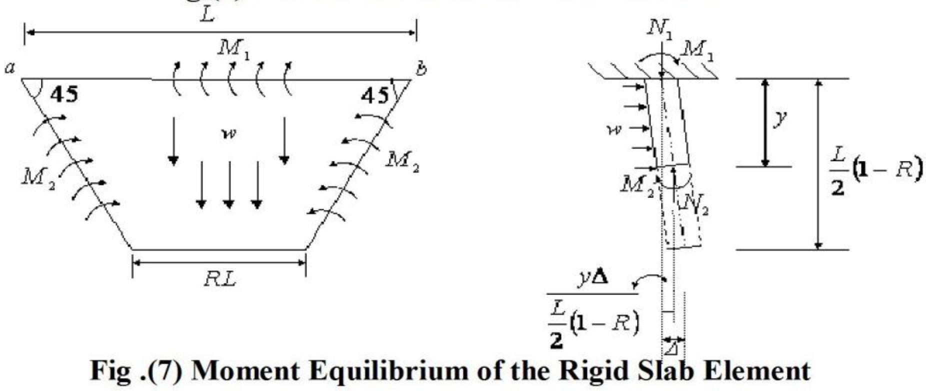


Fig .(7) Moment Equilibrium of the Rigid Slab Element

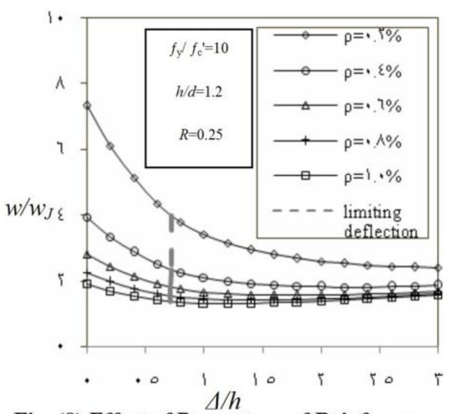


Fig .(8) Effect of Percentage of Reinforcement on w/w_J

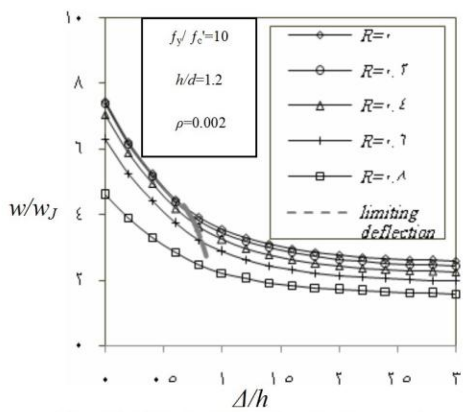


Fig .(9) Effect of Opening Ratio on w/w_J

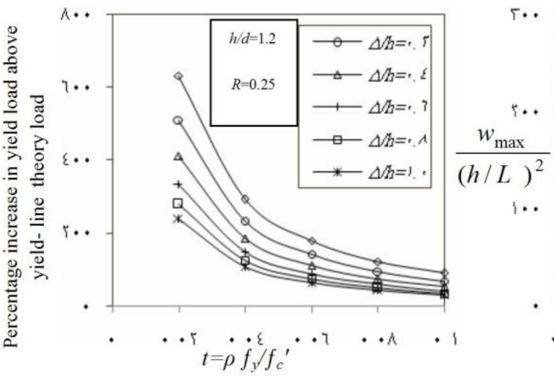


Fig.(10) Variation of the Percentage Increase in the Yield Load above Yield - Line Theory Collapse Load with the Parameter (t) for Different Values of Δ/h

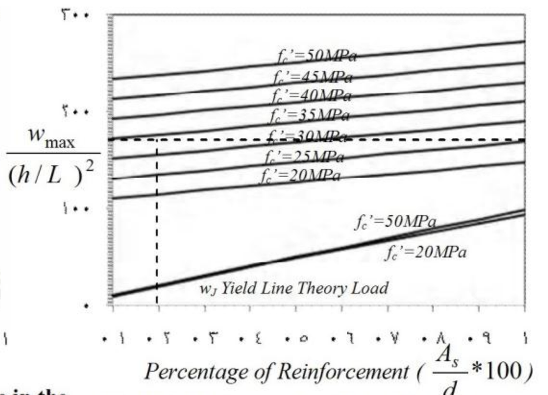


Fig.(11) Maximum Yield Load According to Rigid-Plastic Theory

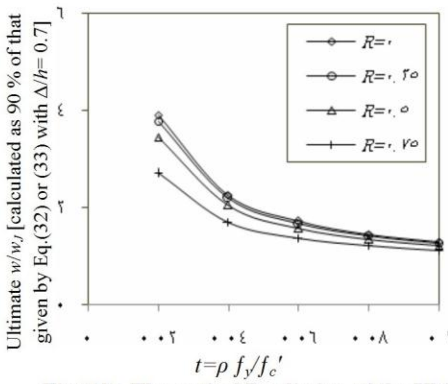


Fig.(12) Theoretical Prediction of the Ultimate Uniform Load

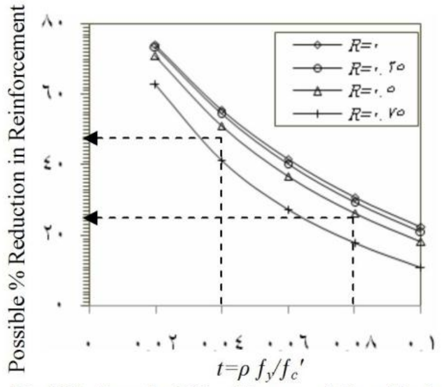


Fig.(13) Practical Significance of Considering Membrane Action in Slab Design

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