Study of Stress Intensity Factor in Corrugated Plate Using Extended Finite Element Method (XFEM)

Dr. Rafil Mahmood Laftah
Engineering College, University of Basrah/Iraq
Email:drrafil74@gmail.com

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ABSTRACT

The presence of crack with different lengths and locations in structure of corrugated plate is studied. XFEM is along with enrichment function based on partition of unity is used for the calculation of stresses due to the presence of crack. Stress intensity factors (K_I) and relations for shape factors (C) are calculated for the four most common crack orientations and loading conditions. Results showed that crack in the flange with load perpendicular to corrugation direction yields higher values of K_I and C in addition to wider spread of stress concentration contours than the web. When loading becomes parallel to corrugation direction, the stress intensity factor and C factor become remarkably less due to the corrugation stiffness. Third mode K_{III} is appeared in the fourth case in addition to the presence of the first mode K_I due to 45^o inclination of the web and applied load direction

Keywords: SIF, XFEM, Corrugated plate, ABAQUS

Nomenclature

a: half crack length, mm
a_i: enrichment degree of freedom
b: web length, mm
c: flange length, mm
C: shape factor
E: Young's modulus, GPa
K_I: mode I stress intensity factor, MPa(mm)$^{1/2}$
K_{III}: mode III stress intensity factor, MPa(mm)$^{1/2}$
N_i: shape function
N_i^*: new set of shape function
r: radial distance from crack-tip, mm
σ: normal stress, MPa
τ: shear stress, MPa
ν: Poisson's ration

INTRODUCTION

Corrugated steel plates have been widely used in many fields of applications due to their good engineering properties. In structural engineering, corrugated plates have many applications like sheet piles, wall or girder web. In the field of building it's been used as facades, roof structures and as the basic component of composite floors. In recent years corrugated plates have been increasingly used as a beam web. Corrugated plate beam web is used in a very specialized application like hybrid bridges, which is type of bridges with concrete decks and steel webs [1].

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

Being subjected to live load it is very important to study the effect of the presence of crack in the body of the corrugated plate, as this might lead to brittle fracture and failure under loads far less than expected.

Extended Finite Elements (XFEM), introduced by Belytschko and Black [2] reduced the time and cost of solving structure problems containing cracks, this is achieved by introducing an enrichment function with additional degrees of freedom at crack area while the rest of the structure is modeled using ordinary FEM. The XFEM is used in this study in ABAQUS software to calculate stress distribution under tensile loading with the presence of crack. ABAQUS is proved to be very powerful and accurate tool used by many researchers and institutes around the world. It been used expensively for the analysis of materials that contain discontinuities like crakes, defects and flaws [3,4].

Boundary element and meshless methods have great importance in solving problems of materials with discontinuities. Unfortunately, these two methods have weakness in dealing with certain problems. Boundary element method has poor results when it comes to strong nonlinearities, or when material is heterogeneous, while meshless method has no firm theoretical foundation [5, 6]. The main advantage of XFEM over other methods is it allows for crack location and propagation inside the elements; cracks with complex geometry can be modeled by structured meshes and can propagate element by element without remeshing, which will greatly save computational cost.

The aim of this work is to study the effect of crack length and applied load direction to the stress intensity factor and shape factor.

Properties and Geometry of Corrugated Plate

Corrugated plate used in the study is carbon steel (Young's Modulus E = 200 GPa and Poisson's ratio ν =0.3). It composed of series of repeated (longitudinal and inclined) panels. The longitudinal panel is often called the flange and the inclined panel is called the web. The dimensions and geometry of the used plate is shown in fig.1 and table 1. It's clear that there are two main directions the first one is perpendicular to the corrugation and the second is in the direction of corrugation. Based on this the case studies are selected to account for the difference in the geometry. Furthermore; the presence of crack is selected carefully to demonstrate the effect of geometry on the stress distribution.

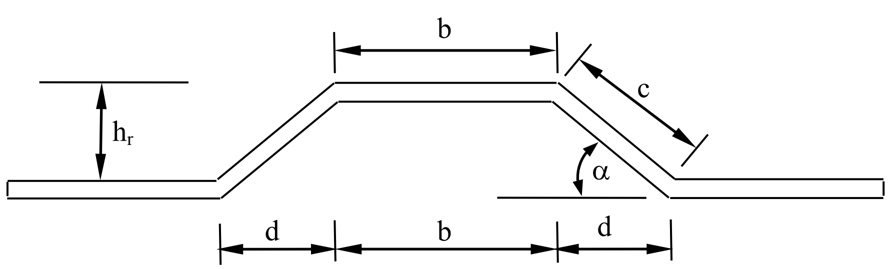
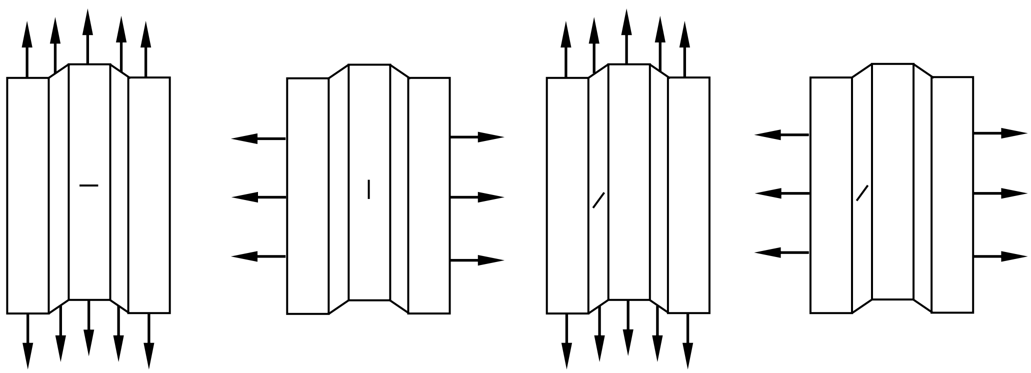


Figure (1) Corrugated plate geometry [7]

Table(1) Dimensions of the used corrugated plate

Symbol	b	c	D	h_r	α
Dimension	147 mm	147 mm	104 mm	104 mm	45^o

Figure 2 presents the location of crack and loading direction for each case study. In the first case the crack is selected to be in the flange and the load is perpendicular to the corrugation direction, while in the second case study the crack is in the flange but in the opposite direction and the load is applied in the corrugation direction. In the third case the crack is presented in the inclined web and the load is applied perpendicular to the corrugation direction, and finally in the fourth case the crack still in the inclined web but load direction becomes in the direction of corrugation of the plate.



(a) **Flange central crack load in y-direction** **(b)** **Flange central crack load in x- direction** **(c)** **Web central crack load in y- direction** **(d)** **Web central crack load in x- direction**

Figure (2) Crack location and load direction for each case

Stress intensity Factors (SIF)

Stress distribution around crack tip [8] for isotropic linear elastic material in the first mode is given by:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 + sin\frac{\theta}{2} sin\frac{3\theta}{2}\right) \quad \text{….(1)}$$

$$\sigma_{xx} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 - sin\frac{\theta}{2} sin\frac{3\theta}{2}\right) \quad \text{…. (2)}$$

Where r is distance measured from crack tip and θ represents the orientation. If the stress distribution is required in crack plane then θ is set equal to zero and equations (1) and (2) are reduced to:

$$\sigma_{yy} = \frac{K}{\sqrt{2\pi r}} \quad \text{…...(3)}$$

$$\sigma_{xx} = \frac{K}{\sqrt{2\pi r}} \quad \text{…..(4)}$$

It is important to mention here that equations (3) and (4) are valid only close to crack tip where the main factor affecting stress distribution is the distance r.

The stress distribution around crack tip for the third mode is given in equations (5) and (6)

$$\tau_{yz} = \frac{-K_{III}}{\sqrt{2\pi r}} cos\frac{\theta}{2} \quad \text{….(5)}$$

XFEM Enriching Function

The XFEM consists of a standard FE space, and the enrichments based on the partition of unity approach. The approximation of the enrichment function u(x) is expressed in Eq. (6).

$$u(x) = \underbrace{\sum_{i\in k} N_i(x)\, u_i}_{\text{Classical}} + \underbrace{\sum_{i\in k^*} N_i^*(x)\, a_i}_{\text{Enriched}} \qquad ....(6)$$

Where $N_i(x)$ is the standard shape function at node i; and k is the set of all nodes in that domain. In contrast k^* is nodes set that is used for enrichment. It is important to mention that $k^* \subset k$ and u_i represents the displacement. $N_i^*(x)$ is the local enrichment function at node i and a_i is the enriched displacement at node i [9].

Case 1: Flange Central Crack with Load Perpendicular to Corrugation

A crack of different sizes is introduced to the web of corrugated plate subjected to tensile load perpendicular to corrugation direction refer to figure (2.a). It is required to study the effect of the presence of crack of different sizes on the stress distribution and the stress intensity factor. The blue curve in figure (3) shows the relation between stress intensity factor in the first mode K_I and the ratio of crack length to flange width (a/b) for twelve crack lengths (10 to 120 mm) with step 10 mm. The effect of geometry or sometimes called the shape factor ($C = K_I/K_o$) for this particular case study is presented in equation (7).

$$C\left(\frac{a}{b}\right) = 0.984 + 0.347\left(\frac{a}{b}\right) - 0.141\left(\frac{a}{b}\right)^2 + 0.008\left(\frac{a}{b}\right)^3 \qquad ....(7)$$

$$K_o = \sigma\sqrt{\pi a} \qquad ....(8)$$

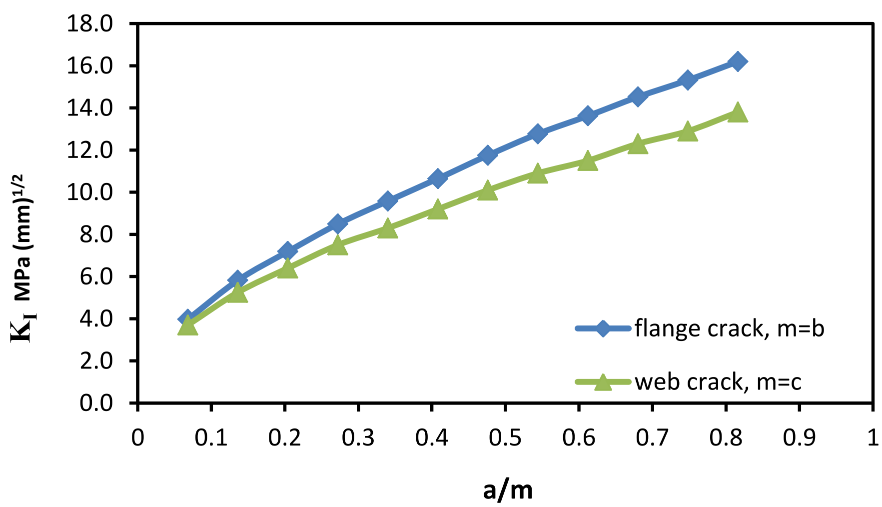


Figure (3) Effect of crack length on the stress intensity factor K_I

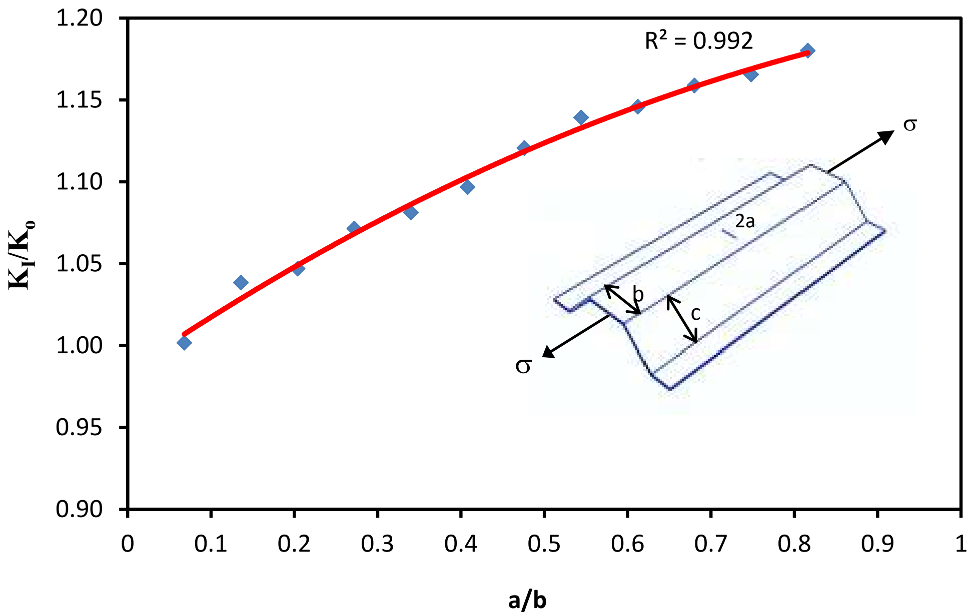
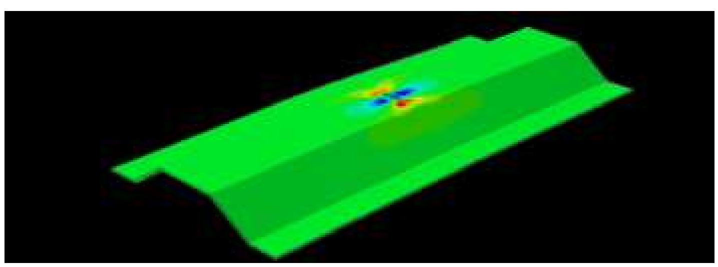


Figure. (4): Effect of crack length on shape factor C



Figure(5). Stress distribution fringes

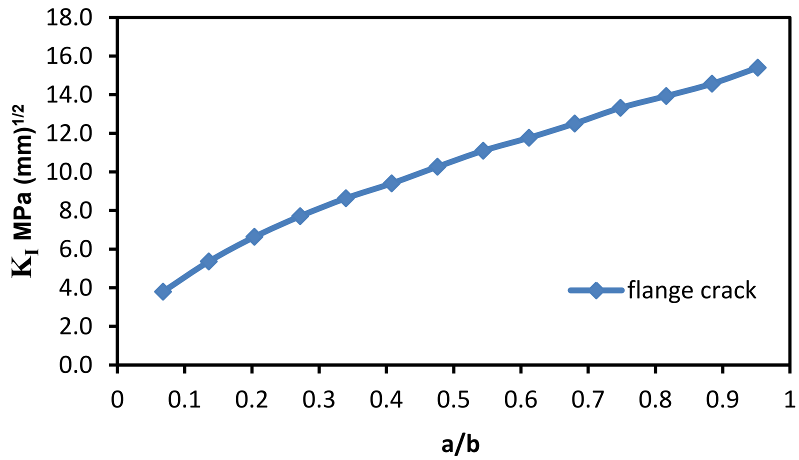
The relation between shape factor and crack length is presented in figure (4) with strong correlation factor R^2. It is clear from the figures (3) and (4) that the value of stress intensity factor and shape factor is increased with the increment of crack length under same tension load. This behavior is similar to the behavior of flat plate under same conditions of varying crack length using same constant load [10]. The fringes and contours of stress distribution around the crack are presented in figure (4). Wide fringes are noted around the crack which indicates the area affected by the presence of the crack with high stress levels around crack tip.

Case 2: Flange Central Crack with Load Parallel to Corrugation

In order to study the effect of corrugation on the stress distribution, a crack is placed in the flange perpendicular to the corrugation direction and the plate is subjected to tensile load in parallel to corrugation direction. Figure (6) shows the relation between stress intensity factor and (a/b). Shape factor ($C=K_I/K_o$) relation with (a/b) is presented in equation (9) and sketched in figure (6). The SIF and C factor noted to be increasing with the increment of crack length with strong correlation factor.

$$C\left(\frac{a}{b}\right)=0.953+0.056\left(\frac{a}{b}\right)-0.027\left(\frac{a}{b}\right)^2+0.061\left(\frac{a}{b}\right)^3 \qquad \ldots(9)$$

Stress distribution fringes shows that the stress contours in the first case are wider spread and higher in values than the second case. The reason for this difference between the two cases may be attributed to the difference in geometry, in the first case the behavior is much like flat plate while in the second case the stiffness inherited in corrugation countered part of the load which led to reduction in stress concentration around crack and the overall stress level.



Figure(6) Effect of crack length on the stress intensity factor K_I

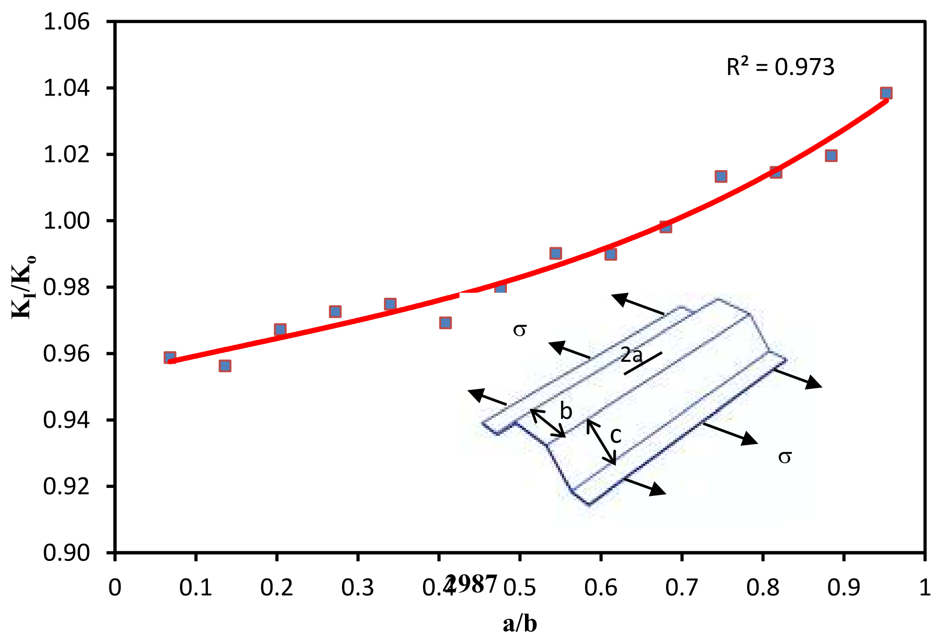


Figure (7) Effect of crack length on shape factor C

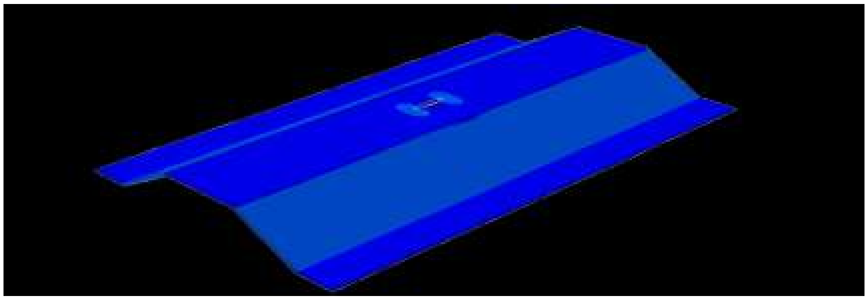


Figure (8) Stress distribution fringes

Case 3: Web Central Crack with Load perpendicular to Corrugation.

In the present case it is intended to study the effect of the presence of crack in the web of the corrugated plate which is inclined by 45 degrees to the flange. A central crack of length varied from 10 to 120 mm with step 10 mm is introduced to the web. The plate is then subjected to an axial force. The effect of crack length on stress intensity factor K_I is presented in figure (3). The figure shows that K_I is increasing with the increment of crack length. Figure (3) also shows that the values of K_I for flange are higher than the web under same conditions of loading and crack lengths. Relation between shape factor and crack (a/c) is presented by figure (9) below and the mathematical representation is given in equation (10)

$$C\left(\frac{a}{c}\right)=0.933-0.015\left(\frac{a}{c}\right)+0.157\left(\frac{a}{c}\right)^2-0.048\left(\frac{a}{c}\right)^3 \quad ..(10)$$

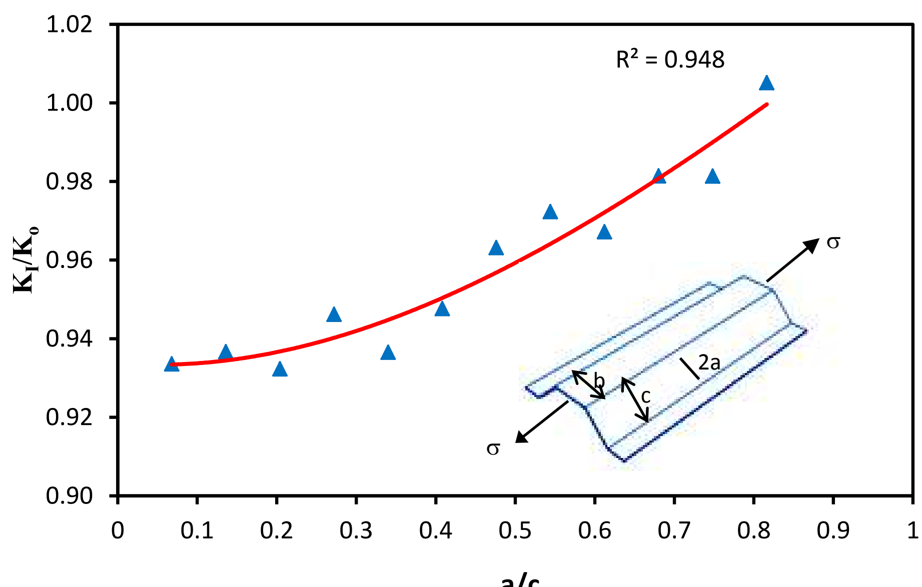


Figure (9) Effect of crack length on shape factor C

It is obvious that the shape factor increases with the increment of crack length. The stress contours are presented in figure (10). Stress contours are quite similar to the contours resulted in the first case study but with less spread area and less stress levels.

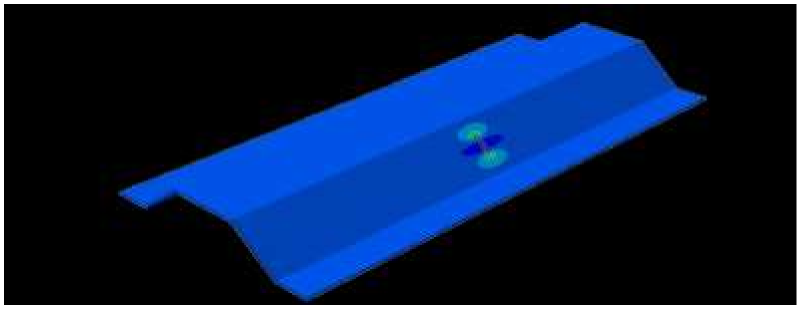


Figure (10) Stress distribution fringes

Case 4: Web Central Crack with Load Parallel to Corrugation

The last case is very important because it will be used to study the effect of the presence of cracks in the web and in the same time it will incorporate the corrugation effect. In this case study the effect of inclination of web panel is quite obvious. Web inclination and load direction lead to the appearance of two modes of deformation with stress intensity factors K_I and K_{III}. Figure (11) shows the relation between stress intensity factors and crack length. It is clear that the curves of the two modes are identical, which is attributed to 45^o angle of the web.
The shape factor C is represented in equations (11) and (12)

$$C=\left(\frac{a}{c}\right)=0.476+0.028\left(\frac{a}{c}\right)-0.013\left(\frac{a}{c}\right)^2+0.030\left(\frac{a}{c}\right)^3 \quad \text{….(11)}$$

$$\begin{aligned} N=I\ ,\ K_o&=\sigma\sqrt{\pi a} \\ N=III\ ,K_o&=\tau\sqrt{\pi a} \end{aligned} \quad \text{...(12)}$$

Figure (12) shows the relation between shape factors and crack length for the two modes of deformation which is again identical for the two modes. The stress distribution firings show clearly how the two modes are acting. From figure (13) the effect of first deformation mode (opening) and the effect of third mode (tearing) are very clear and distinguishable.

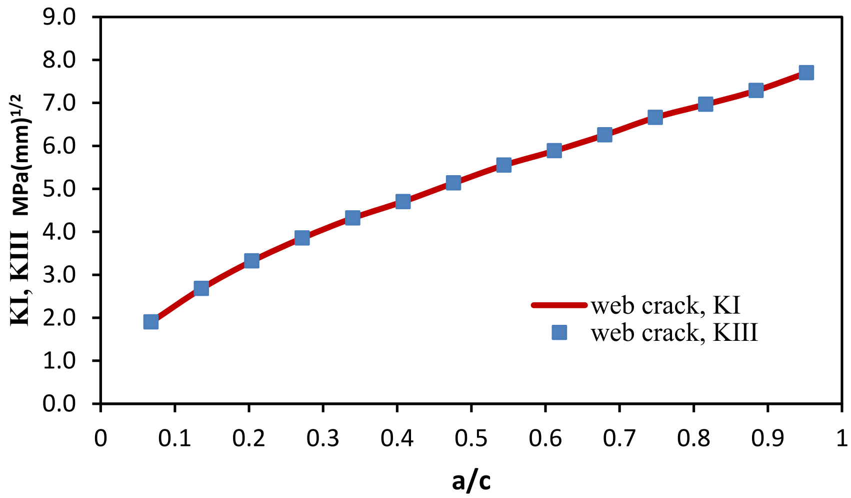


Figure (11) Effect of crack length on the stress intensity factors K_I and K_{III}

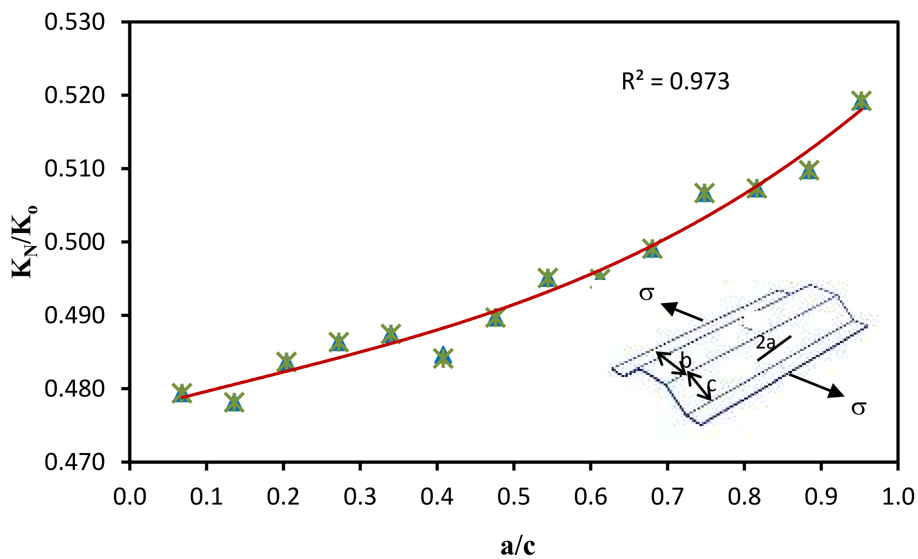


Figure (12) Effect of crack length on the shape factor

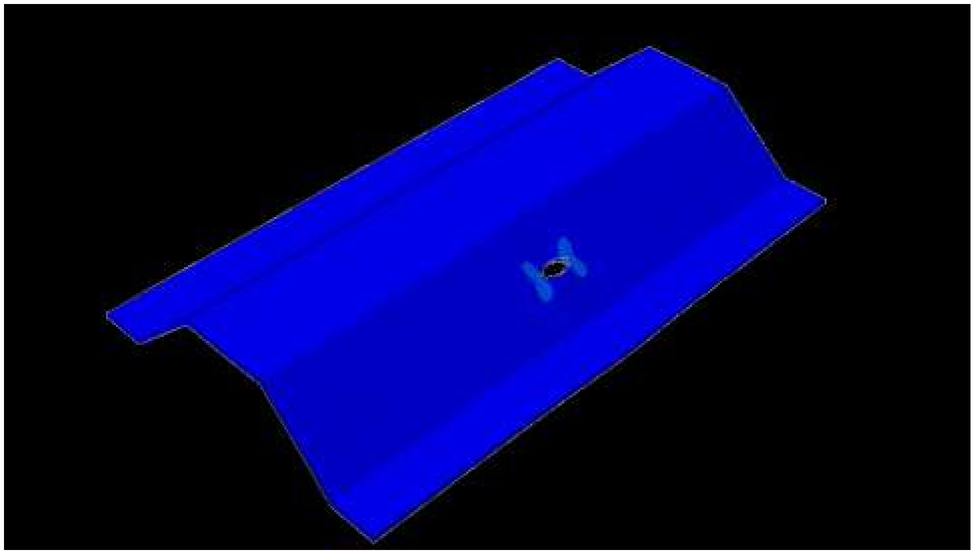


Figure (13) Stress distribution fringes

CONCLUSIONS

Stresses and stress intensity factors are noticed to be higher when the load is applied perpendicular to corrugation. This is due to the presence of first mode of deformation. Stress distribution fringes are also wider and spreader in these two cases. The reason that stresses, stress intensity factors and stress distribution fringes are less when the load is applied in the corrugation direction may be attributed to the stiffness of the corrugation geometry. According to this the stress fringes tend to localize in these cases. The fourth case showed an interesting result which is the presence of the third mode of deformation and hence a value for third mode of stress intensity factor K_{III}. The presence of this mode reduces the value of the first mode K_I. The presence of K_{III} reduced the value of K_I because the load is distributed on the two modes. Shape factor which indicates the effects of shape and loading condition for each case study is presented in the form of equation and curve.

REFERENCES

[1] Kovesdi, B., Dunai, L., "Determination of the patch loading resistance of girders with corrugated webs using nonlinear finite element analysis", Computers and Structures, 89, 2010-2019, 2011.

[2] Belytschko, T., Black, T., "Elastic crack growth in finite elements with minimal remeshing.", International Journal for Numerical Methods in Engineering", 45(5):601– 620, 1999.

[3] E. Giner, N. Sukumer, J.E. Tarancon and F.J. Fuenmayor, "An Abaqus Implementation Of The Extended Finite Element Method", Engineering Fracture Mechanics, Vol.76, pp. 347–368, 2009.

[4] Michael James McNary, MSc. Thesis, "Implementation Of The Extended Finite Element Method (XFEM) In The Abaqus Software Package ", Georgia Institute of Technology, 2009.

[5] Yan, Xiangqiao, "A boundary element analysis for stress intensity factors of multiple circular arc cracks in a plane elasticity plate", Applied Mathematical Modeling, Vol . 34, Issue 10, 2722-2737, 2010.

[6] Belytschko, T. Y., Lu Y., and Gu, L., "Element-Free Galerkin Methods", International Journal for Numerical Methods in Engineering, Vol. 37, pp. 229–256, 1994.

[7] Richard S., Hassam H.,Wagdy G., Robert G., Mohamed E., "Corrugated Web Girder Shape and Strength Criteria", National Center for Engineering Research on Advance Technology for Large Structural Systems,ATLSS Report No. 03-18, Civil and Environmental Engineerun, Lehigh University, 2003.
[8] Ewalds, H. L., Wanhill, R. J. H., "Fracture Mechanics" Edward Arnold, DUM, (1983).
[9] Keswani, K., Singh, K. L., Arokkiaswamy, " Computation of SIF and Crack Growth Simulation using XFEM Techniques", International Conference on Advanced Research in Mechanical Engineering, 2012.
[10] Khalaf, Hassanien I., "Crack Propagation in Plane Stress Problems by Using Experimental and Extended Finite Element Method (XFEM)", Ph.D. thesis, University of Basrah, 2015.