CALCULATION OF THE INELASTIC LONGITUDINAL ELECTRON SCATTERING OF ²⁰Ne AND ²⁴Mg NUCLEI

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حساب عوامل التشكل للأستطارة الإلكترونية الطولية غير المرنة ²⁰Ne و²⁴Mg

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الخلاصة:

والمتضمنة انتقال كثافة الشحنة (F(q)'s) درست الاستطارة الالكترونية الطولية غير المرنة اخذين بنظر الاعتبار التشوه في الأنماط التجميعية النووية إلى جانب كثافة الانتقال لأنموذج القشرة و مع صيغة لتوزيعات كثافة Tassie استقطاب القلب لكثافة الانتقال حسبت بالاعتماد على شكل أنموذج) . SRC الشحنة النووية للجسيمين في الحالة الأرضية والمتظمنة تأثير دالة ترابطهما قصيرة المدى لاحظنا بان تأثير استقطاب القلب الذي يمثل نمط تجميعي يكون جو هريا للحصول على توافق جيد بين و القيم العملية للنواتين للنواتين يو القيم العملية للنواتين للنواتين ال

ABSTRACT

The inelastic longitudinal electron scattering form factors F(q)'s, an expression for the transition charge density are studied where the deformation in nuclear collective modes is taken into consideration besides the shell model transition density. The core polarization transition density is evaluated by adopting the shape of Tassie model together with form of the ground state two-body charge density distributions which included the effect of short range correlation(SRC). It is noticed that the core polarization effects which represent the collective modes are essential in obtaining a good agreement between the calculated inelastic longitudinal F(q)'s and those of experimental data for ²⁰Ne and ²⁴Mg nuclei.

1-INTRODUCTION

Inelastic scattering of medium energy electron provides a well-understood probe of the charge, current and magnetization densities which characterized nuclear excitations. In light nuclei, where the plan-wave Born approximation is quate accurate, provided a simple correction to the momentum transfer for Coulomb distortion is made, the connection between the measured form factors and the transition densities is direct and is simply expressed as Bessel transform. Also, it is for light nuclei that the most extensive microscopic calculations of the transition densities can be performed and tested[1].

Large basis space projected Hartree-Fock wave functions were used by Amos and Steward [2] to calculate the longitudinal and transverse form factors from the excitations of 2^+_1 and 4^+_1 states in ${}^{12}C$, ${}^{20}Ne$ and ${}^{24}Mg$. The result obtained using such large basis space models of structure were compared with limited basis space (shell-model) predictions to show that momentum-transferdependent corrections can be quite diverse. Karataglidis et. al. [3] compared between the calculation of transverse electric form factor using the standard expression for the electric multipole operator and those obtained by invoking current conservation. The results of E2 transitions in ¹²C, ²⁰Ne, ²⁴Mg and ²⁸Si were found that the form factors yielded by the various operators differ significantly when the conventional $0\hbar\omega$ shell model wave function is used. The charge density distribution of ²⁴Mg, ²⁸Si and ³²S nuclei were calculated by Mashaan [4] using the wave function of a harmonic oscillator on the assumption that the occupation number of the states in real nuclei differ from the prediction of the simple shell model. The elastic electron scattering form factors of the considered nuclei were calculated using the ground state charge density distribution. Coulomb form factors of C4 transitions in eveneven N = Z sd-shell nuclei (20 Ne, 24 Mg, 28 Si and 32 S) have been discussed by Radhi [5] taking into account higher-energy configurations outside the sd-shell model space which are called core polarization effects. Higher configurations are taken into account through a microscopic theory, which allows particle-hole excitations from the 1s and 1p shells core orbits and also from the 2s1d-shell orbits to the higher allowed orbits with excitations up to $4\hbar\omega$. The effect of core polarization was found essential in both the transition strengths and momentum transfer dependence of form factors, and gives a remarkably good agreement with the measured data with no adjustable parameters. The calculations were based on the Wildenthal interaction for the sd-shell model space and on the modified surface delta interaction (MSDI) for the core polarization effects. Shell-model wave functions obtained from a unified

treatment of the structure of the positive parity states in sd- shell nuclei have been used by Flaih [6] to calculate the feature of the inelastic transition from $J^{\pi}(0^+ \rightarrow 2^+ \text{ and } 4^+)$ states in this region.

The purpose of the present work are to calculate the longitudinal C2 and C4 form factors for ²⁰Ne and ²⁴Mg nuclei depending on the ground state two body charge density distributions which included the effect of short range correlation (SRC). The Wildenthal (W) [7] interactions are used to get the sd-shell model space wave functions. The two-particle wave functions are those of the harmonic oscillator (HO) potential with size parameter b chosen to reproduce the measured root mean square charge radius of the nuclei considered in this work. The results will be compared with the available experimental data and for different range of momentum transfer(q).

2-THEORY

2-1 Inelastic Longitudinal Form Factors

Inelastic longitudinal electron scattering form factors involving angular momentum J and momentum transfer q which can be written as [8].

where $\hat{T}_{J}^{L}(q)$ is the longitudinal electron scattering operator. The nuclear states have well defined isospin $T_{i/f}$, therefore the form factors of eq (1) may be written in terms of the matrix elements reduced in both angular momentum and isospin [9].

(2)

where T is restricted by the following selection rule:

and $T_Z = \frac{Z - N}{2}$. The bracket () in eq (2) is the three -j symbol and the reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system including the configuration mixing are given in terms of the One Body Density Matrix (OBDM) elements times the single particle matrix elements of the longitudinal operator [10], i.e.

$$\left\langle f \| \hat{T}_{JT}^{L} \| i \right\rangle = \sum_{a,b} OBDM^{JT}(i, f, J, a, b) \left\langle b \| \hat{T}_{JT}^{L} \| a \right\rangle$$
..... (4)

The OBDM elements are calculated in terms of the isospin-reduced matrix elements [7], i.e.

$$OBDM(\tau_{Z}) = (-1)^{T_{f}-T_{z}} \begin{pmatrix} T_{f} & 0 & T_{i} \\ -T_{Z} & 0 & T_{Z} \end{pmatrix} \sqrt{2} \quad \frac{OBDM(\Delta T = 0)}{2} + \tau_{Z}(-1)^{T_{i}} \begin{pmatrix} T_{f} & 1 & T_{i} \\ -T_{Z} & 0 & T_{Z} \end{pmatrix} \sqrt{6} \frac{OBDM(\Delta T = 1)}{2}$$

..... (5)

The OBDM(ΔT) is defined [7] as :

$$OBDM(i, f, j, j', \Delta T) = \frac{\left\langle f \parallel \left[a_{j}^{+} \times \tilde{a}_{j'} \right]^{J, \Delta T} \parallel i \right\rangle}{\sqrt{2J+1} \sqrt{2\Delta T+1}} \qquad \dots \dots$$

(6)

The operator a_j^+ creates a neutron or proton in the single nucleon state j and the operator $\tilde{a}_{j'}$ annihilates a neutron or proton in the single nucleon state j'.

2-2 Core – Polarization Effects

The model space matrix elements is not adequate to describe the absolute strength of the observed gamma-ray transition probabilities, because of the polarization in nature of the core protons by the model space protons and neutrons [11]. Therefore the many particle reduced matrix elements of the longitudinal operator, consists of two parts one is for the model space and the other is for core Polarization matrix element [12].

$$\left\langle f \left\| \hat{T}_{J}^{L}(\tau_{Z},q) \right\| i \right\rangle = \left\langle f \left\| \hat{T}_{J}^{ms}(\tau_{Z},q) \right\| i \right\rangle + \left\langle f \left\| \hat{T}_{J}^{core}(\tau_{Z},q) \right\| i \right\rangle$$
.....(7)

where the model space matrix element in eq.(7) has the form [12].

$$\left\langle f \left\| \hat{T}_{J}^{L}(\tau_{Z},q) \right\| i \right\rangle = e_{i} \int_{0}^{\infty} d\mathbf{r} \, \mathbf{r}^{2} \, j_{J}(q\mathbf{r}) \stackrel{ms}{\rho}_{J,\tau_{Z}}(i,f,\mathbf{r})$$
.....(8)

The model space transition density $\stackrel{ms}{\rho}_{J,\tau_Z}(i, f, \mathbf{r})$ is expressed as the sum of the product of the OBDM times the single particle matrix elements, and is given by [7].

$$\overset{ms}{\rho}_{J,\tau_{Z}}(i,f,\mathbf{r}) = \sum_{jj'(ms)}^{ms} OBDM(i,f,J,j,j',\tau_{z}) \left\langle j \| \mathbf{Y}_{J} \| j' \right\rangle R_{nl}(\mathbf{r}) R_{n'l'}(\mathbf{r})$$

$$(9)$$

The core- polarization matrix element in eq. (7) takes the following form [12].

where ρ_J^{core} is the Core- Polarization transition density which depends on the model used for core polarization. To take the core- polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density becomes

In the present work, the shape of the Tassie Model (TM)[13] is employed for core-polarization.

2-3 Tassie – Model

This model has been used to describe gamma-transition and the excitation of nuclei by electron scattering. It is the multiple analysis of the inelastic scattering. For a uniform charge distribution this model is reduced to the usual

liquid drop model. Tassie–Model is an attempt to a model with more elasticity and modification that permits for a non-uniform charge and mass density distribution. According to this model, the core- polarization transition density depends on the ground state charge density of the nucleus. In this work, the ground state charge density is formulated in terms of the two-body charge density for all occupied shells including the core. According to the collective modes of nuclei, the core polarization transition density is given by the Tassie shape [13].

where N is a proportionality constant and ρ_o is the ground state two – body charge density distribution, which is given

and i and j are all the required quantum numbers, i.e.

 $i \equiv n_i, \ell_i, j_i, m_i, t_i, m_{t_i}$ and $j \equiv n_j, \ell_j, j_j, m_j, t_j, m_{t_j}$

where the functions $f(\mathbf{r}_{ij})$ are the two – body short range correlation (SRC). In this work, a simple model form of short range correlation of Ref. [14] will be adopted, i.e.

$$f(\mathbf{r}_{ij}) = 1 - \exp[-\beta(\mathbf{r}_{ij} - \mathbf{r}_{c})^{2}]$$

The Coulomb form factor for this model becomes:-

$$F_J^L(q) = \sqrt{\frac{4\pi}{2J_i + 1}} \frac{1}{Z} \begin{cases} \int_0^\infty r^2 j_J(qr) \rho_J^{ms}(i, f, r) dr \\ + N \int_0^\infty dr r^2 j_J(qr) r^{J-1} \frac{d\rho_o(i, f, r)}{dr} \end{cases} F_{cm}(q) F_{fs}(q)$$
.....(15)
The radial integral $\int_0^\infty dr r^{J+1} j_J(qr) \frac{d\rho_o(i, f, r)}{dr}$ can be written as:

$$\int_{0}^{\infty} \frac{d}{d\mathbf{r}} \left\{ \mathbf{r}^{J+1} j_{J}(q\mathbf{r}) \rho_{o}(i, f, \mathbf{r}) \right\} d\mathbf{r} - \int_{0}^{\infty} d\mathbf{r} (J+1) \mathbf{r}^{J} j_{J}(q\mathbf{r}) \rho_{o}(i, f, \mathbf{r}) - \int_{0}^{\infty} d\mathbf{r} \mathbf{r}^{J+1} \frac{d}{d\mathbf{r}} j_{J}(q\mathbf{r}) \rho_{o}(i, f, \mathbf{r})$$

..... (16)

where the first term gives zero contribution, the second and the third term can be combined together as

$$-q \int_{0}^{\infty} d\mathbf{r} \, \mathbf{r}^{J+1} \rho_o(i, f, \mathbf{r}) \left[\frac{d}{d(q\mathbf{r})} + \frac{J+1}{q\mathbf{r}} \right] j_J(q\mathbf{r})$$
.....(17)

From the recursion relation of spherical Bessel function:

Therefore, the form factor of eq. (15) takes the form:

$$F_{J}^{L}(q) = \left(\frac{4\pi}{2J_{i}+1}\right)^{1/2} \frac{1}{Z} \left\{ \int_{0}^{\infty} r^{2} j_{J}(qr) \rho_{J_{t_{z}}}^{ms} dr - Nq \int_{0}^{\infty} dr r^{J+1} \rho_{o} j_{J-1}(qr) \right\} \times F_{cm}(q) F_{fs}(q) \qquad (20)$$

The proportionality constant N can be determined from the form factor evaluated at q=k, i.e. substituting q=k in eq. (20), we obtain

$$N = \frac{\int_{0}^{\infty} d\mathbf{r} \, \mathbf{r}^{2} \, j_{J}(k\mathbf{r}) \, \rho_{Jt_{Z}}^{ms}(i, f, \mathbf{r}) - F_{J}^{L}(k) \, Z_{\sqrt{\frac{2J_{i}+1}{4\pi}}}}{k \int_{0}^{\infty} d\mathbf{r} \, \mathbf{r}^{J+1} \rho_{o}(i, f, \mathbf{r}) \, j_{J-1}(k\mathbf{r})}$$
......(21)

The reduced transition probability B(CJ) is written in terms of the form factor in the limit q = k (photon point) as [10].

$$B(CJ) = \frac{\left[(2J+1)!!\right]^2 Z^2 e^2}{4\pi k^{2J}} \left| F_J^L(k) \right|^2$$

(22)

In eq(22), the form factor at the photon point q=k is related to the transition strength B(CJ). Thus using eq(22) in eq(20) leads to give

 $j_j(k\mathbf{r}) = \frac{(k\mathbf{r})^J}{(2J+1)!!}$

where [10]:

$$J_{J-1}(k\mathbf{r}) = \frac{(k\mathbf{r})^{J-1}}{(2J-1)!!}$$

(24) Introducing eq.(24) into eq.(23), we obtain:

$$N = \frac{\int_{0}^{\infty} d\mathbf{r} \, \mathbf{r}^{J+2} \, \rho_{Jt_{z}}^{ms}(i, f, \mathbf{r}) - \sqrt{(2J_{i}+1)B(CJ)}}{(2J+1)\int_{0}^{\infty} d\mathbf{r} \, \mathbf{r}^{2J} \, \rho_{o}(i, f, \mathbf{r})}$$
.....(25)

The proportionality constant N can be determined by adjusting the reduced transition probability B(CJ) using eq. (25) with the experimental value of B(CJ).

3-RESULTS, DISCUSSIONS AND CONCLUSIONS

The inelastic longitudinal C2 form factors of ²⁰Ne nucleus are presented in figure (1) and of ²⁴Mg nucleus in figures (2) and (3). Here, the calculated longitudinal C2 form factors are plotted as function of the momentum transfer (q) for the transitions [7,15,16,17], $(J_i^{\pi} T_i \rightarrow J_f^{\pi} T_f) 0^+ 0 \rightarrow 2_1^+ 0$ (E_x=1.634MeV,B(C2)=292.07±37.72e².fm⁴)in

²⁰Ne, 0⁺ 0 \rightarrow 2⁺₁ 0 (E_x=1.37MeV,B(C2)=428.9±8.74e².fm⁴)and 0⁺ 0 \rightarrow 2⁺₂ 0 (E_x =4.238MeV, B(C2)=22.37±0.05 e².fm⁴) in ²⁴Mg. In these figures, the dash curves represent the contribution of the model space where the configuration mixing is taken into account, the dash- dotted curves represent the core polarization contribution, the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects into consideration and the circle symbols represent the experimental data of ²⁰Ne and ²⁴Mg. The OBDM elements for the above transitions are given in table (1) for ²⁰Ne and ²⁴Mg. These figures show that the contribution of the model space can not reproduce the experimental data since it underestimates the data for all values of momentum transfer. Considering the effect of core polarization together with the model space (the solid curves), leads to give an enhancement to the longitudinal C2 form factors and consequently to make the calculated results to be in a satisfactory description with those of the experimental data for all values of momentum transfer *q*.

The inelastic longitudinal C4 form factors of ²⁰Ne and ²⁴Mg nuclei are presented in figures (4) and (5) respectively. Here, the calculated longitudinal C4 form factors are plotted as function of the momentum transfer (q) for the transitions [7,15,16,17], $(J_i^{\pi} T_i \rightarrow J_f^{\pi} T_f)$, 0⁺0 \rightarrow 4⁺₁0 (E_x=4.25 MeV, B(C4)=38±8x10³ e².fm⁸) in ²⁰Ne and 0⁺0 \rightarrow 4⁺₁0 (E_x =6.0MeV, B(C4)=43± 6 x10³ e².fm⁸) in ²⁴Mg. The OBDM elements for the above transitions are given in table (2) for ²⁰Ne and ²⁴Mg. These figures show that the contribution of the model space can not reproduce the experimental data since it underestimates the data for all values of momentum transfer.

Finally It is concluded that the model space and the core polarization effects, which represent the collective modes, are essential in obtaining a remarkable agreement between the calculated longitudinal C2 and C4 form factors and those of experimental data.

Table(1):The	one-body	transition	density	matrix	elements	for	0^+ to
2^+ trans	sitic	ons in th	ne ²⁰ Ne and	l ²⁴ Mg nucl	ei[7].				

Nucleus	E _x (MeV)	ΔT									2j-2j'
			5-5	5-1	5-3	1-5	1-3	3-5	3-1	3-3	
²⁰ Ne	1. 634	0.0	0.4010	0.4399	0.0882	0.3	757 0.1	.533 -().1032	-0.2177	0.0947

²⁴ Mg	1.37	0.0	-0.6176	-0.3232	-0.3197	-0.4255	-0.1523	0.3155	0.1473	-0.0653
	4.238		0.1700	0.0552	0.0990	0.0609	-0.0799	-0.273	9 -0.0065	0.1850

Table(2):The one-body transition density matrix elements for 0^+ to 4^+ transitions in the 20 Ne and 24 Mg nuclei[7].

Nucleus	Ex	ΔT	2j-2j'					
	(MeV)		5-5	5-3	3-5			
²⁰ Ne	4.25	0.0	-0.4106	-0.257	0.2931			
²⁴ Mg	6.0	0.0	0.2394	0.2739	-0.4697			



Figure (1): Inelastic longitudinal C2 form factor for ²⁰Ne nucleus.



Figure (2): Inelastic longitudinal C2 form factor for ²⁴Mg nucleus.



Figure (3): Inelastic longitudinal C2 form factor for ²⁴Mg nucleus.



Figure (4): Inelastic longitudinal C4 form factor for ²⁰Ne nucleus.

¹²



Figure (5): Inelastic longitudinal C4 form factor for ²⁴Mg nucleus.

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