An Equation Related To Jordan *-Centralizers

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معادلة ترتبط في تمركزات *-جوردان

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Abstract

Let R be a *-ring, an additive mapping T: R $\rightarrow$R is called A left (right) Jordan *-centralizer of a *-ring R if satisfies $T(x^2)=T(x)\, x^*$ ($T(x^2)= x^*T(x)$) for all $x \in$ R. A Jordan *-centralizer of R is an additive mapping which is both left and right Jordan *-centralizer. The purpose of this paper is to prove the result concerning Jordan *-centralizer. The result which we refer state as follows: Let R be a 2-torsion free semiprime *-ring and let T: R $\rightarrow$ R be an additive mapping such that $2T(x^2) = T(x)\, x^* + x^*\, T(x)$ holds for all $x \in$ R. In this case, T is a Jordan *-centralizer

ألمستخلص

لتكن R حلقة-*, تدعى الدالة التجميعية $T: R\rightarrow R$ تمركزات-*جوردان اليسرى(اليمنى) إذا حققت الشرط الأتي : لكل x في R ($T(x^2) = x^*T(x)$) $T(x^2) = T(x)x^*$, وتسمى تمركزات-*جوردان اذا كانت T تمركزات-*جوردان اليسرى واليمنى, في هذا البحث سنبرهن الاتي: لتكن R حلقة-* شبه أوليه طليقة الالتواء من النمط ٢ ولتكن $T: R\rightarrow R$ دالة تجميعية تحقق الشرط الاتي: $2T(x^2) = T(x)x^*+x^*T(x)$ لكل x في R. فان T تكون دالة تمركزات-* جوردان .

1. Introduction

Throughout, R will represent an associative ring with center Z(R). A ring R is n-torsion free, if $nx = 0$, $x \in$ R implies $x = 0$, where n is a positive integer. Recall that R is prime if $a\text{R}b = (0)$ implies $a = 0$ or $b = 0$, and semiprime if $a\text{R}a = (0)$ implies $a =0$. An additive mapping $x \rightarrow x^*$ on a ring R is called an involution if $(xy)^* = y^*\, x^*$ and $(x)^{**} = x$ for all $x, y \in$ R. A ring equipped with an involution is called *-ring (see [1]). As usual the commutator $xy - yx$ will be denoted by $[x, y]$. We shall use basic commutator identities

$[xy, z] = [x, z]y + x[y, z]$ and $[x, yz] = [x, y]z + y[x, z]$ for all $x,y,z \in R$, (see [1, P.2]). Also we write xo $y=xy+yx$ for all $x, y \in R$ (see [1]). An additive mapping d: R$\to$R is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all pairs $x,y \in R$, and is called a Jordan derivation in case $d(x^2) = d(x)x + xd(x)$ is fulfilled for all $x \in R$(see [2]). Every derivation is a Jordan derivation, but the converse is in general not true. A classical result of Herstein [3] asserts that every Jordan derivation on a prime ring of characteristic different from 2 is a derivation. Cusack [4] generalized Herstein's theorem to 2-torsion free semiprime ring. A left (right) centralizer of R is an additive mapping T: R$\to$R which satisfies $T(xy) = T(x)y$ ($T(xy) = xT(y)$) for all $x, y \in R$. A centralizer of R is an additive mapping which is both left and right centralizer. A left (right) Jordan centralizer of R is an additive mapping T: R$\to$R which satisfies $T(x^2) = T(x)x$ ($T(x^2)= xT(x)$) for all $x \in R$. A Jordan centralizer of R is an additive mapping which is both left and right Jordan centralizer (see [5,6,7, and 8]). Every centralizer is a Jordan centralizer. B. Zalar [8] proved the converse when R is 2- torsion free semiprime ring. Inspired by the above definition we define. A left (right) reverse *-centralizer of a *-ring R is an additive mapping T: R$\to$R which satisfies $T(yx)=T(x)y^*$ ($T(yx) =x^*T(y)$) for all $x,y \in R$. A reverse *-centralizer of R is an additive mapping which is both left and right reverse *-centralizer. A left (right) Jordan *-centralizer of R is an additive mapping T: R$\to$R which satisfies $T(x^2)=T(x)\, x^*$ ($T(x^2)= x^*T(x)$) for all $x \in R$. A Jordan *-centralizer of R is an additive mapping which is both left and right Jordan *-centralizer. Every reverse *-centralizer is a Jordan *-centralizer. In this work we will study an Identity on a Jordan *-centralizers of semiprime *-rings. We will prove in case T: R$\to$R be an additive mapping, satisfies $2T(x^2) = T(x)\, x^* + x^*\, T(x)$ holds for all $x \in R$, where R be a 2-torsion free semiprime *-ring, then T is a Jordan *-centralizers.

2. The Main Results

If T: R $\to$ R is a Jordan *-centralizer, where R is an arbitrary *-ring, then T satisfies the relation $2T(x^2) = T(x)\, x^* + x^*\, T(x)$ for all $x \in R$. It seems natural to ask whether the converse is true. More precisely, we are asking whether an additive mapping T on a *-ring R satisfying $2T(x^2) = T(x)\, x^* + x^*\, T(x)$for all $x \in R$, is a Jordan *-centralizer. It is our aim in this paper to prove that the answer is affirmative in case R is a 2-torsion free semiprime *-ring.

Theorem 2.1. Let R be a 2-torsion free semiprime *-ring and let T: R→R be an additive mapping such that $2T(x^2) = T(x)\, x^* + x^*\, T(x)$ holds for all $x \in$ R. In this case T is a Jordan *-centralizer.

For the proof of the above theorem we shall need the following.

Lemma 2.2. [6]. Let R be a semiprime ring. Suppose that the relation $axb + bxc = 0$ holds for all $x \in$ R and some a, b, c $\in$ R. In this case $(a + c)xb = 0$ is satisfied for all $x \in$ R.

Proof of Theorem 2.1: We have

$$2T(x^2) = T(x)\, x^* + x^*T(x) \quad \text{for all } x \in R, \tag{1}$$

We intend to prove the relation

$$[T(x), x^*] = 0 \quad \text{for all } x \in R \tag{2}$$

In order to achieve this goal we shall first prove a weaker result that T satisfies the relation

$$[T(x), x^{*2}] = 0 \qquad \text{for all } x \in R \tag{3}$$

Since the above relation can be written in the form $[T(x), x^*]x^*+x^*[T(x),x^*]= 0$, it is obvious that T satisfies the relation (3) if T is satisfies (2) .

Putting in the relation (1) $x^* + y^*$ for x one obtains

$$2T((xy+yx)^*)=T(x^*)y+ xT(y^*) + T(y^*)x + yT(x^*) \quad \text{for all } x,y\in R. \tag{4}$$

Our next step is to prove the relation

$$8T(xyx) = T(x)((yx)^* + 3(xy)^*) + ((xy)^* + 3(yx)^*)T(x) + 2x^*T(y)\, x^*$$
$$-x^{*2}\, T(y) -T(y)\, x^{*2} \qquad \text{for all } x, y \in R \tag{5}$$

For this purpose, we put in the relation (4) $2(xy + yx)$ for y, then using (4) we obtain

$$4T((x(xy+yx)+(xy+yx)x)^*) = 2T(x^*)(xy+yx) + 2xT((xy+yx)^*) + 2T((xy+yx)^*)x + 2(xy + yx)T(x^*) = 2T(x^*)\,(xy + yx)+ xT(x^*)y + x^2\, T(y^*) + xT(y^*)x + (xy)T(x^*) + T(x^*)(yx)+ xT(y^*)x+ T(y^*)\, x^2 + yT(x^*)\, x + 2(xy + yx)\, T(x^*)$$

Thus, we have

$$4T((x(xy+yx) + (xy+yx)x)^*) = T(x^*)(2xy+3yx) +(3xy+2yx)\, T(x^*) +$$
$$xT(x^*)y + yT(x^*)x + 2xT(y^*)x + x^2T(y^*)$$
$$+ T(y^*)\, x^2 \qquad \text{for all } x, y \in R, \tag{6}$$

On the other hand, using (4) and (1), we obtain

$$4T((x(xy+yx)+(xy+yx)x)^*)=4T((x^2y+yx^2)^*)+8T((xyx)^*)=2T(x^{*2})y+2x^2\, T(y^*)+2T(y^*)x^2+2y\, T(x^{*2})+8T((xyx)^*)=T(x^*)(xy)+xT(x^*)y+2x^2\, T(y^*)+2T(y^*)x^2+ yT(x^*)x+(yx)T(x^*)+8T((xyx)^*)$$

$$\text{for all } x,y\in R \qquad (7)$$

By comparing (6) with (7) we arrive at (5). Let us prove the relation

$$T(x)(xyx-2yx^2-2x^2y)^*+(xyx-2x^2y-2yx^2)^*T(x)+x^*T(x)(xy+yx)^*+(xy+yx)^*$$
$$T(x)x^* + x^{*2}\,T(x)y^* + y^*T(x)x^{*2} = 0 \qquad \text{for all } x,y\in R. \qquad (8)$$

Putting in (4) $8(xyx)^*$ for y^* and x^* for x using (5), we obtain

$$16T(x^2yx+xyx^2)=8T(x)(xyx)^*+8x^*T(xyx)+8T(xyx)x^*+8(xyx)^*T(x)=$$
$$8T(x)(xyx)^*+x^*T(x)(yx+3xy)^*+(xyx+3yx^2)^*T(x)+2x^{*2}T(y)x^*-x^{*3}T(y)-x^*T(y)x^{*2}+$$
$$T(x)(xyx+3x^2y)^*+(xy+3yx)^*T(x)x^*+2x^*T(y)x^{*2} - x^{*2}\,T(y)x^* - T(y)\,x^{*3}+ 8(xyx)^*T(x)$$

We have, therefore

$$16T(x^2yx+xyx^2)=T(x)(9xyx+3x^2y)^*+(9xyx+3yx^2)^*T(x)+x^*T(x)$$
$$(yx+3xy)^* +(xy + 3yx)^*T(x)x^* + x^{*2}\,T(y)x^* + x^*T(y)x^{*2} - T(y)x^{*3}$$
$$- x^{*3}T(y) \qquad \text{for all } x, y\in R. \qquad (9)$$

On the other hand, we obtain first using (5) and then after collecting some terms using (4)

$$16T(x^2yx+xyx^2)=16T(x(xy)x)+16T(x(yx)x)=2T(x)(3x^2y+xyx)^*+2(3xyx+x^2y)^*$$
$$T(x)+4x^*T(xy)x^*-2x^{*2}\,T(xy)-2T(xy)x^{*2}+2T(x)(3xyx+yx^2)^*+$$
$$2(3yx^2+xyx)^*T(x)+4x^*T(yx)x^*-2x^{*2}\,T(yx)-2T(yx)x^{*2}=$$
$$T(x)(6x^2y+2yx^2+8xyx)^*+(8xyx+6yx^2+2x^2y)^*T(x)+4x^*T(xy+yx)x^*-2x^{*2}\,T(xy+yx)-$$
$$2T(xy+yx)x^{*2}=T(x)(6x^2y+2yx^2+8xyx)^*+(8xyx+6yx^2+ 2x^2y)^*T(x) + 2x^*T(x)(xy)^* + 2x^{*2}$$
$$T(y)x^* + 2x^*T(y)x^{*2} + 2(yx)^*T(x)x^* - x^{*2}\,T(x)y^* -x^{*3}T(y) - x^{*2}\,T(y)x^* - (y\,x^2)^*T(x) -$$
$$T(x)(\,x^2y)^* - x^*T(y)x^{*2}$$
$$- T(y)x^{*3}-y^*T(x)x^{*2} \qquad \text{for all } x,y\in R,$$

We have, therefore

$$16T(x^2yx + xyx^2) = T(x)\,(5x^2y + 2yx^2 + 8xyx)^* + (5yx^2+2x^2y+8xyx)^*$$
$$T(x)+2x^*T(x)(xy)^*+2(yx)^*T(x)x^*+x^{*2}\,T(y)x^*+x^*T(y)x^{*2}-x^{*2}\,T(x)y^*- y^*T(x)x^{*2}-x^{*3}T(y)-$$
$$T(y)x^{*3} \qquad \text{for all } x,y \in R. \qquad (10)$$

By comparing (9) with (10), we obtain (8). Replacing in (8) y by xy, we obtain

$$T(x)(x^2yx-2xyx^2-2x^3y)^*+(x^2yx-2x^3y-2xyx^2)^*T(x)+x^*T(x)(xyx+x^2y)^*+ \quad (xyx+x^2y)^*\,T(x)$$
$$x^*+ x^{*2}\,T(x)\,(xy)^* +(xy)^*\,T(x)\,x^{*2} = 0 \qquad \text{for all } x,y\in R. \qquad (11)$$

Right multiplication of (8) by x^*gives

$$T(x)\,(x^2yx - 2xyx^2 - 2x^3y)^* +(\,xyx - 2x^2y -2yx^2)^*\,T(x)\,x^*+ x^*T(x)$$
$$(xyx + x^2y)^* + (xy + yx)^*\,T(x)\,x^{*2} + x^{*2}\,T(x)\,(xy)^* + y^*\,T(x)x^{*3}$$
$$= 0 \quad \text{for all } x, y\in R \qquad (12)$$

Subtracting (12) from (11), we obtain

$$(xyx)^*[x^*,T(x)]+2(x^2y)^*[T(x),x^*]+2(yx^2)^*[T(x),x^*]+(xy)^*[x^*,T(x)]x^*+ (yx)^*[x^*,T(x)]x^*+y^*[x^*,T(x)]x^{*2} = 0, \quad \text{for all } x, y \in R.$$

This reduces after collecting the first and the five terms together to

$$(yx)^*[x^{*2},T(x)]+2(x^2y)^*[T(x),x^*]+2(yx^2)^*[T(x),x^*]+(xy)^*[x^*,T(x)]x^*+ y^*[x^*,T(x)]x^{*2} = 0 \quad \text{for all } x, y \in R. \quad (13)$$

Substituting $y\,(T(x))^*$ for y in the above relation gives

$$x^*T(x)y^*[x^{*2},T(x)] + 2T(x)(x^2y)^*[T(x),x^*] +2x^{*2}\, T(x)y^*[T(x),x^*] + T(x) (xy)^*[x^*,T(x)]x^*+ T(x)y^*[x^*,T(x)]\, x^{*2} = 0 \quad \text{for all } x, y \in R. \quad (14)$$

Left multiplication of (13) by $T(x)$ leads to

$$T(x)(yx)^*[x^{*2},T(x)]+2T(x)(x^2y)^*[T(x),x^*]+2T(x)(yx^2)^*[T(x),x^*]+T(x) (xy)^*[x^*,T(x)]x^*+ T(x)y^*[x^*,T(x)]x^{*2} = 0, \text{ for all } x, y \in R \quad (15)$$

Subtracting (15) from (14), we arrive at

$$[T(x),x^*]y^*[T(x),x^{*2}]-2[T(x),x^{*2}]y^*[T(x), x^*] =0 \qquad \text{for all } x, y \in R.$$

From the above relation and Lemma 1.2.3 it follows that

$$[T(x),x^*]y^*[T(x),x^{*2}] = 0 \quad \text{for all } x, y \in R. \quad (16)$$

From the above relation one obtains easily

$$([T(x), x^*]\, x^*+ x^*[T(x),x^*])\, y^*[T(x), x^{*2}] = 0 \quad \text{for all } x, y \in R.$$

Replace y by y^*, we get

$$[T(x),x^{*2}]\, y\, [T(x),x^{*2}] = 0 , \quad \text{for all } x, y \in R.$$

This implies (3). Substitution $x + y$ for x in (3) gives

$$[T(x), y^{*2})]+[T(y),x^{*2}]+[T(x),(xy+yx)^*]+[T(y),(xy+yx)^*] = 0 \quad (17)$$

Putting in the above relation $-x$ for x and comparing the relation so obtained with the above relation, we obtain

$$[T(x),(xy + yx)^*] + [T(y),x^{*2}] = 0 \quad \text{for all } x, y \in R. \quad (18)$$

Putting in the above relation $2(xy+yx)$ for y we obtain according to (4) and (3)

$$0 =2[T(x) ,(x^2y + yx^2 + 2xyx)^*] + [T(x)y^* + x^*T(y) + T(y)x^* + y^*T(x), x^{*2}] =2\, x^{*2}\, [T(x) , y^*] + 2\, [T(x) ,y^*]\, x^{*2} + 4\, [T(x) ,(xyx)^*] + T(x)\, [y^*,x^{*2}] + x^*[T(y),x^{*2}] + [T(y),x^{*2}]x^*+[y^*,x^{*2}]T(x) \qquad \text{for all } x, y \in R,$$

Thus, we have

$$2x^{*2}\, [T(x),y^*] + 2[T(x),y^*]x^{*2} + 4[T(x),(xyx)^*] + T(x)[y^*,x^{*2}] +[y^*,x^{*2}]T(x)+x^*[T(y),x^{*2}]+[T(y),x^{*2}]x^*=0 \quad \text{for all } x,y \in R. \quad (19)$$

For $y = x$ the above relation reduces to

$$x^{*2}[T(x),x^*] + [T(x),x^*]x^{*2} + 2[T(x),(x^2x)^*] = 0$$

This gives

$$x^{*2}[T(x),x^*] + 3[T(x),x^*]x^{*2} = 0, \text{ for all } x \in R.$$

According to the relation $[T(x), x^*]x^* + x^*[T(x), x^*] = 0$ (see (3)) one can replace in the above relation $x^{*2}[T(x), x^*]$ by $[T(x), x^*]x^{*2}$, which gives

$$[T(x), x^*]x^{*2} = 0, \text{ for all } x \in R \quad . \tag{20}$$

And

$$x^{*2}[T(x), x^*] = 0, \text{ for all } x \in R \tag{21}$$

We have also,

$$x^*[T(x), x^*]x^* = 0 \quad \text{for all } x \in R \tag{22}$$

Because of (18) one can replace in (19) $[T(y),x^{*2}]$ by $-[T(x),(xy+yx)^*]$, which gives

$$0 = 2x^{*2}[T(x),y^*] + 2[T(x),y^*]x^{*2} + 4[T(x),(xyx)^*] + T(x)[y^*,x^{*2}] +$$
$$[y^*,x^{*2}]T(x) - x^*[T(x),(xy+yx)^*] - [T(x),(xy+yx)^*]x^* = 2x^{*2}[T(x),y^*]$$
$$2[T(x),y^*]x^{*2} + 4[T(x),x^*](xy)^* + 4x^*[T(x),y^*]x^* + 4(yx)^*[T(x),x^*]$$
$$+T(x)[y^*,x^{*2}] + [y^*,x^{*2}]T(x) - x^*[T(x),x^*]y^* - x^{*2}[T(x),y^*] - x^*$$
$$[T(x),y^*]x^* - (yx)^*[T(x),x^*] - [T(x),x^*](xy)^* - x^*[T(x),y^*]x^*$$
$$- [T(x),y^*]x^{*2} - y^*[T(x),x^*]x^* = 0 \quad \text{for all } x,y\ x \in R.$$

We have, therefore

$$x^{*2}[T(x),y^*] + [T(x),y^*]x^{*2} + 3[T(x),x^*](xy)^* + 3(yx)^*[T(x),x^*]$$
$$+2x^*[T(x),y^*]x^* + T(x)[y^*,x^{*2}] + [y^*,x^{*2}]T(x) - x^*[T(x),x^*]y^* -$$
$$y^*[T(x),x^*]x^* = 0, \quad \text{for all } x, y \in R, \tag{23}$$

The substitution xy for y in (23) gives

$$0 = x^{*2}[T(x),(xy)^*] + [T(x),(xy)^*]x^{*2} + 3[T(x),x^*](x^2y)^* + 3(xyx)^*[T(x),x^*] +$$
$$2x^*[T(x),(xy)^*]x^* + T(x)[(xy)^*,x^{*2}] + [(xy)^*,x^{*2}]T(x) - x^*[T(x),x^*](xy)^* -$$
$$(xy)^*[T(x),x^*]x^* = x^{*2}[T(x),y^*]x^* + (yx^2)^*[T(x),x^*] + y^*[T(x),x^*]x^{*2}$$
$$+[T(x),y^*]x^{*3} + 3(xyx)^*[T(x),x^*] + 3[T(x),x^*](x^2y)^* + 2x^*[T(x),y^*]x^{*2} + 2(yx)^*[T(x), x^*]x^* +$$
$$T(x)[y^*, x^{*2}]x^* + [y^*, x^{*2}]x^*T(x) - x^*[T(x), x^*](xy)^* - (xy)^*[T(x), x^*]x^*$$
$$\text{for all } x, y \in R,$$

Which reduces because of (20) and (21) to

$$x^{*2}[T(x),y^*]x^* + (yx^2)^*[T(x),x^*] + [T(x),y^*]x^{*3} + 3(xyx)^*[T(x),x^*] +$$
$$3[T(x),x^*](x^2y)^* + 2x^*[T(x),y^*]x^{*2} + 2(yx)^*[T(x), x^*]x^* + T(x)[y^*, x^{*2}]x^* +$$

$$[y^* , x^{*2}]x^*T(x) - x^*[T(x) , x^*] (xy)^* =0 \quad \text{for all} \quad x,y \in R. \qquad (24)$$

Right multiplication of (23) by x^*gives

$$x^{*2}[T(x),y^*]x^*+[T(x),y^*]x^{*3}+3[T(x),x^*](x^2y)^*+3(yx)^*[T(x),x^*]x^*+2x^*$$
$$[T(x),y^*]x^{*2}+T(x)[y^*,x^{*2}]x^*+[y^*,x^{*2}]\ T(x)x^*-x^*[T(x),x^*](xy)^*-$$
$$y^*[T(x),x^*]x^{*2}= 0 \quad \text{for all } x, y \in R, \qquad (25)$$

Subtracting (25) from (24), we obtain

$$[y^*,x^{*2}][x^*,T(x)]+ 3(yx)[x^*,[T(x),x^*] + 2(yx)^* [T(x) , x^*] x^*+y^*[T(x),x^*]$$
$$x^{*2}+(yx^2)^*[T(x),x^*] = 0 \quad \text{for all } x, y \in R,$$

Which reduces because of (21), (20) to

$$2(yx^2)^*[T(x),x^*]+ 3(yx)^*[x^*,[T(x),x^*] + 2(yx)^* [T(x) , x^*] x^*=0 \quad \text{for all } x,y\in R.$$

Replacing in the above relation - $[T(x),x^*]x^*$by $x^*[T(x),x^*]$, we obtain

$$(yx^2)^*[T(x), x^*]+ 2(yx)^* [T(x) , x^*] x^*=0 \quad \text{for all } x, y \in R.$$

Because of (3), (20), (21) and (22) the relation (13) reduces to $(yx^2)^* [T(x),x^*] = 0$ for all $x, y \in R$, which gives together with the relation above $(xyx)^*[T(x),x^*] = 0$ for all $x,y \in R$, whence it follows

$$x^*[T(x),x^*]y^*x^*[T(x),x^*] = 0 \quad \text{for all } x, y \in R.$$

Thus, we have

$$x^*[T(x) , x^*] = 0, \text{ for all } x \in R. \qquad (26)$$

Of course, we have also

$$[T(x) , x^*] x^*= 0 \quad \text{for all } x \in R. \qquad (27)$$

From (26) one obtains (see the proof of (18))

$$y^* [T(x) , x^*] + x^*[T(x) , y^*] + x^*[T(y) , x^*] = 0 \quad \text{for all } x,y \in R.$$

Left multiplication of the above relation by $[T(x),x^*]$ gives because of (27)

$$[T(x) , x^*] y^* [T(x) , x^*] = 0 \quad \text{for all } x, y \in R,$$

Whence it follows

$$[T(x) , x^*] = 0 \quad \text{for all } x \in R. \qquad (28)$$

Combining (28) with (1), we obtain

$$T(x^2) = T(x) x^* \quad \text{for all } x \in R.$$

And also

$$T(x^2) = x^*T(x) \quad \text{for all } x \in R.$$

Which means that T is a Jordan *-centralizer. The proof of the Theorem is complete.

If R is prime ring, we get the following corollary

Corollary 2.3. Let R be a 2-torsion free prime *-ring and let T: R $\rightarrow$ R be an additive mapping such that $2T(x^2) = T(x)x^* + x^*T(x)$ holds for all $x \in$ R. In this case, T is a Jordan *-centralizer.

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