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Deformation parameters and nuclear radius of Zirconium (Zr) isotopes using the Deformed Shell Model

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عوامل التشوه وأنصاف الاقطار النووية لنظائر الزركونيوم (Zr) باستخدام نموذج القشرة المشوهة

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الخلاصة:

تم في هذا البحث دراسة مؤثرات عوامل التشوهات النووية رباعية القطب (β_2,δ)، العزوم الذاتية رباعية القطب (Q_o)، معدل الجذر التربيعي لنصف القطر النووي $<r^2>^{1/2}$ والاحداثيان الكبير والصغير (a,b) للقطع الناقص الذي يمثل شكل التشوه النووي بالاضافة الى الفرق بينهما (ΔR) باستخدام برنامج تم استحداثه بنظام الـ Matrix Laboratory لمعادلات نموذج القشرة المشوهه للنظائر الزوجية-زوجية لعنصر الزركونيوم (Zr) ذات الاعداد الكتلية ($A=80 \rightarrow 104$).

Abstract:

In this search we studied the most important deformation parameters (□$_2$,□), intrinsic Quadrupole moments (Q_o), root mean square of nuclear radius and major and minor of ellipsoid axises (a,b) in addition to the different between them (□R), all of these parameters were calculated to even-even Zirconium (Zr) isotopes (Z=40) for atomic number ($A=80 \rightarrow 104$), using deformed shell model equations as a simple new program in Matrix Laboratory system.

Introduction:

In the first half of the 20^{th} century a model was introduced to explain the observed shell structure in nuclei. Closed shells occur at proton and neutron "magic" numbers of 2, 8, 20, 28, 50, 82 and 126 where there are large energy gaps between successive nuclear orbitals. At these shell closures, the binding energy of the last nucleon is much larger than the corresponding value in the neighboring nuclei[1]. For nuclei to rotate they must be non-spherical so that they have a preferred axis. For deformed nuclei assuming a constant nuclear volume (incompressibility) and real solutions, This means that such nuclei are axially symmetric, either oblate or prolate[1,2].

The model that describes axially symmetric nuclei is called the ***Deformed Shell Model***. In this model the Schrödinger equation is solved using a potential that describes, as closely as possible, the actual shape of the nucleus, and it best describes spherical nuclei with a Woods-Saxon [3] potential coupled with a spin-orbit potential. This spin-orbit term arises from a coupling between the intrinsic angular momentum (s) and orbital angular momentum (ℓ) of the individual nucleons, such that j=ℓ+s. The energy levels of each j-shell are (2j+1) degenerate, labeled by m_j (the projection of j)[1,4]. The quadrupole deformation parameter (β_2), can be related to the axes of the spheroid, the larger the value of (β_2) the more deformed the nucleus. Positive and negative values correspond to prolate and oblate shapes respectively, see Figure bellow[1,5].

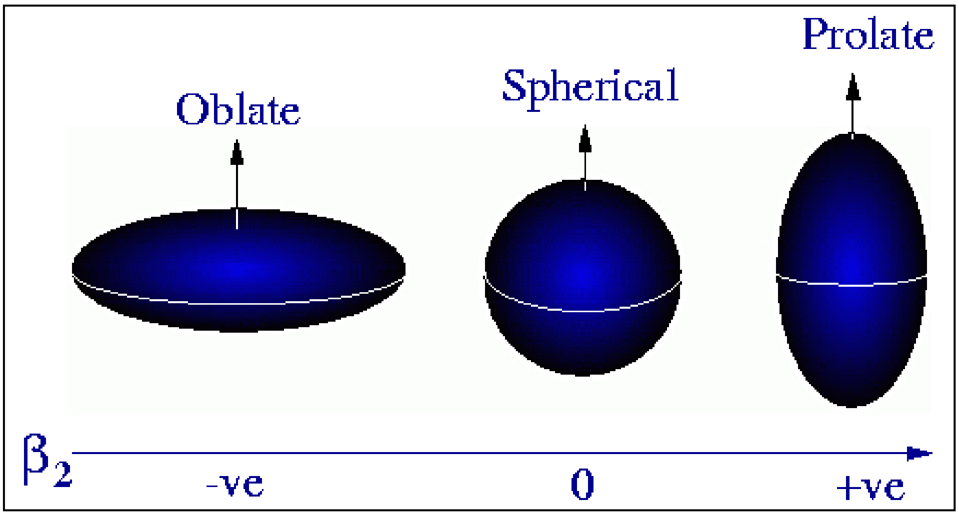

Figure (1): Diagram showing oblate, spherical and prolate shapes. The arrows for the oblate and prolate shapes indicate the symmetry axis.

The quadrupole deformation parameter (β) which is degree of nucleus shape difference from sphere, intrinsic Quadrupole moments (Q_o), nuclear radius <r> and small and large ellipsoid axises (a,b) in addition to the difference between them (ΔR), all of these parameters were calculated to even-even Zirconium (Zr) isotopes (Z=40) for atomic number (A=80 to 104), in this range there is one magic number (N=50) for ^{90}Zr, The number of valence proton (neutron) pairs N_π (N_ν) is counted from nearest magic number which to be formed total number of boson ($N=N_\pi+N_\nu$)[6].

Theoretical Part:

Nucleus quadrupole deformation parameter (□$_2$) was obtained using the following equation [1]:

$$\beta_2 = (4\pi/3 Z R_o^2)[B(E2; 0^+ \rightarrow 2_1^+)/e^2]^{1/2} \qquad (1)$$

where $B(E2; 0^+ \rightarrow 2_1^+)$ is reduced probability for E2- transition from ground 0^+ to first excited 2_1^+ state and:

$$Ro = 1.2 A^{1/3} \text{ fm} \qquad (2)$$

The intrinsic quadrupole moment of evenly charged ellipsoid can be described by equation[7,8]: $Q_o = 2/5 Z (b^2 - a^2)$ (3)

Where a and b are the small and large ellipsoid axises. Since quadrupole deformation parameter δ (degree of nucleus shape difference from sphere) can be written as[7,8]:

$$\delta = 0.3 (b^2 - a^2)/2<r^2> \qquad (4)$$

Where mean-squared charge distribution radius average is equal to:

$$<r^2> = (b^2 + 2a^2)/5 \qquad (5)$$

Equation (3) would be written as follows:

$$Q_o = 4/3 Z <r^2> \delta \qquad (6)$$

The nucleus quadrupole deformation parameter values δ could be calculated by:

$$\delta = 0.75 Q_o /(Z <r^2>) \qquad (7)$$

The values under discussion β_2 (eq.1) and δ (eq.7) are connected as[6]:

$$\delta = 0.946\beta_2 \qquad (8)$$

Value of $<r^2>$ was evaluated using the following expressions[7,8]:

$$\langle r^2\rangle=0.63R_o^2(1+10/3(\pi a_o/R_o)^2)/(1+(\pi a_o/R_o)^2)\ (\grave{A}\leq 100) \qquad (9)$$

$$\langle r^2\rangle = 0.63(1.2A^{1/3})^2\ (\grave{A}>100) \qquad (10)$$

Which takes into account effects of light nuclei surface diffusion properties.

Parameters of radial Woods-Saxon potential form-factor[8]:

($R_o=1.07A^{1/3}$ fm and $a_o=0.55$ fm)

Were obtained from the data on fast electrons scattering.

From equations 3,4 and 5 we obtained:

$$a=\sqrt{\frac{\langle r^2\rangle}{3}\left(5-\frac{2\delta}{0.3}\right)} \qquad (11)$$

$$b=\sqrt{5\times\langle r^2\rangle-2a^2} \qquad (12)$$

The different between major and minor of ellipsoid axises (a,b) can be write as[6]:

$\Delta R=\delta * R_o$ or $\Delta R = b-a$ (13)

Results and Discussions:

In this work we had studied the nuclear deformation parameters of even-even Zirconium (Zr; A=80→104) isotopes using deformed and spherical shell model. This study past over quadrupole deformation parameter (□□□,□□□□□intrinsic quadrupole moments (Q_o), root mean square radii $\langle r^2\rangle^{1/2}$, major and miner ellipsoid axises and the different between them for each isotope sunder study. Table-1 listed the isotopes were used in the present work according to their atomic mass number, and total number of boson taken from interacting boson model, which is a good parameter about how far or near the isotope from magic number for both protons and neutrons, therefore ^{90}Zr isotope have a lowest (N=5) it's a magic number N□=0 (closed shell for neutrons number) which have a very low values in quadrupole deformation parameters (□□□,□□□ and intrinsic quadrupole moments (Q_o) corresponding with reduced probability for E2-transition from ground 0^+ to first excited 2_1^+ state B(E2; 0^+□2_1^+) for transition energy in (KeV) and ^{104}Zr isotope have a largest (N=12) a very high values in (□□□□□□□□ and (Q_o) comparing with experimental data as shown in table-1.

The main objective is to clarify the effective of the results of deformation parameters using the equations from deformed and spherical shell model comparing with total number of boson came from interacting boson model, in this search we used the deformed shell model which given a good details about nuclear shapes, therefore its and another equations using to calculate the most important deformation parameter which led us to imagination of the shape of nuclei as how far from oblate and prolate (see figure-1).

In figure-2 explains the agreement between the available experimental data[9,10] and theoretical calculation and we show the same meaning up when drawing between □□□values and atomic mass number (A) for all isotopes of Zr. While figure-3 found the same behavior of the relationship of □□□between Q_o and A because of the equations[6,8].

Figure-4 shows the relevance of electric quadrupole transitions probability B(E2;0^+□2_1^+) and □□□□comparing with experimental data(11,12). All of these figures verifying that the isotope take a high values of deformation when its far from magic number of ^{90}Zr and they has a prolate shape because of the +ve values of experimental data B(E2) and □□□□see equation-1□□

Table (1): Total number of boson N, transition energy E□, reduced probability for E2-transition B(E2), intrinsic quadrupole moments Q_o and quadrupole deformation parameters □□□□ comparing with experimental data.

isotope	Total No. of boson (N)	Experimental Data				Present Work		
		ransition Energ (KeV) [9]	$B(E2;0^+\rightarrow2^+)$ (e^2b^2)[10,11]	Q_o (b) [11,12]	□□ [10,12]	Q_o (b)	□□	□
$^{80}_{40}Zr_{40}$	10	289.900	0.7600	------	------	2.9701	0.3415	0.3230
$^{82}_{40}Zr_{42}$	9	407.300	0.5300	3.020	0.3670	2.4724	0.2805	0.2654
$^{84}_{40}Zr_{44}$	8	540.000	0.4380	2.100	0.2510	2.2406	0.2509	0.2374
$^{86}_{40}Zr_{46}$	7	751.750	0.2800	1.290	0.1510	1.7861	0.1975	0.1868
$^{88}_{40}Zr_{48}$	6	1057.030	0.2600	1.600	0.1850	1.7161	0.1874	0.1773
$^{90}_{40}Zr_{50}$	5	2186.274	0.0690	0.783	0.0894	0.8816	0.0951	0.0900
$^{92}_{40}Zr_{52}$	6	934.490	0.0789	0.913	0.1027	0.9401	0.1002	0.0948
$^{94}_{40}Zr_{54}$	7	918.750	0.1015	0.810	0.0900	1.0635	0.1121	0.1060
$^{96}_{40}Zr_{56}$	8	1750.498	0.1120	0.730	0.0800	1.1143	0.1161	0.1098
$^{98}_{40}Zr_{58}$	9	1222.930	0.1580	------	------	1.3202	0.1360	0.1286
$^{100}_{40}Zr_{60}$	10	212.530	1.2300	3.340	0.3550	3.6745	0.3744	0.3541
$^{102}_{40}Zr_{62}$	11	151.770	1.6700	4.060	0.4270	4.3015	0.4305	0.4072
$^{104}_{40}Zr_{64}$	12	140.300	2.0000	------	------	4.7073	0.4650	0.4399

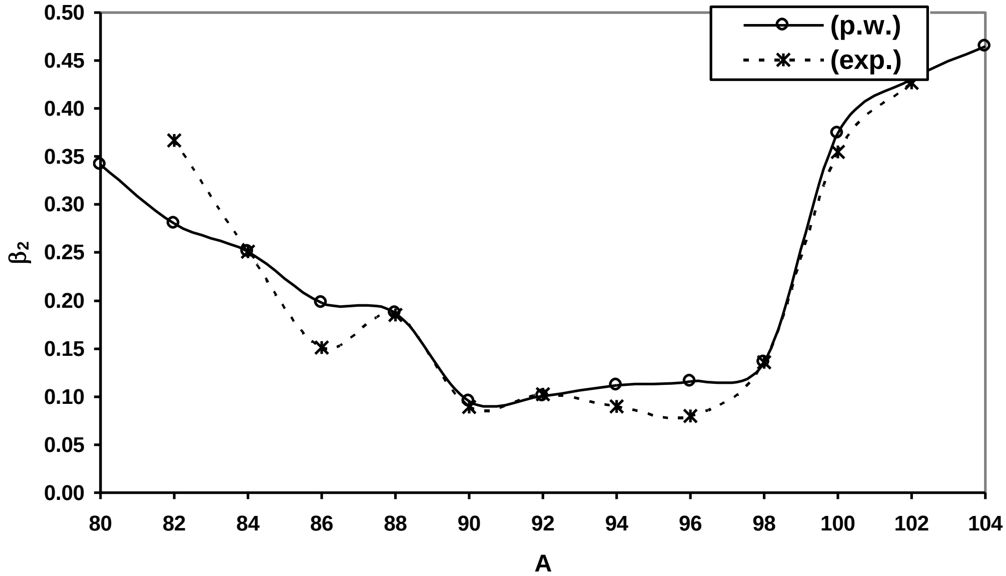

Figure (2): Comparison between calculated and experimental[10,12] β_2-values with atomic number for Zr isotopes.

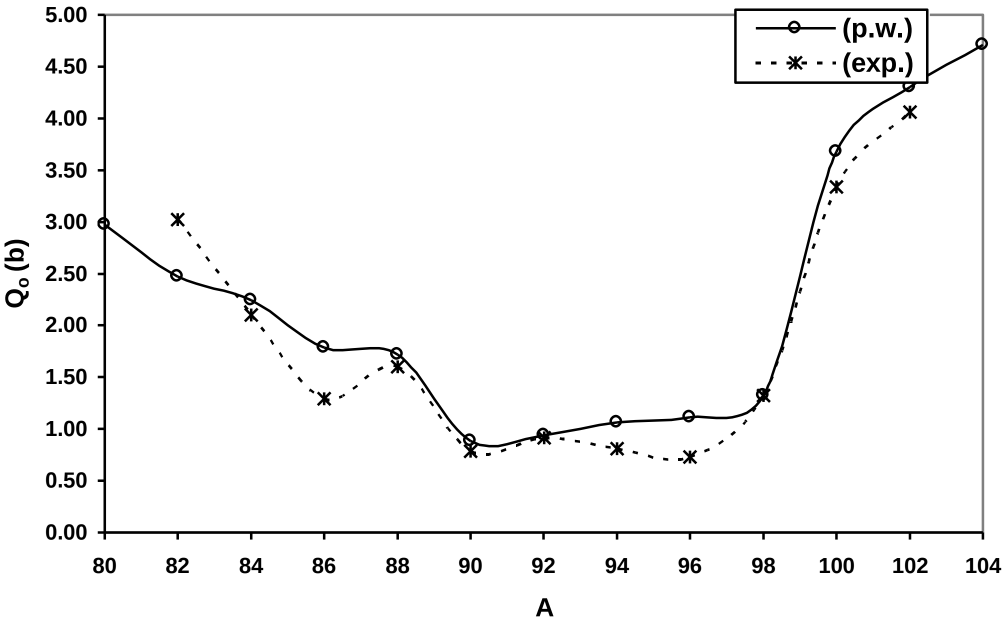

Figure (3): Comparison between calculated and experimental[11,12] Q_o-values with atomic number for Zr isotopes.

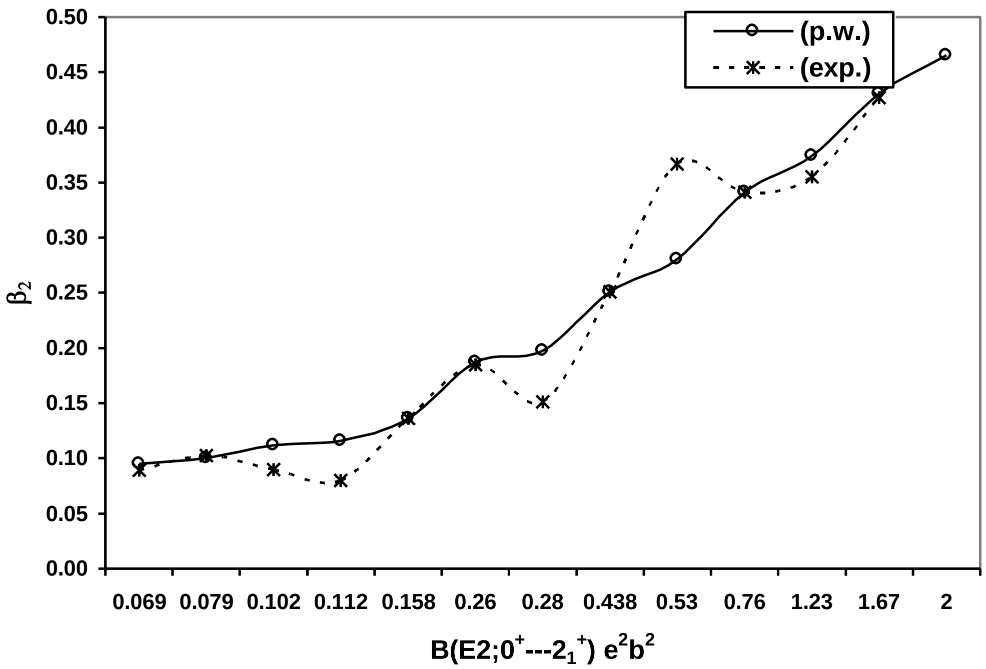

Figure (4): Quadrupole deformation parameters □2 as a function of reduced probability for E2-transition compared with experimental data[10,11,12].

Table-2 listed the hard nuclear radii parameter R_o (eq.-2), root mean square radii (eq.-9,10) compared with experimental data[13], small and large ellipsoid axises (a,b) (eq.-11,12) and the different between them in two way see eq.-13.

Figure-5 show the root mean square radii as a function of atomic mass number (A) compared with experimental data[13] and in the second (figure-6) we find the relation between □R by two methods with the atomic mass number (A) which show that the isotopes Zr (A=90,92,94,96) has a shape nearly from spherical but the other arising to prolate especially Zr (A=80,100,102,104) has a higher prolate shapes.

Table (2): Hard nuclear radii R_o, root mean square radii $<r^2>^{1/2}$ compared with experimental data, small and large ellipsoid axises (a,b) and the different between them □R from two equations.

isotope	R_o (fm)	Present Work					Experimental Data[13]
		$<r^2>^{1/2}$	a	b	ΔR_1	ΔR_2	$<r^2>^{1/2}$
$^{80}_{40}Zr_{40}$	5.1706	4.1521	1.9849	3.5890	1.6042	1.6703	4.1805
$^{82}_{40}Zr_{42}$	5.2134	4.1797	2.1217	3.4490	1.3273	1.3834	4.1820
$^{84}_{40}Zr_{44}$	5.2554	4.2069	2.1891	3.3838	1.1946	1.2475	4.2065
$^{86}_{40}Zr_{46}$	5.2968	4.2337	2.3018	3.2514	0.9496	0.9897	4.2307
$^{88}_{40}Zr_{48}$	5.3376	4.2600	2.3284	3.2337	0.9053	0.9464	4.2546
$^{90}_{40}Zr_{50}$	5.3777	4.2860	2.5037	2.9761	0.4689	0.4839	4.2782
$^{92}_{40}Zr_{52}$	5.4172	4.3116	2.5055	3.0005	0.4950	0.5137	4.3182
$^{94}_{40}Zr_{54}$	5.4562	4.3369	2.4913	3.0449	0.5537	0.5784	4.3552
$^{96}_{40}Zr_{56}$	5.4946	4.3619	2.4911	3.0657	0.5747	0.6034	4.4098
$^{98}_{40}Zr_{58}$	5.5325	4.3865	2.4610	3.1335	0.6724	0.7117	4.4531
$^{100}_{40}Zr_{60}$	5.5699	4.4107	1.9698	3.7807	1.8109	1.9725	4.5482
$^{102}_{40}Zr_{62}$	5.6068	4.4503	1.8411	3.9334	2.0923	2.2833	4.5864
$^{104}_{40}Zr_{64}$	5.6432	4.4792	1.7568	4.0278	2.2710	2.4826	4.6163

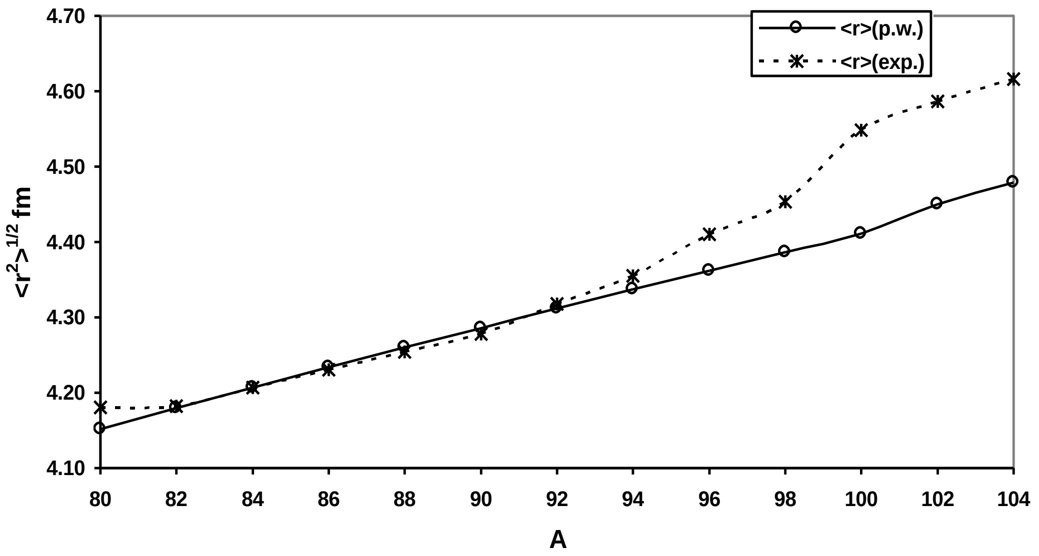

Figure (5): Root mean square radii $<r^2>^{1/2}$ as a function of (A) compared with experimental data[13].

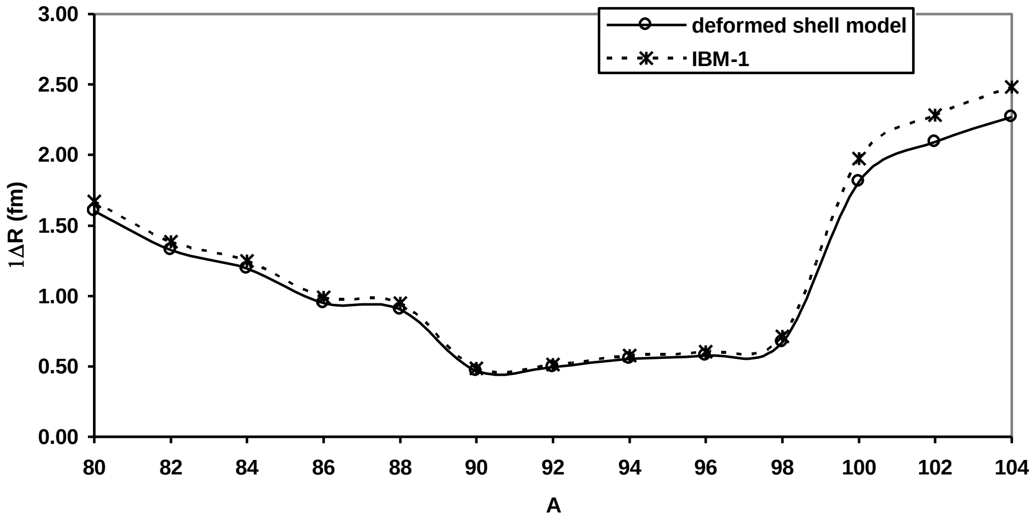

Figure (6): The difference between major and minor of ellipsoid axises (□R) from two methods (deformed shell model and IBM-1) as a function with (A).

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