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A NEW ANALYTICAL REPRESENTATION FOR CREEP CURVES OF PLASTICS

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تمثيل تحليلي جديد لمخططات الزحف في اللدائن

بتول مردان فيصل – كلية الهندسة / جامعة واسط

ألخلاصة:

تم في هذا البحث التوصل إلى معادلات جديده للحصول على التنبؤ الأفضل للتصرف اللزج-المرن للبوليميرات ألصلبه وذلك بإدخال (stress invariants) في نظرية التكاملات المتعدده. في الجزء التحليلي لحالة تحميل الشد الأحادي المحور وجد أن الزحف يمكن أن يجزأ إلى ثلاث مركبات. تمثل ألمركبه الأولى والثانية الزحف الناتج من (hydrostatic and deviatoric stresses) على التوالي بينما تمثل ألمركبه ألثالثه الزحف الناتج من التأثـير المشـــــترك لـ (hydrostatic and deviatoric stresses). اما لحالة تحميل الالتواء فقد وجد ان كل مركبات الزحف تعتمد على (deviatoric stresses) لذا فقد تم إيجاد معادلات جديده بدلالة قانون القوه وهي ((hydrostatic, deviatoric and synergistic kernels). ألمقارنه بين النتائج ألعمليه لدراسات سابقه ونتائج نظريه التكاملات المتعدده أعطت مقاربه جيده عند حالات تحميل مختلفه ولمواد البولي بروبلين، البولي اثيلين والبولي مثيل ميثاكريليت وبمعدل خطأ مقداره 0.05%.

Abstract

In order to have the best prediction of the non-linear viscoelastic behaviour of solid polymers, a new constitutive relationship has been obtained by inserting the stress invariants (I_1) and (I_2') into the formulation of multiple integral representation (MIR). The analytical part for the uniaxial tensile loading, it was found that creep could be separate into three components. First and second components, represent the contributions of the hydrostatic and deviatoric stresses respectively, while the third component represent the contribution of the synergistic effect of the hydrostatic and deviatoric stresses. Also, for the case of pure shear loading it was found that all components of creep are dependent on the deviatoric stress. Thus, new kernels have been obtained which called the hydrostatic, deviatoric and synergistic kernels, in terms of power law. Good agreement has been obtained from the comparison between the experimental of other studies and theoretical results of MIR under

different types of loading for PP, PE and PMMA. This comparison proved that MIR gives the prediction with an average error of 0.05%.

Notations

F_{ij}	A non-linear tensile function
t	Time (s)
e	Strain
I_1, I_2'	Stress invariants
K_1, K_2, K_3	Hydrostatic, deviatoric and synergistic tensile kernel functions
G_1, G_2	Deviatoric shear kernel functions
e_{11}	Tensile strain
e_{12}	Shear strain
n	Time constant
PMMA	Polymethyl methacrylate
PE	Polyethylene
MIR	Multiple integral representation
PP	Polypropylene
tr	Trace
$\nu_0,..\nu_3$ and $\pi_1,..\pi_5$	Time functions including the material constant
τ	Shear stress
ξ	Time parameter (s)
σ	Tensile stress (N/m^2)
I	Unit matrix
σ_{ij}	Stress tensor
e_{ij}	Strain tensor

1. Introduction:

Polymers are the fastest growing class of engineering materials in volume of usage and now firmly established in many load-bearing duties. The structure of polymers below glass transition is non-equilibrium inhomogeneous[1], so that the stress and strain analysis is complicated[1, 2]. Also, the deformation behaviour of thermoplastics under mechanical loads depends to a great extent on time. The influence of time makes the dimensioning of plastic components considerably more complicated than other materials[3].

Different attempts of researchers investigated the creep behaviour of plastics from different sights. The first attempt to study the effect of hydrostatic (I_1) and deviatoric (I'_2) stresses was by Buckley and McCrum[4] whose found that the linear response of solid polymers is related to the hydrostatic stress while the non-linear response is related to the deviatoric stress. Resen[5] designed a biaxial creep machine and performed many experiments under tension, torsion and combined tension-torsion loading to study the separate roles of (I_1) and (I'_2). Also, for the same purpose, Jabbar[6] and Zai′bel[7] performed many experiments under combined tension-internal pressure and combined tension-torsion- internal pressure loading respectively. Also Za'ibel[7] proposed an analytical procedure by using the finite and boundary element formulations to predicate the onset of non-linear creep, recovery and stress relaxation. Oliveira and Creus[8] used a numerical method for modeling the failure behaviour of composite laminates in the presence of large displacements and creep. The modeling of material behaviour included thermal, and viscoelastic effects, using an efficient state variable representation. Thus, the procedure can be used to analyze buckling, creep, buckling and creep including damage. Then they have been extended this procedure to study the nonlinear viscoelasticity of thin-walled beams in composites materials[9] and ageing in fiber reinforced polymer composites[10]. Resen and Faisal[11] used MIR to predict the creep response of PMMA under combined tension-torsion loading. They found that MIR gives a good perdition for long term of creep under combined loading.

As we knew (I_1) represents the components of the applied stresses which cause the linear response while (I'_2) represents the components of the applied stresses which cause the nonlinear response. Until now MIR is depend on the applied stresses and don't use the stress invariants. Also most researches are limited to experimental study and to the linear and the onset of the nonlinear range. Thus, the aim of this work is to insert (I_1) and (I'_2) into the relations of MIR and predict the non-linear creep of semicrystalline and amorphous solid polymers which help us to separate the linear and nonlinear response and know to which range of strain we should be used these materials in engineering application.

<u>2. Theory:</u>

2.1 Constitutive Relationship:

The strain of polymeric materials can be expressed in the form of multiple integrals due to the dependency of the strain at any time on all stress history [2].

$$e(t) = F_{ij}\left[\frac{d\sigma(\xi)}{d\xi}\right]_{-\infty}^{t} \qquad \ldots(1)$$

where σ and e are the stress and strain tensors respectively, F_{ij} is a non-linear tensile function, and ξ is an arbitrary time.

This behaviour could be described by containing the stress terms up to third order. Thus, for the case of step loading, the strain tensor is given by the following equation becomes [2]:

$$e = I\left[\pi_1 tr\sigma + \pi_2 tr(\sigma\sigma) + \pi_3 tr\sigma\, tr\sigma + \pi_4 tr\sigma\, tr(\sigma\sigma) + \pi_5 tr\sigma\, tr\sigma\, tr\sigma\right]$$

$$+\left[\nu_o + \nu_1 tr\sigma + \nu_2 tr(\sigma\sigma) + \nu_3 tr\sigma\, tr\sigma\right]\sigma \qquad \ldots(2)$$

where e = e_{ij} is the strain tensor.

π_i and ν_j are the time functions including the material constant [2].

i = 1, …, 5

j = 0, …, 3

tr = trace

2.2 Loading Programs:

2.2.1 Uniaxial tensile loading:

For this case of uniaxial tensile loading, the stress tensor and it’s traces are:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad tr\sigma = \sigma_{11}, \quad tr\sigma\sigma = \sigma_{11}^2 \qquad \ldots(3)$$

Where $\sigma_{11} = \sigma$ and σ is the tensile stress.

Also, the stress invariants for this case is given by:

$$I_1 = \sigma_{11} \quad \ldots(4a)$$

$$I'_2 = \frac{\sigma_{11}^2}{3} \quad \ldots(4b)$$

Substituting Eqs. (3) and (4) into Eq. (2), the tensile strain $e_{11}(t)$ can be obtained as follows:

$$\begin{aligned} e_{11}(t) &= \pi_1 I_1 + 3\pi_2 I'_2 + 3\pi_3 I'_2 + 3\pi_4 I_1 I'_2 + 3\pi_5 I_1 I'_2 + \\ &\quad \nu_0 I_1 + 3\nu_1 I'_2 + 3\nu_2 I_1 I'_2 + 3\nu_3 I_1 I'_2 \\ &= K_1 I_1 + K_2 I'_2 + K_3 I_1 I'_2 \end{aligned} \quad \ldots(5)$$

Where $K_1 = \pi_1 + \nu_0$

$$K_2 = 3\times(\pi_2 + \pi_3 + \nu_1)$$

$$K_3 = 3\times(\pi_4 + \pi_5 + \nu_2 + \nu_3)$$

Where $K_1, K_2 \text{and} K_3$ are the hydrostatic, deviatoric and synergistic tensile kernel functions respectively.

According to Eq. (5), three pure tension tests at different stress levels are required to determine the unknown kernels $K_1, K_2 \text{and} K_3$.

2.2.2 Pure shear loading:

For this case, the stress tensor and its traces are given by the following equation:

$$\sigma_{ij} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{tr}\sigma = 0, \quad \mathrm{tr}\sigma\sigma = 2\tau^2 \quad \ldots(6)$$

Where τ is the shear stress, but:

$$I_1 = 0 \quad \ldots(7a)$$

$$I'_2 = \tau^2 \quad \ldots(7b)$$

Substituting Eqs. (6) and (7) into Eq. (2), the shear $e_{12}(t)$ strain can be obtained as follows:

$$e_{12}(t) = \left(\nu_0 + 2\nu_2 I_2'\right)\sqrt{I_2'}$$

$$= G_1\sqrt{I_2'} + G_2\left(I_2'\right)^{3/2} \qquad \ldots(8)$$

Where $G_1 = \nu_0$

$G_2 = 2\nu_2$

$G_1 \text{and} G_2$ are the deviatoric shear kernel functions.

It can be noted (from Eq. (8)) that two pure shear tests at different stress levels are required to determine these unknown kernels.

3. Results and Discussion:

3.1 Uniaxial tensile loading:

The behaviour of semicrystalline PE and PP at 20ºC and amorphous PMMA at 30ºC was studied over time range of 3, 30 and 3 hr respectively. The block diagram of the program which has been written is shown in Fig 1. The responses of three uniaxial tests at different stresses[5, 12] (Table 1) have been substituted into Eq. (5) to determine the hydrostatic K_1, deviatoric K_2 and synergistic K_3 tensile functions by using Guassin elimination. Then, least square method has been used for fitting these functions in terms of power law as given in Table 2. It was found that time exponents for PE, PP and PMMA are 0.125, 0.07 and 0.065 respectively which are similar to those found by Faisal[13]. These functions are presented in Fig. 2, 3 and 4.

Fig. 2 shows that K_1 and K_2 increase non-linearly with time while K_3 decrease non-linearly with time. In contract Figs. 3 and 4 show that K_1 and K_3 increase non-linearly with time whereas K_2 decreases non-linearly with time. These increasing or decreasing can be noted from the log scale of the time which depend on the type of material if it is amorphous or semicrystalline. The change is only in the behaviour of K_2 and K_3 which can be attributed to the decreasing of the degree of crystallization[2].

Substituted these functions (Table 2) into Eq. (5) to find the tensile strains at different hydrostatic (I_1) and deviatoric (I'_2) stress level. These determined strains were compared with experimental results[5, 12] as shown in Figs. 5, 6 and 7. A good agreement between the experimental[5, 12] and MIR results has been obtained with a tensile error of 2.892835×10^{-4}. From these figures it can be noted that tensile strain increase non-linearly with time and increasing of I_1 and I'_2 caused a step shift in these curves because I_1 and I'_2 depend on the applied stresses. Since I'_2 depends on I_1, so this shift occurred due to increment in I_1 that can be consider the main factor for the case of unaixial tensile loading which agree with the results of Resen[5].

3.2 Pure shear loading

For the case of pure shear loading, the behaviour of PMMA at 30ºC was studied over the time range of 3 hr. by substituting the responses of two pure shear tests [5] at different stresses (Table 3) and by using block diagram of the program which is shown in Fig 1. The deviatoric shear functions G_1 and G_2 have been determine by using the same procedure that mentioned in section 3.1.

It was found that time exponent of these functions is the same as that found for the tensile functions of PMMA (Table 2). These functions are also present in Fig. 8, which indicate that both G_1 and G_2 are increase nonlinearly with time.

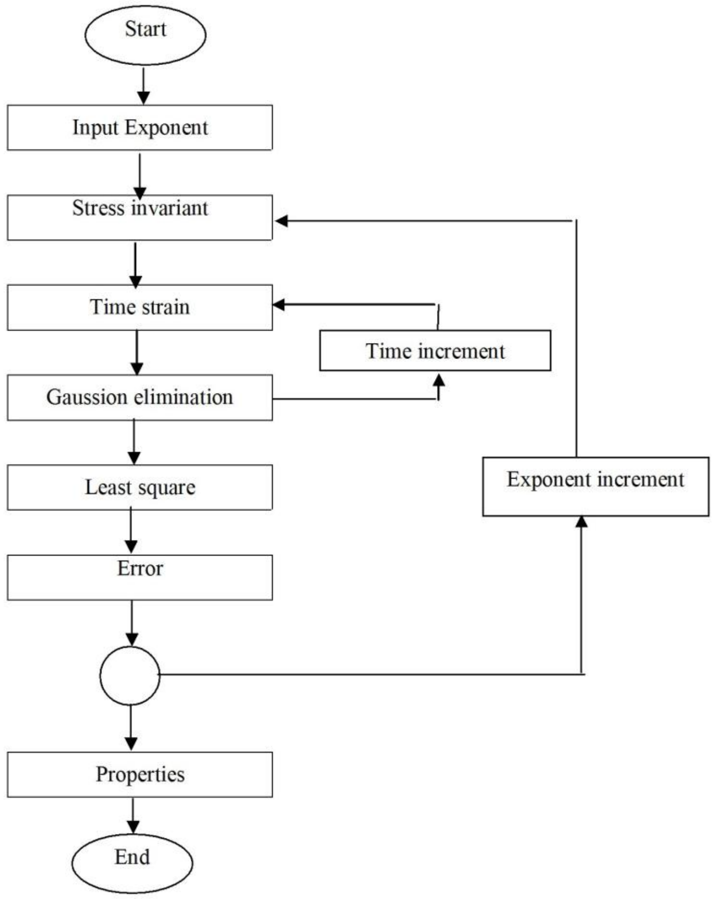
Substituting G_1 and G_2 into Eq. (8) to find the shear response at different values of I'_2. Fig. 9 shows the comparison between the experimental [5] and MIR results. A good agreement between these results has been obtained with a shear error of 1.219314×10^{-3}. Also it can be noted that the increasing of I'_2 caused of step shift in the curves of shear strains, which indicate that I´2 is the main factor for the case of pure shear loading.

From the comparison shown in Figs. 5, 6, 7 and 9, it can be noted that a good agreement was obtained by using the formulation that derived in this paper. Also the total error in tensile and shear strains was 0.05%.

4.Conclusions:

New constitutive equations have been obtained by inserting I_1 and I'_2 into the MIR. For the case of uniaxail tensile, the functions have been obtained are the hydrostatic, deviatoric and synergistic functions for PE, PP and PMMA. While only the deviatoric functions have been obtained for the case of pure shear loading. Also it was found that tensile strain depends on I_1 while shear strain in highly dependent on I'_2. The obtained error in the first case was 2.892835×10^{-4} and 1.219314×10^{-3} for the second case. The total error in tensile and shear strains was 0.05%.

Fig. 1 The block diagram of MIR



Main subroutine	**Function**
Stress invariants	To enter stresses ivariants date required for calculation of Gaussian elimination
Time, Strain	To enter time and a corresponding strain at stresses given in stress subroutine
Gaussian	To find kernel functions
Time increment	To control increments if time is intended to be in incermental form.
Least square	To find the kernel functions in the form of power law
Error	To calculate the error between the experimental and MIR results.
Properties	To obtain creep compliance and compressibility function in terms of power law. Also, to calculate their values and Poisson's ratio at any required time and stress.

Table 1 The hydrostatic and deviatoric stresses

that used to fined K_1 , K_2 and K_3

Material	No. Of Test	**I_1 (MPa)**	**I'_2(MPa^2)**
PE	1	0.5520	0.5071040
	2	1.1030	0.4055360
	3	1.6550	0.9130080
PP	1	1.3780	0.6326910
	2	4.1360	5.7021650
	3	6.8970	15.856203
PMMA	1	12.247	49.996336
	2	24.247	195.97200
	3	34.600	399.05300

Table 2 Hydrostatic, deviatoric and synergistic tensile functions for different materials.

Material	**N**	**Tensile functions**
PE	0.125	K_1 = 0.1809797E-3 + 568.7743E-3 t^n K_2 = 100.40550E-3 + 276.9348E-3 t^n K_3 = 49.481490E-3 + 38.00241E-3 t^n
PP	0.07	K_1 = - 14.1241E-3 + 132.25400E-3 t^n K_2 = 6.960681E-3 – 27.102280E-3 t^n K_3 = -1.556835E-3 + 4.621695E-3 t^n
PMMA	0.065	K_1 = 0.52972650E-3 + 2.0826460E-3 t^n K_2 = 0.70489650E-3 – 0.6110778E-3 t^n K_3 = -0.03216576E-3 + .03867906E-3 t^n

Table 3 Hydrostatic and deviatoric stresses that used to

find G_1 and G_2 for PMMA.

Material	**No. of Test**	**I_1 (MPa)**	**I_2 (MPa^2)**
PMMA	1	0	299.29
	2	0	49.985

Table 4 Deviatoric shear functions for PMMA.

Material	n	Shear functions
PMMA	0.065	$G_1 = 0.09663496E\text{-}3 + 0.26425420E\text{-}3\ t^n$ $G_2 = -0.02205132E\text{-}3 + 0.02946145E\text{-}3\ t^n$

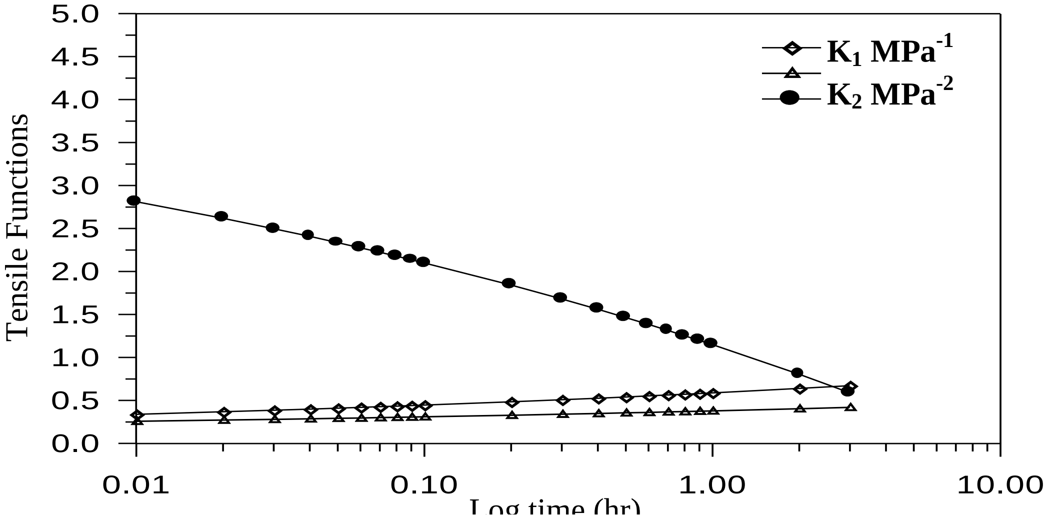


Fig. (2) Hydrostatic, deviatoric and synergistic tensile kernel functions of PE at 20°C

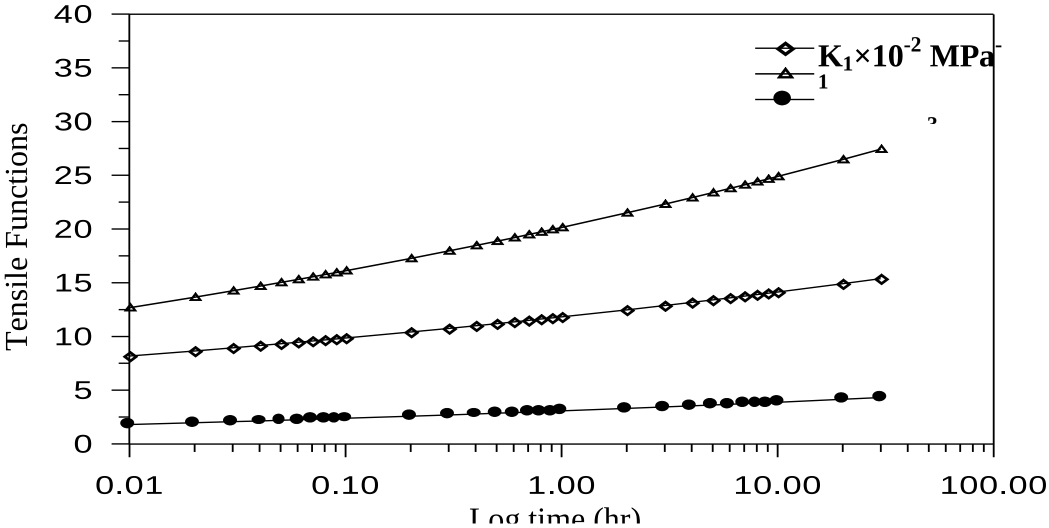


Fig. (3) Hydrostatic, deviatoric and synergistic tensile kernel functions of PP at 20°C

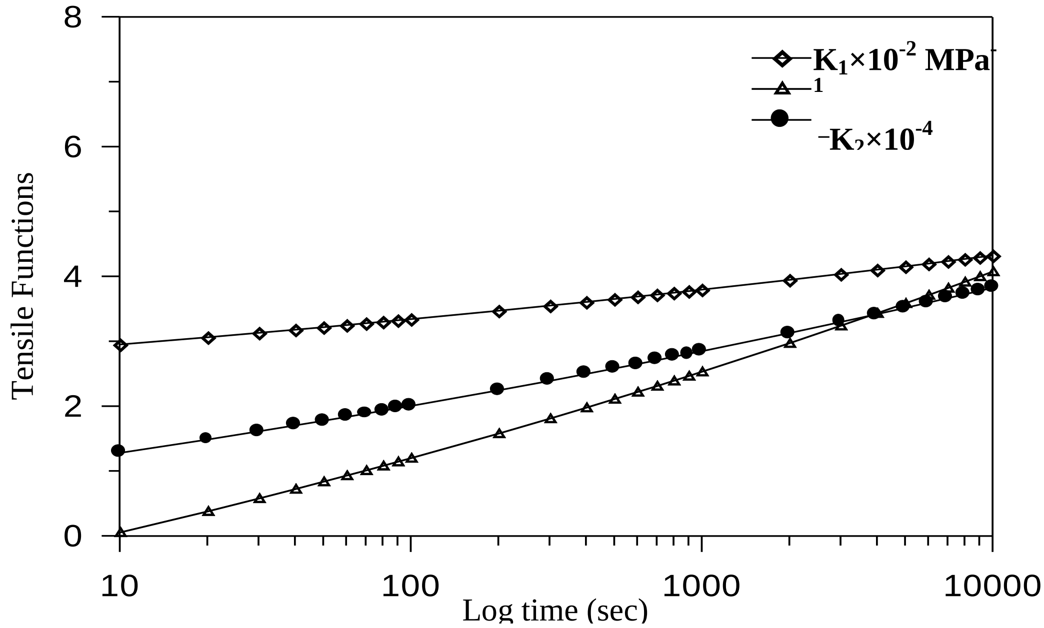


Fig. (4) Hydrostatic, deviatoric and synergistic tensile kernel functions of PMMA at 30°C.

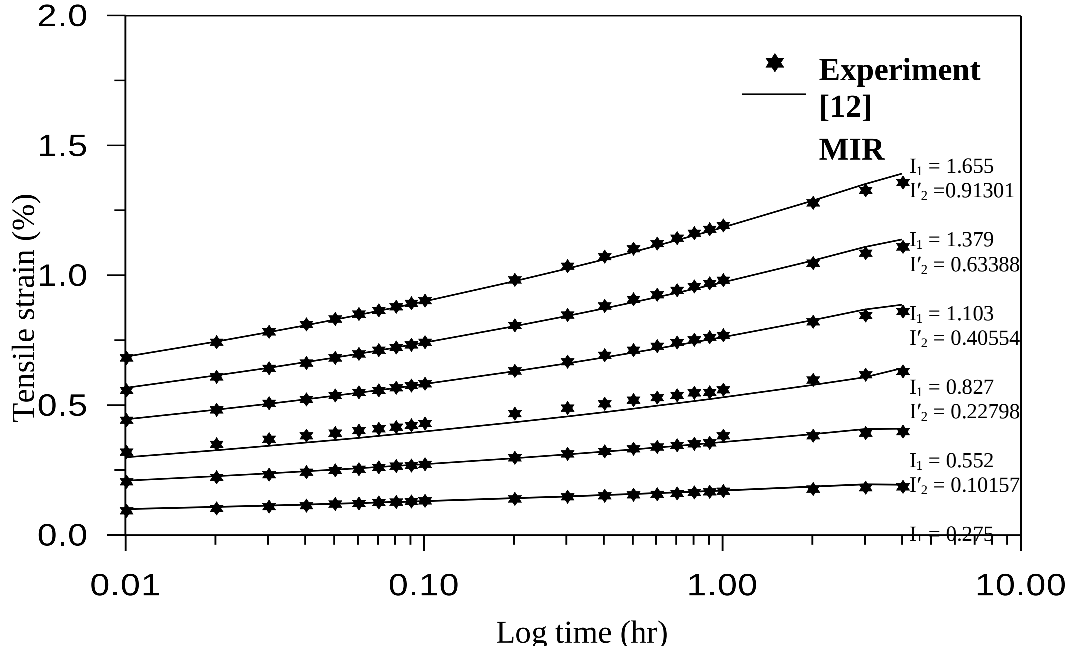


Fig. (5) Creep curves of PE for different hydrostatic and deviatoric stress at 20°C.

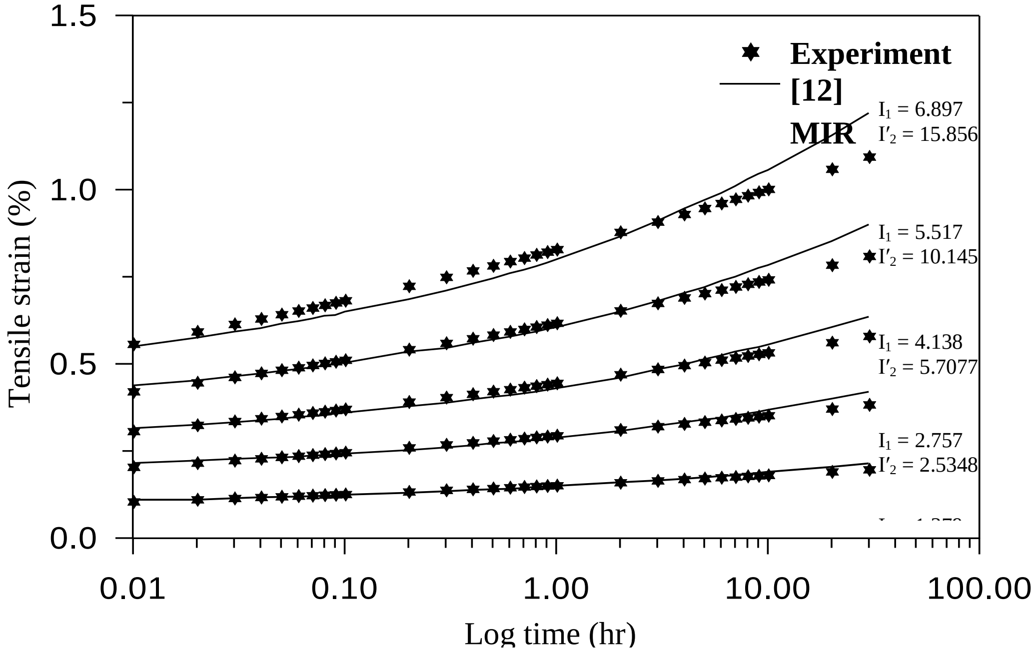


Fig. (6) Creep curves of PP for different hydrostatic and deviatoric stress at 20°C.

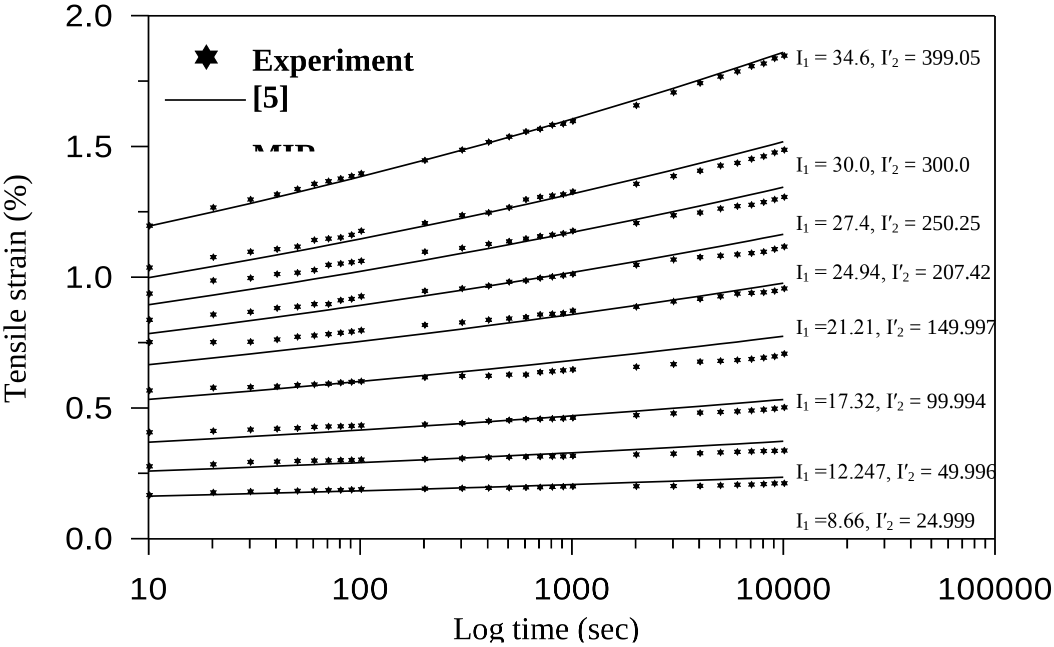


Fig. (7) Creep curves of PMMA for different hydrostatic and deviatoric stress at 30°C.

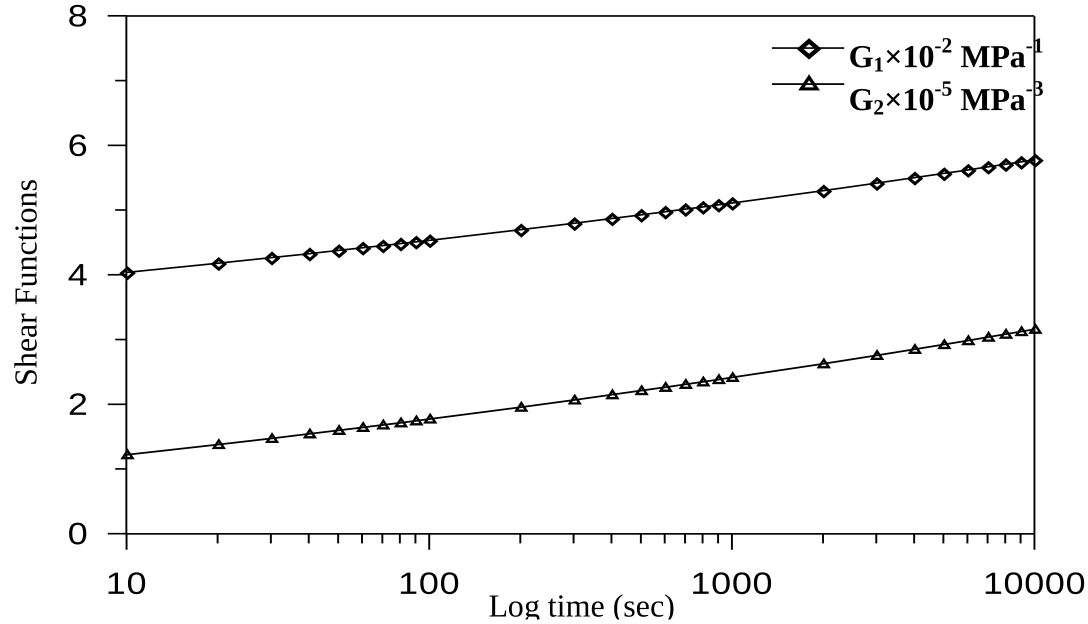


Fig. (8) Deviatoric shear kernel functions of PMMA at 30°C.

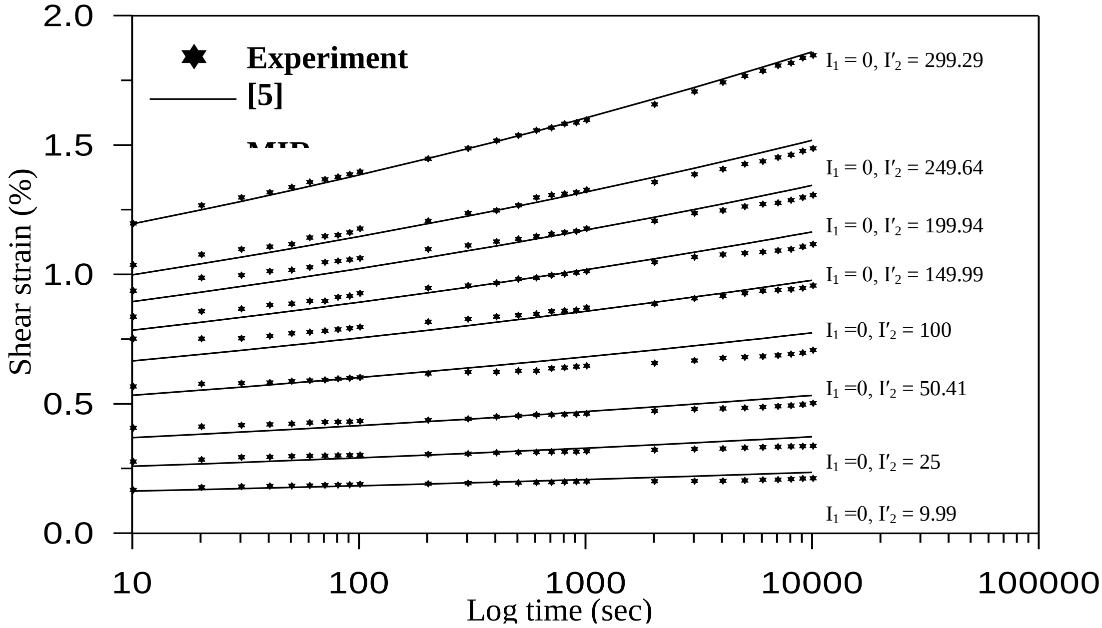


Fig. (9) Creep curves of PMMA for different hydrostatic and deviatoric stress at 30°C.

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