



AN APPROACH FOR SINGLE-TONE FREQUENCY ESTIMATION USING DFT INTERPOLATION WITH PARZEN WINDOWING

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ABSTRACT

In many applications, including radar, radio and television, medical, industrial, and others, frequency estimate of a periodic sinusoidal signal is a crucial step in the signal processing process. Due to its simplicity of usage in digital systems, the interpolation-based signal frequency estimation algorithm is now frequently employed. Because of its performance speed, interpolation methods in the analysis rely on the fast Fourier transform (FFT). This paper proposes an approach for single-tone frequency estimate utilizing DFT interpolation with Parzen windowing in order to increase the accuracy of frequency estimation. In addition, compared to Li and Dian algorithm, the proposed method has a lower computing complexity and more steady performance. To minimize undesirable effects brought on by spectrum leakage from the FFT procedure, suitable windows have been investigated. To investigate the viability of the suggested method, three windows Flattop, Parzen, and Bohman, were applied to the simulation signal. When compared to the other windows, the Parzen window with the proposed algorithm outperformed them with a maximum frequency estimation error of 0.00003 compared to 0.0001 and 0.0002 for the Dian and Li algorithms, respectively, when the Sample Size was 8192.

KEYWORDS: Frequency estimation, windowing function, FFT, interpolation, algorithm, periodic signal.

1. INTRODUCTION

An important task of frequency estimation in modern signal processing in many applications such as radar, radio and television channels, medical and industrial fields, and others (Dan, et al., 2012; Vaseghi, 2008). Current frequency estimation tools can be basically divided into two classifications: algorithms of the time-domain and algorithms of the frequency-domain. First classification involve algorithms of the maximum likelihood (ML) (Chen and Braga-Neto, 2013; Agüero, et al., 2010), algorithms of the autocorrelation (Tu and Shen, (2017); Cao (2012)), algorithms of the linear prediction (Alku, et al., 2013), and algorithms of the least squares (Nandi, 2012). However, due to the large number of calculation requisite, these algorithms are tough to use in real-time implementations. Second classification usually performed on the basis of discrete Fourier transform DFT (Fan and Qi, 2018; Belega and Petri, 2015; Belega and Petri, 2014; Fan et al., 2021; Serbes, 2019; Aboutanios and Mulgrew, 2005; Candan, 2013; Fang et al., 2012). These algorithms ordinarily have teeny calculation requisite. Hence, they are convenient for real-time implementations. Frequency estimates based on DFT typically have two stages: rough search and accurate search. Rough search is utilized to locate the maximum strength of the DFT of samples using an easy maximum search steps. The accurate search allows a relative variance of the signal frequency from rough estimation by definite methods of the interpolation. The variance between various algorithms of the interpolation fall only in the second stage. The real sine wave model is too recurrently used in empirical implementations, and real sine wave frequency estimation is more complex than complex sine wave because of the issue of a spectrum leak from a component of the negative signal spectrum. Plenty of researchers offered its algorithms to real sine wave (Dutra et al., 2014; Li and Kui, 2008; Ding et al., 2010). Dutra et al., 2014) Offers an estimator such as ML build by matching of the spectrum. The algorithm bypass the spectral leakage problem by including it in the model of the spectrum. However, this algorithm requires a comprehensive search that requires a lot of computation.

Li and Kui, (2008) introduced a new interpolation method, pertinent to the complex spectrum of many windows, a complex formula for obtaining the frequency of the component shown in equation (1) and (2). This complex spectral approach (CSBA) is less sensitive to spectral leakage than the modular approach. Frequency correction amount δ is respectively

$$\delta = \pm \frac{\alpha}{\alpha-1} \quad (1)$$

$$\delta = \pm \frac{2\alpha+1}{\alpha-1} \quad (2)$$

Where

$$\alpha = \begin{cases} \frac{U_{k-1}}{U_k}, & \text{if } U_{k-1} \geq U_{k+1} \\ \frac{U_k}{U_{k+1}}, & \text{if } U_{k-1} < U_{k+1} \end{cases} \quad (3)$$

U_k, U_{k-1} and U_{k+1} : are spectral lines with the maximum amplitudes and two spectral lines left and right with the largest amplitudes respectively, That is, the complex ratio of the two spectral lines with the largest amplitudes. δ - The offset of the normalized fractional frequency, $\delta = (-0.5, 0.5)$.

Ding et al., (2010) execute also a fine search using the maximum spectral line and two spectral lines to the left and right of the maximum as shown below:

$$\delta = \frac{U_{k+1} - U_{k-1}}{U_k + U_{k-1} + U_{k+1}} \quad (4)$$

This paper proposes an accurate frequency evaluation of the real sine wave based on DFT. The proposed estimator is depend on interpolation of the three maximum point of the DFT spectral line and its suitable with most windows, Flat top, Parzen and Bohman windows were tested and compared. Computer simulation results show that the behavior of proposed algorithm is superior to that of Ding and Li algorithms. The algorithm analysis was performed showing that the calculations complexity are lower and the performance is more stable depending on the frequency evaluation error. The rest of this paper is as follows. In the second section the proposed algorithm described. In the third section the results and discussion are described, Conclusions presented in the last section.

2. PROPOSED MODEL

The general procedure of the interpolation estimator are shown in Fig. 1.

Consider a sine wave as a discrete sequence:

$$p[n] = A \sin(2\pi F_0 n t + \varphi) \quad n = 0, 1, 2, \dots, N - 1 \quad (5)$$

Where N is the Sample Size. A, F_0 and φ are the amplitude, frequency, and initial phase of sine wave, respectively.

If the number of period of the signal are integer, then the frequency of the signal can be expressed as

$$F_0 = (K_{\max} \frac{F_s}{N}) \quad (6)$$

Where K_{\max} is the index value of the discrete frequency of the maximum component in spectrum of DFT (see Fig. 2) and F_s is the sampling frequency. The frequency resolution:

$$\Delta F = \frac{F_s}{N} \quad (7)$$

ΔF : is the distance between two spectral lines.

Applying the windowing function on $p[n]$:

$$P[n] = p[n] * w[n] \quad (8)$$

The following windowing function are used in this paper: Flattop, Parzen, and Bohman window:

2.1. Flattop Window

Flattop window have very low passband ripple. This window drawbacks gives the broad bandwidth and poor frequency resolution. Flat top windows are summations of cosines shown in eq. 5 (Gade and Henrik, 1987) [20]:

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) - a_3 \cos\left(\frac{6\pi n}{N-1}\right) + a_4 \cos\left(\frac{8\pi n}{N-1}\right) \quad (9)$$

The coefficients for this window are:

$$a_0 = 0.21557895, a_1 = 0.41663158, a_2 = 0.277263158, a_3 = 0.083578947, a_4 = 0.006947368$$

2.2. Parzen window

Parzen windows are piecewise-cubic approximations of Gaussian windows. This window usually using to minimize side lobe levels, but they tend to have a heavy scalloping. The Parzen (Harris, 1978) window is defined as:

$$w(n) = \begin{cases} 1 - 6\left(\frac{|n|}{\frac{N-1}{2}}\right)^2 \pm 6\left(\frac{|n|}{\frac{N-1}{2}}\right)^3, & 0 \leq |n| \leq \frac{N-1}{4} \\ 2\left(1 - \frac{|n|}{\frac{N-1}{2}}\right)^3, & \frac{N-1}{4} < |n| \leq \frac{N-1}{2} \end{cases} \quad (10)$$

Where $n = 0, 1, 2, \dots, N-1$. The periodic window is N -periodic.

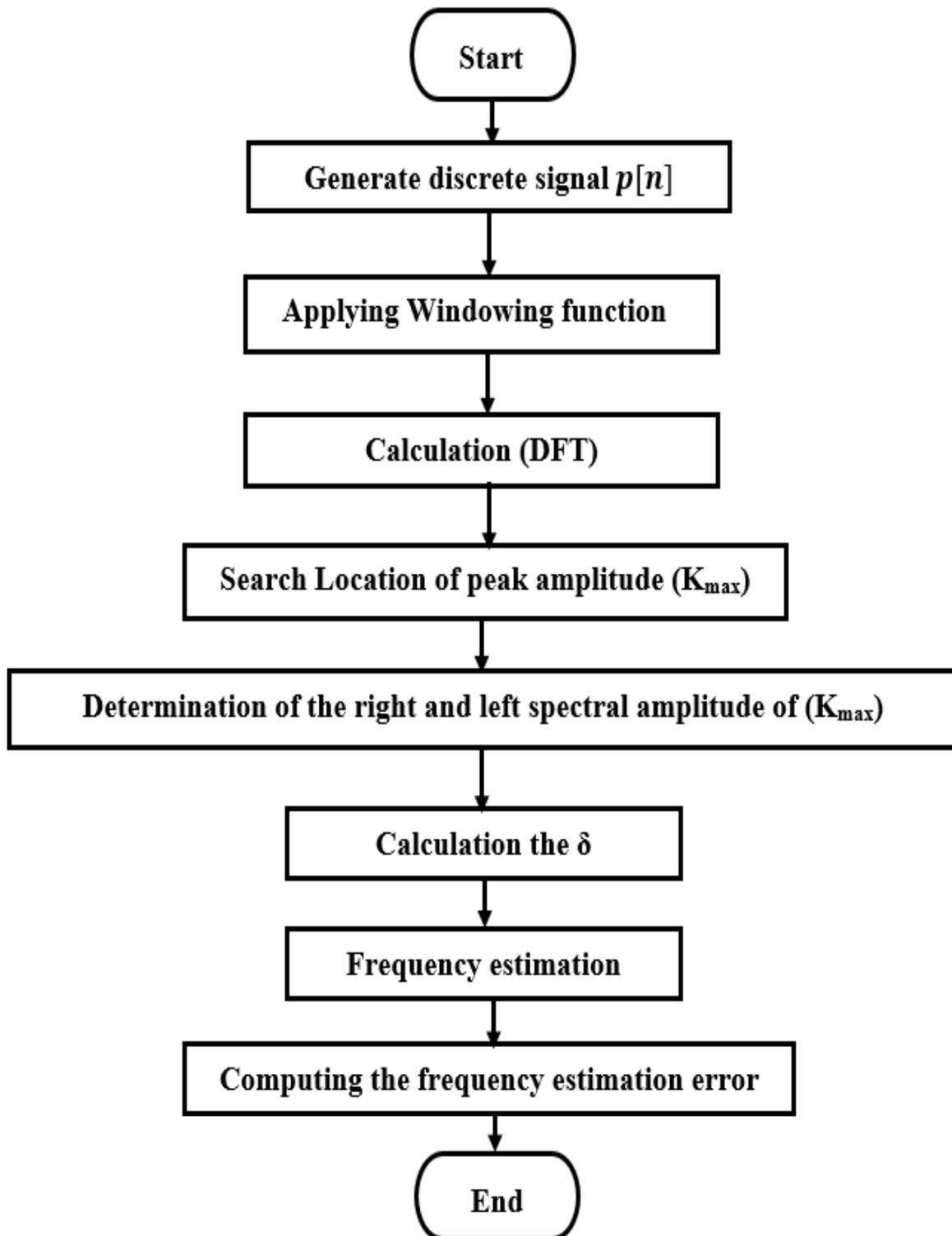


Fig. 1. Procedure of the general interpolation method.

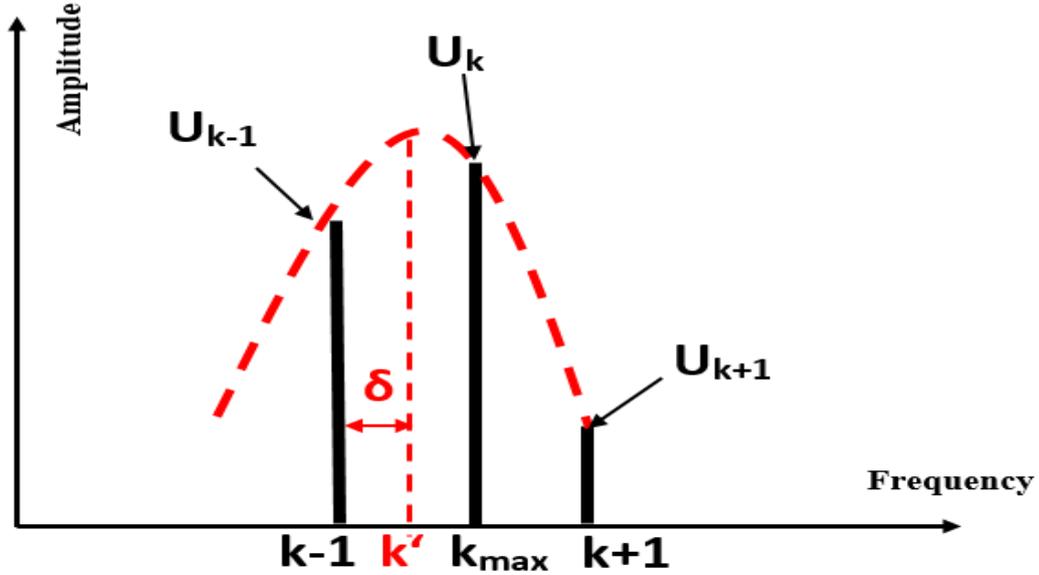


Fig. 2. The three points in the DFT spectrum used for interpolation.

2.3. Bohman window

This window is a product of two cosine lobes of half the duration. In the time domain, it is the product of a triangular window and a single cycle of a cosine with a term added to set the first derivative to zero at the boundary. The window is defined in equation (Harris, 1978):

$$\omega(x) = (1 - |x|)\cos(\pi|x|) + \frac{1}{\pi}\sin(\pi|x|), \quad -1 \leq x \leq 1 \quad (11)$$

Where x is a length vector of linearly spaced values.

The analysis algorithm examined the spectral characteristics of the signal. In a lot of applications and for the purpose of spectroscopy. DFT is also often referred to as (FFT) and is the algorithm that achieves DFT. In DFT first the sliding window mechanism is applied to the input signal $p[n]$. That is, the signal will be soft at its ends. The choice of analysis window is a well-developed topic and affects the spectral resolution of the analysis. The DFT for signal $p[n]$ is given by:

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} P(n) \cdot e^{-j\left(\frac{2\pi kn}{NT}\right)} \quad (12)$$

Where T is sampling period and k is the DFT bin number with $0 \leq k \leq N - 1$. The important step in the analysis stage is to assign the detected peaks and amplitude. The different formula available of the Li, Dian and proposed algorithms are the main scope of in this paper.

As shown in Fig. 1, the block diagram of the computational steps begins with generating a signal and take the sampling process for it with a number of samples (16-8192), then applying the time window to this sampled signal to reduce spectral dispersion, after that calculate the (FFT) of the output, subsequently find the sample that carries the highest amplitude, which that falls desired frequency. But if the number of cycles taken in the sampling process is an integer number of cycles, therefore this means that the required frequency is Located between two samples that carry the highest amplitude, as shown in Fig. 2. After this, compute δ the difference between K_{\max} and $k + 1$ or $k - 1$ according to the highest amplitude by equation (13) to sum it with the k and we find the required frequency and evaluate the error according to equation (15)

$$\delta = \frac{U_{k+1} - U_{k-1}}{U_k} \quad (13)$$

The signal frequency is calculated by:

$$F_{Est} = \frac{(\delta + K_{\max})Fs}{N} \quad (14)$$

Where $K' = (\delta + K_{\max})$

Estimate of the error of the frequency estimation by:

$$Error = \frac{(F_{Est} - F_c)}{F_c} \quad (15)$$

3. RESULT AND DISCUSSION

The parameters for computer simulations for result in Figs. 3-5 are as follows: The value of phase elected between $[-\pi, \pi]$ with step $=10^\circ$, $F_s/F_c=4$, Sample size is (16,32,64,.....,8192) respectively, sampling frequency = (16,32,64,.....,8192) respectively and the used windows are Flattop, Parzen, and Bohman respectively , All the presented results in Figs. 3-5 compared between three algorithms Li, Dian and proposed algorithms with relative maximum frequency estimation error and Sample size. The optimum result of simulations are when Parzen window used. The performance of this algorithms is similar when using a Flat top window, but the difference is clear when using the Bohman and Parzen windows. The systematic maximum errors reach (0.0002, 0.0001 and 0.00003) for Flat top, Bohman and Parzen windows respectively when sample size = 8192. Further, the results shows that our algorithm is better

than other algorithms for all sample size. The error decreases gradually with increasing sample size.

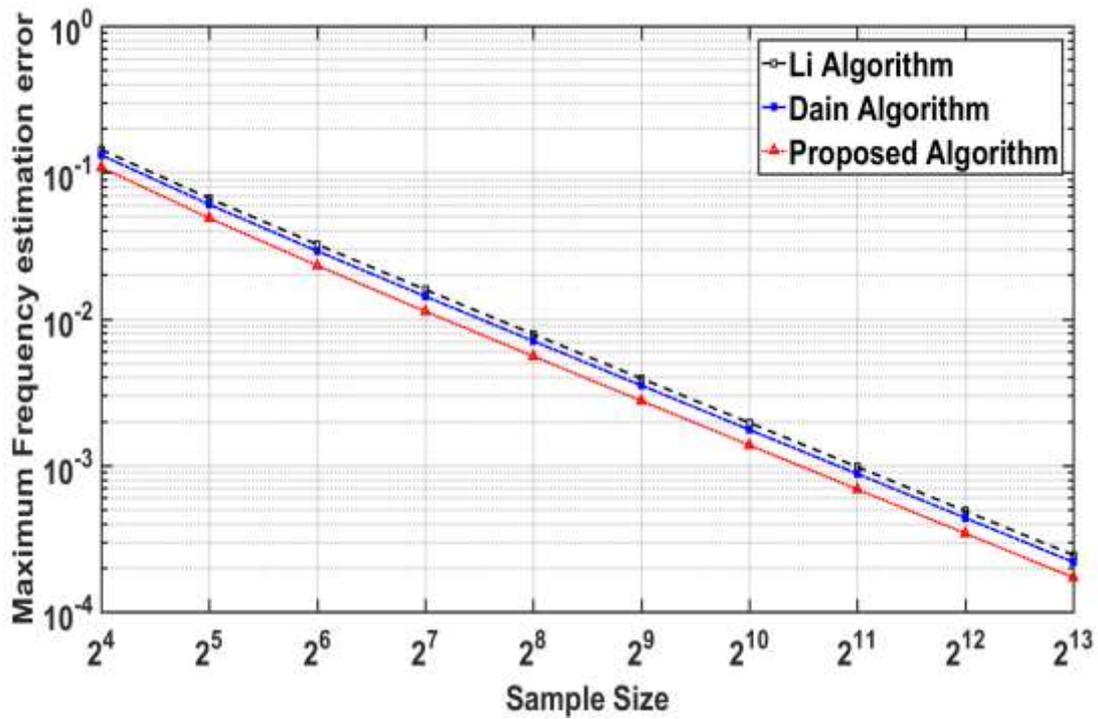


Fig. 3. Relation between relative maximum frequency estimation error and Sample Size for Flat-top window.

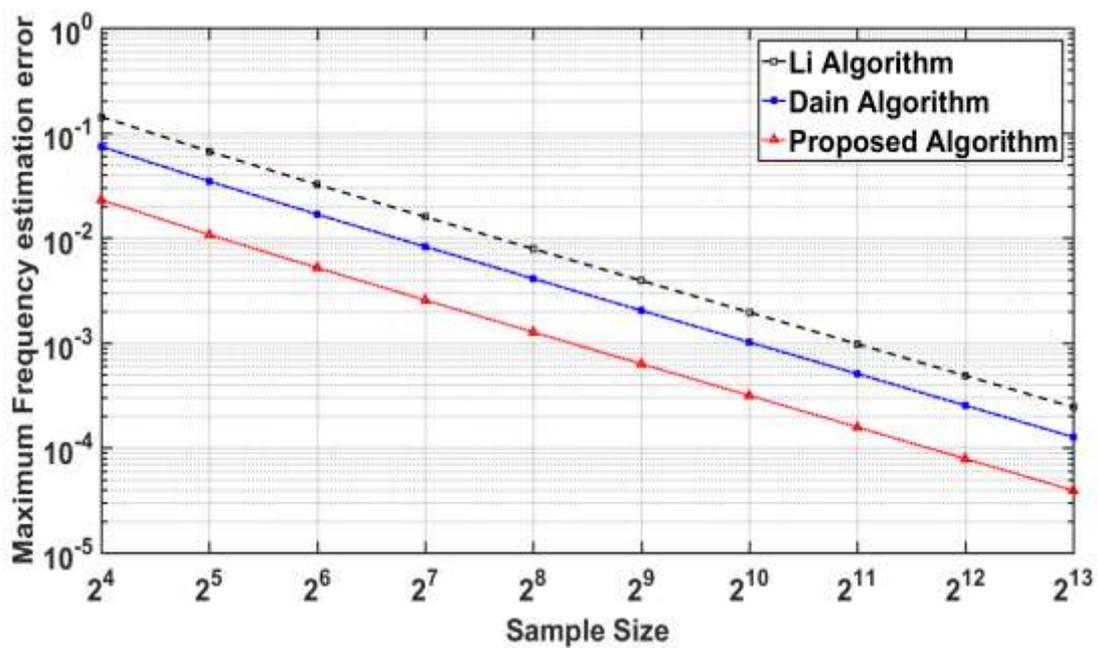


Fig. 4. Relation between relative maximum frequency estimation error and Sample Size for Parzen window.

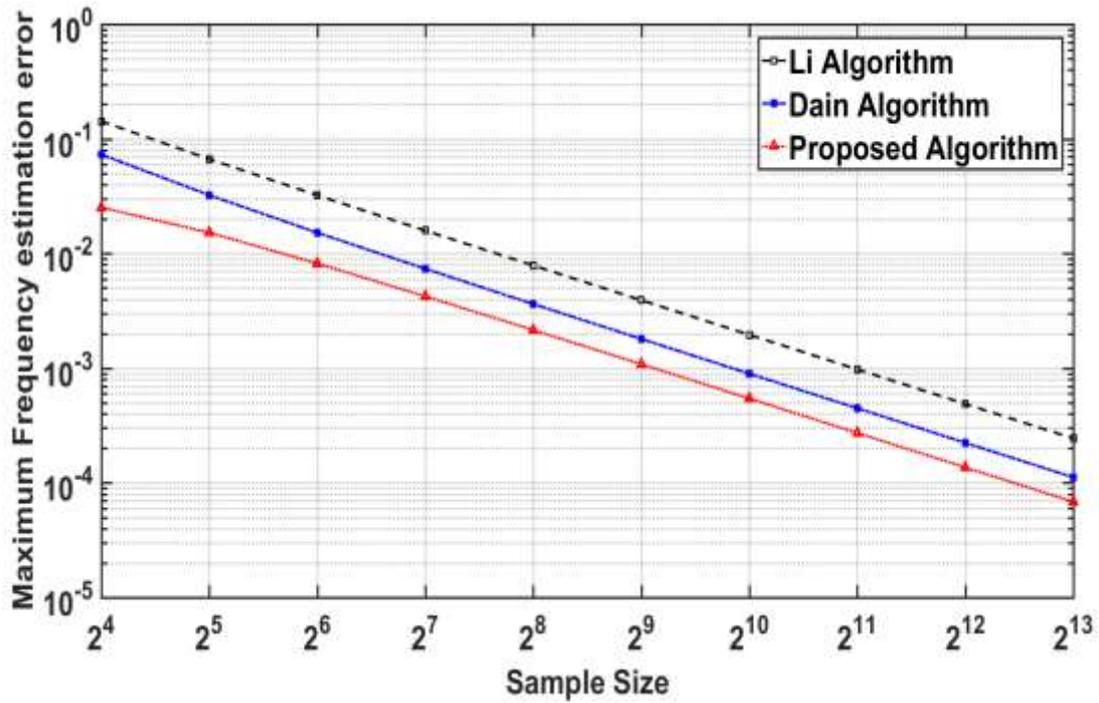


Fig. 5. Relation between relative maximum frequency estimation error and Sample Size for Bohman window.

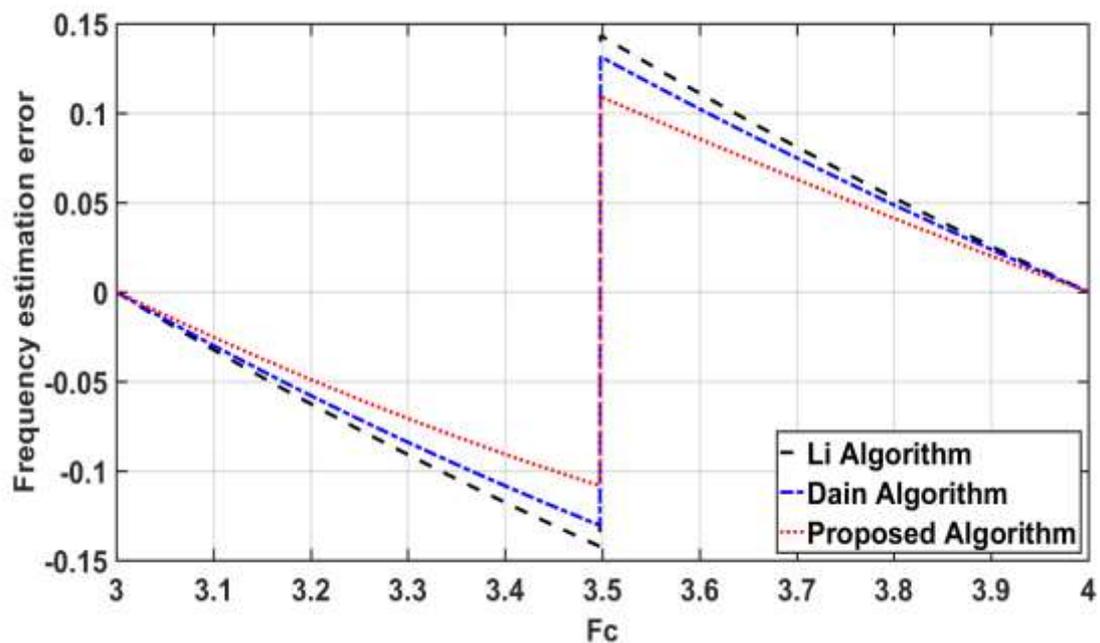


Fig. 6. Relation between relative error and Fc at 16 samples (Flat top window).

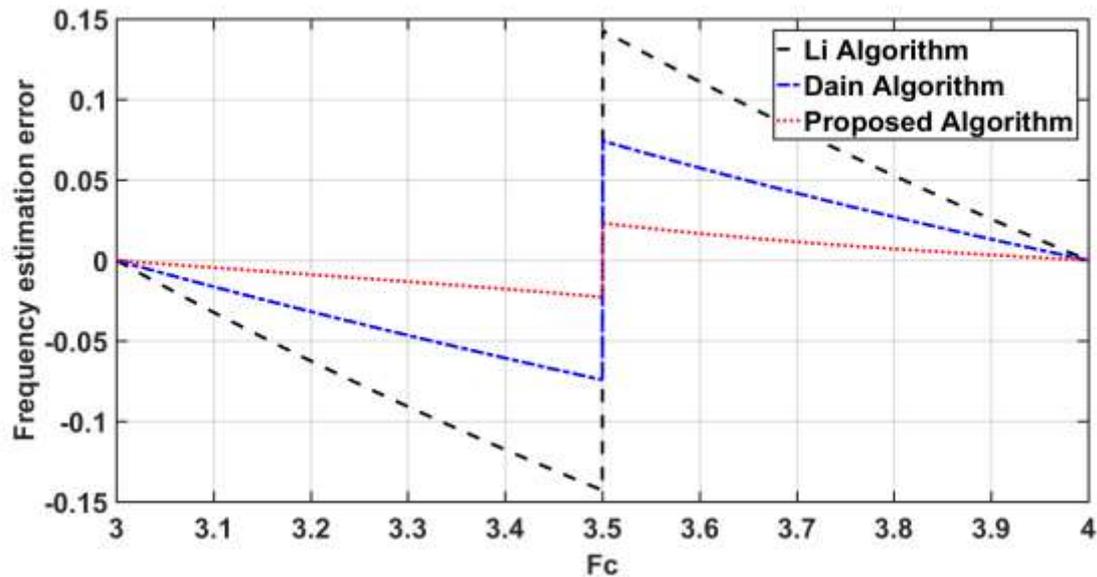


Fig. 7. Relation between relative error and F_c at 32 samples (Parzen window).

To illustrate the effects of the non-integer number of signal period especially when closely to 3.5 Hz, we have generated compare with a signal length $N = 16$ and 32, frequency spacing between each components equal to 1 Hz with signal frequency = (3 to 4) Hz. Figs. 6-7 plots this effect of the Flat top and Parzen windows respectively with respect to frequency estimation error with F_c . The Figures clearly shows that when the signal frequency near from 3.5 Hz the highest frequency error occurs. Furthermore, it is also shown that Parzen window reduce this error. Where the errors at $N=16$ were 0.14, 0.13 and 0.10 for Li, Dian and proposed algorithms respectively when Flat top window used, Where errors become (0.14, 0.07 and 0.02) using the Barzen window for Li, Dian and proposed algorithms respectively. This means that the times of error reduction is 5 times in the proposed algorithm and twice in the Dine algorithm.

4. CONCLUSION

In this paper, we propose a fine and effective algorithm based on three-point interpolation of DFT samples to evaluate the fundamental frequency. The results shows that our algorithm performs better than all the other algorithms, especially when the Parzen window used. Our algorithm also shows high efficiency when the number of samples is 8192 and the error drops to $(3.0 * 10^{-5})$ using the Parzen window.

We recommend that our approach can also be used as a configuration for other iterative algorithms, improving overall performance in frequency evaluation. Future work will be interesting to combine this weighted algorithm with other algorithms to achieve more accurate results.

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