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Control of the detuning and Rabi frequency on the behavior of soliton fiber laser

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سيطرة اللاتنغيم وتردد رابي على سلوك السوليتون لليزر الألياف

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الملخص:
يستعرض هذا البحث سلوك السوليتون في وسط الليزر المصنع من ليف زجاجي المطعم بايونات اليربيوم باستخدام نموذج يستعمل البصريات الكمية ونظام الثلاث مستويات .نبدأ من معادلة الحركة لمؤثرات الكثافة الالكترونية.نشتق معادلة شرود نكر اللاخطية والتي تضم عامل اللاتنغيم وتردد رابي والتشتت الليفي واللاخطية لليف.نستعمل حل تحليلي لتوضيح أن الموجات السوليتونية ممكن أن تتواجد وان تكون مستقرة بعد أربع دورات في المرنان الحلقي ومحصلة الكسب تعتمد على اللاتنغيم وتردد رابي.

Abstract :
This research explore the behavior of soliton in Er $^{+3}$ fiber laser medium by using a model that use the Quantum optics and three levels system . Starting from the equation of motion of the density operator, the researcher derive the nonlinear Schrödinger equation which include the detuning , Rabi frequency , the group velocity dispersion coefficient of the host fiber and γ the host nonlinearities and use analytic solutions to show that solitary waves can exist and be stable after four loops in ring cavity and the net gain depends on detuning and Rabi frequency .

Introduction:
The interaction of a strong bichromatic field with three levels atomic transition is fundamental to a number of research disciplines, including nonlinear optics, quantum optics, and laser theory, one of this research is behavior of laser pulse in Erbium-doped fiber which is consisted of a short section of fiber that has a small amount of the rare earth element (Erbium)added to it. The principle involved here is that Erbium ions are able to exist in various electronic energy states .Now the question arises, why this Erbium

doped Fiber was chosen as a candidate for these applications. The main reason lies there in, that this material shows suitable transition at 1550 nm, and this wavelength is of extraordinary importance for the communication technology with glass fibers. This wavelength falls in the so-called second absorption window. Such pulses propagate unchanged over long distances in the absence of loss. However, optical fibers are inherently lossy, and some types of gain mechanism are required to compensate for the loss. A common technique consists of doping the silica fiber with rare-earth ions and pumping them optically to realize the optical gain.

For the past few years, R.-J. Essiambre and G. P. Agrawal(1995), have analytically expressed two conditions for periodic amplification of short solitons (T_{FWHM} from 1 to 5 ps) and numerically solved this set of coupled nonlinear equations in terms of the soliton width and mean frequency for different amplifier spacings and gain bandwidths[1]. L.W. Liou and G. P. Agrawal (1996), use numerical simulations to show that solitary waves can exist provided there is enough broadband loss such that the net gain is negative far away from the gain peak[2]. G. Shaulov et. al. (1999), found that for a specific parameter range the solitary wave-type solutions exist and can be expressed in analytic form, including a new gray-pulse solution[3]. Thomas Carruthers et. al. (2000), have used Harmonically mode-locked Er-fiber soliton lasers as a source of high-repetition-rate picosecond pulses in high-speed communications[4], Eduardo J. S. Fonseca et. al. (2002), have investigated the interaction between a pair of solitons that originate from the breakup of a high-order soliton propagating through a cylindrical waveguide in the presence of three-level resonance associated with a dopant[5].. Hojoon Lee. and Agrawal G.P. (2003), studied numerically the nonlinear switching characteristics of optical pulses transmitted through fiber Bragg gratings. The nonlinear coupled-mode equations were solved numerically for pulse widths ranging from 50ps to 10ns or more[6]**.** J. Swiderski et.al(2004), have built two experimental laser set-ups based on neodymium- and ytterbium-doped active media. A Yb3+-silica fiber laser has been cladding pumped at 937 nm by a InGaAs semiconductor laser diode and generated 4 W cw output power with slope efficiency of 73 ±3%. However, Nd^3doped fiber laser generated over 10 W cw output power with a slope efficiency of 63%[7] Lai W.J.,at.al.(2004),investigated bi-directional optical wave propagations in a dual-pumped Erbium doped fiber ring laser without isolator, and observe optical bistabillity behavior[8]. Erin Hammond (2005) studied operation of single mode and multimode fiber lasers. The advantages of fiber lasers over traditional solid-state lasers are discussed along with fairly recent advances in higher output powers associated with fiber lasers.[**9]** Lixin Xu, (2006) demonstrated a 40-GHz actively mode-locked Erbium-doped fiber laser that incorporates an electro-absorption modulator and a linear optical amplifier. Stable pulses with peak power of 46 mW and pulse width of 2.8 ps are obtained when pumped with 100 mw[10]. Younis Al-zahy (2006) studies some of nonlinear effects as self phase modulation (SPM), cross phase modulation (XPM) ,four wave mixing (FWM) in passively mode locking and generation soliton from noise [11]. S. A. Ponomarenko1 and G. P. Agrawal (2007) obtain exact self-similar solutions to an inhomogeneous nonlinear Schrödinger equation, describing propagation of optical pulses in fiber amplifiers with distributed dispersion and gain[12]. My goal ,it is found analytic solution applies to nonlinear gain medium. By modeling the doped optical fiber as a gain medium with equation of motion of the density operator.

Problem formulution:

When an Erbium ion is in an excited energy state, a photon of light can stimulate it to give up some of its energy to the light beam and return to a more stable lower energy state. This is called stimulated emission. In such applications, a pump laser diode generates a high powered beam of light at a wavelength such that the Erbium ions will absorb it and jump to an excited state. The amplification process is as follows: First, the ions at the ground level are excited by the pump to a transition energy level. Due to this level's short lifetime, the ions spontaneously transit to the metastable level *E2*, which has a long lifetime in the order of 10 ms. The metastable band is narrow enough to be roughly homogenously populated even at ambient temperature. As a consequence, any ion used for stimulated emission is quickly replaced by another ion, which typically results in homogenous gain broadening.

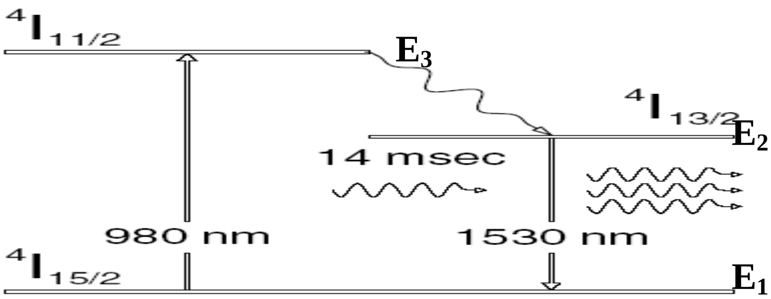

Fig. 1 Energy-levels of Erbium in Silica Glass

Fig. (1) shows the well known level system, the pump transition occurs between the states $^4I_{15/2} \longrightarrow {}^4I_{11/2}$, followed by a quick transfer between the states $^4I_{11/2} \longrightarrow {}^4I_{13/2}$ and finally as irradiative transition back to the ground state $^4I_{15/2}$,with a comparatively extremely long life time of 14 msec, this system fulfils the requirements for the production of the desired population inversion[13,14 and 15]. Fig. 2 shows the basic operation of a forward-pumped EDF, with signal amplification, pump absorption, and ASE generation simultaneously taking place within each incremental cylinder of the EDF core.

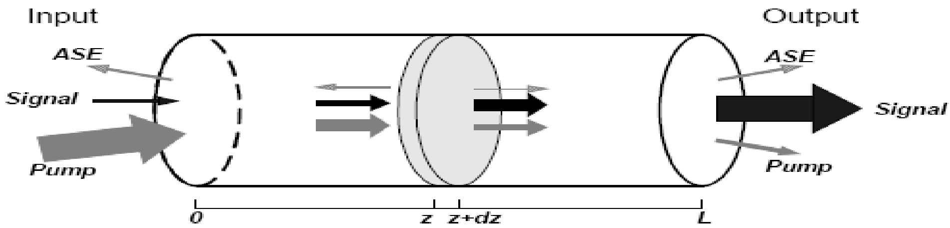

Fig. 2 Basic EDFL Operation

The equation of motion of the density operator is generally given by Eq. (2) as

$$\hat{H} = \hat{H}_0 + \hat{H}^{\backslash} + \hat{H}_R \qquad (1)$$

$\hat{H}_0$ is the Hamiltonian in the absence of external forces, $\hat{H}^{\backslash}$ is the interaction Hamiltonian being linear in the applied electric Field of the light, and where the new term $\hat{H}_R$ describes the various relaxation processes that brings the system into the thermal equilibrium whenever external forces are absent[16,17].

Equation (1) can be analyzed by means of the equation of motion of the density operator $\hat{\beta}$.

$$\frac{d\hat{\beta}}{dt} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\beta}\right] \qquad (2)$$

$$\frac{d\hat{\beta}}{dt} = -\frac{i}{\hbar}\left[\left(\hat{H}_0 + \hat{H}^{\backslash} + \hat{H}_R\right), \hat{\beta}\right] \qquad (3)$$

The relaxation process of the medium towards thermal equilibrium can be described

By:

$$\left[\hat{H}_R, \hat{\beta}\right] = -i\hbar\hat{\Gamma}\left(\hat{\beta} - \hat{\beta}_\circ\right) \qquad (4)$$

where $\hat{\beta}_\circ$ is the thermal equilibrium density operator of the system. This phenomenological introduced operator $\hat{\Gamma}$ describes the relaxation of the medium, and can be considered as being independent of the interaction

Hamiltonian. Here the operator $\hat{\Gamma}$ has the physical dimension of an angular frequency, and its matrix elements can be considered as giving the time constants of decay for various states of the system.
By taking the perturbation series for the density operator as:

$$\hat{\beta}(t) = \hat{\beta}_o + \hat{\beta}_1(t) + \hat{\beta}_2(t) + ... + \hat{\beta}_n(t) \tag{5}$$

From Eqs.(3 and 5) we obtains the system of equations

$$i\hbar \frac{d\hat{\beta}_o}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}_o\right]$$

$$i\hbar \frac{d\hat{\beta}_1(t)}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}(t)\right] + \left[+\hat{H}_I, \beta_o\right] - i\hbar\hat{\Gamma}\hat{\beta}_1(t)$$

$$i\hbar \frac{d\hat{\beta}_2(t)}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}_2(t)\right] + \left[+\hat{H}_I, \beta(t)_1\right] - i\hbar\hat{\Gamma}\hat{\beta}_2(t)$$

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$$i\hbar \frac{d\hat{\beta}_n(t)}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}_n(t)\right] + \left[+\hat{H}_I, \beta_{n-1}(t)\right] - i\hbar\hat{\Gamma}\hat{\beta}_n(t) \tag{6}$$

For the three-level system, the equation of motion can be expressed in terms of the matrix elements of the density operator as:

$$i\hbar \frac{d\beta aa}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}(t)\right]_{aa} + \left[+\hat{H}_I(t), \beta(t)\right]_{aa} - \left[\hat{H}_R, \hat{\beta}\right]_{aa} \tag{7}$$

$$i\hbar \frac{d\beta ab}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}(t)\right]_{ab} + \left[+\hat{H}_I(t), \beta(t)\right]_{ab} - \left[\hat{H}_R, \hat{\beta}\right]_{ab} \tag{8}$$

$$i\hbar \frac{d\beta_{bb}}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}(t)\right]_{bb} + \left[+\hat{H}_I(t), \beta(t)\right]_{bb} - \left[\hat{H}_R, \hat{\beta}\right]_{bb} \tag{9}$$

$$i\hbar \frac{d\beta cc}{dt} = \frac{1}{i\hbar}\left[\hat{H}_o, \hat{\beta}(t)\right]_{cc} + \left[+\hat{H}_I(t), \beta(t)\right]_{cc} - \left[\hat{H}_R, \hat{\beta}\right]_{cc} \tag{10}$$

β_{aa}, β_{bb} and β_{cc}, the density of atoms in states a, b and c respectively. Starting with the thermal-equilibrium part of the commutators in the right-hand sides(first term) of Eqs. (7, 9 and 10), the diagonal elements given by:

$$\begin{aligned}\left[\hat{H}_o, \hat{\beta}(t)\right]_{aa} &= \langle a|H_o\beta|a\rangle - \langle a|\beta H_o|a\rangle \\ &= \sum_k \langle a|H_o|k\rangle\langle k|\beta|a\rangle - \sum_j \langle a|\beta|j\rangle\langle j|H_o|a\rangle \\ &= \mathrm{E}_a\beta_{aa} - \mathrm{E}_a\beta_{aa} \\ &= \left[\hat{H}_o, \hat{\beta}(t)\right]_{bb} = \left[\hat{H}_o, \hat{\beta}(t)\right]_{cc} = \mathrm{o}\end{aligned} \tag{11}$$

The commutator in the right-hand sides(first term) of Eq. (8), the off-diagonal elements given by:

$$\left[\hat{H}_o, \hat{\beta}(t)\right]_{ab} = \langle a|H_o\beta|b\rangle - \langle a|\beta H_o|b\rangle$$

$$= \sum_k \langle a|H_o|k\rangle\langle k|\beta|b\rangle - \sum_j \langle a|\beta|j\rangle\langle j|H_o|b\rangle$$
$$= \mathrm{E}_a\beta_{ab} - \mathrm{E}_b\beta_{ab} = -\hbar\Omega_{ab}\beta_{ab} \quad (12)$$

For the commutators in the right-hand sides of Eqs.(7,9 and 10),(second term) involving the interaction Hamiltonian, similarly have for the diagonal elements:

$$\left[\hat{H}_I(t), \hat{\beta}(t)\right]_{aa} = \langle a|\eta\mathrm{E}(r,t)\hat{\beta}|a\rangle - \langle a|\hat{\beta}\eta\mathrm{E}(r,t)|a\rangle$$
$$= \mathrm{E}(r,t)\left(\sum_k \langle a|\eta|k\rangle\langle k|\beta|a\rangle - \sum_j \langle a|\beta|j\rangle\langle j|\eta|a\rangle\right)$$
$$= E(r,t)\left\{\eta_{ab}\beta_{ba} - \beta_{ab}\eta_{ba}\right\} = -\left[\hat{H}_I(t), \hat{\beta}(t)\right]_{bb} \quad (13)$$

$$\left[\hat{H}_I(t), \hat{\beta}(t)\right]_{ab} = \langle a|\eta\mathrm{E}(r,t)\hat{\beta}|b\rangle - \langle a|\hat{\beta}\eta\mathrm{E}(r,t)|b\rangle$$
$$= \mathrm{E}(r,t)\left(\sum_k \langle a|\eta|k\rangle\langle k|\beta|b\rangle - \sum_j \langle a|\beta|j\rangle\langle j|\eta|b\rangle\right)$$
$$= -\{\beta_{bb} - \beta_{aa}\}\eta_{ab}\mathrm{E}(r,t) \quad (14)$$

η is the transition dipole moment

For the commutators describing relaxation processes last term in Eqs.(7,9,and 10), the diagonal elements are given as:

$$\left[\hat{H}_R, \hat{\beta}\right]_{aa} = -i\hbar\left(\beta_{aa} - \beta_o(a)\right)/T_a \quad (15)$$
$$\left[\hat{H}_R, \hat{\beta}\right]_{bb} = -i\hbar\left(\beta_{bb} - \beta_o(b)\right)/T_b \quad (16)$$
$$\left[\hat{H}_R, \hat{\beta}\right]_{cc} = -i\hbar\left(\beta_{cc} - \beta_o(c)\right)/T_c \quad (17)$$

where T_a , T_b and T_c are the decay rates towards the thermal equilibrium at respective level, and where $\beta_o(a), \beta_o(b)$ and $\beta_o(c)$ are the thermal equilibrium values of β_{aa}, β_{bb} and β_{cc}, respectively(i. e. the thermal equilibrium population densities of the respective level). The off-diagonal elements are similarly given as:

$$\left[\hat{H}_R, \hat{\beta}\right]_{ab} = -i\hbar\left(\beta_{ab}\right)/T_2 \quad (18)$$
$$\left[\hat{H}_R, \hat{\beta}\right]_{ba} = -i\hbar\left(\beta_{ba}\right)/T_2 \quad (19)$$

T_2 is the time constant for loss of phase coherence between individual atoms of the ensemble when E (r,t) is turned off or dephasing time of the dipole moment.

As the above matrix elements of the commutators involving the various terms of the Hamiltonian are inserted into the right-hand sides of Eqs. (7,8 , 9and 10), one obtains the following system of equations for the matrix elements of the density operator:

$$i\hbar\frac{d\beta_{aa}(t)}{dt} = -\left\{\beta_{ba} - \beta_{ab}\right\}\eta_{ab}\mathrm{E}(r,t) - i\hbar\left(\beta_{aa} - \beta_o(a)\right)/T_a \quad (20)$$

$$i\hbar \frac{d\beta_{ab}(t)}{dt} = -\hbar\Omega_{ab}\beta_{ab} - \{\beta_{bb} - \beta_{aa}\}\eta_{ab}\mathrm{E}(r,t) - i\hbar(\beta_{ab})/T_2 \tag{21}$$

$$i\hbar \frac{d\beta_{bb}(t)}{dt} = \{\beta_{ba} - \beta_{ab}\}\eta_{ab}\mathrm{E}(r,t) - i\hbar(\beta_{bb} - \beta_o(b))/T_b \tag{2٢}$$

$$i\hbar \frac{d\beta_{cc}(t)}{dt} = -i\hbar(\beta_{cc} - \beta_o(c))/T_c \tag{2٣}$$

Assume the light to be linearly polarized and quasimonochromatic, of the form:

$$\mathrm{E}(r,t) = \mathrm{E}(r)\cos(\omega t) \tag{2٤}$$

$$\frac{d\beta_{aa}(t)}{dt} = i\{\beta_{ba} - \beta_{ab}\}\mu\cos(\omega t) - (\beta_{aa} - \beta_o(a))/T_a \tag{2٥}$$

$$\frac{d\beta_{ab}(t)}{dt} = i\Omega_{ab}\beta_{ab} + i\{\beta_{bb} - \beta_{aa}\}\mu\cos(\omega t) - (\beta_{ab})/T_2 \tag{2٦}$$

$$\frac{d\beta_{bb}(t)}{dt} = -i\{\beta_{ab} - \beta_{ba}\}\mu\cos(\omega t) - (\beta_{bb} - \beta_o(b))/T_b \tag{2٧}$$

$$\frac{d\beta_{cc}(t)}{dt} = -(\beta_{cc} - \beta_o(c))/T_c \tag{2٨}$$

where the Rabi frequency μ, defined in terms of the spatial envelope of the electrical field and the transition dipole moment as:

$$\mu = \eta\mathrm{E}(r)/\hbar \tag{2٩}$$

The equations of motion by taking a new variable σ_{ab} according to the variable substitution:

$$\beta_{ab} = \sigma_{ab}\exp\lfloor i(\Omega_{ab} - \Delta)t\rfloor \tag{٣٠}$$

where $\Delta = \Omega_{ab} - \omega$ is the detuning of the angular frequency of the light from the transition frequency $\Omega_{ab} = (\mathrm{E}_b - \mathrm{E}_a)/\hbar$

$$\frac{d\beta_{aa}(t)}{dt} = i\{\sigma_{ab}\exp(-i(\Omega_{ab} - \Delta)t) - \sigma_{ab}\exp i(\Omega_{ab} - \Delta)t\}\mu\cos(\omega t) - (\beta_{aa} - \beta_o(a))/T_a \tag{3١}$$

$$\frac{d\sigma_{ab}(t)}{dt} = i\Delta\sigma_{ab} + i\mu\cos(\omega t)\{\beta_{bb} - \beta_{aa}\}\exp[-i(\Omega_{ab} - \Delta)t] - (\sigma_{ab})/T_2 \tag{3٢}$$

$$\frac{d\beta_{bb}(t)}{dt} = -i\{\sigma_{ba}\exp[-i(\Omega_{ba} - \Delta)t] - \sigma_{ab}\exp i(\Omega_{ab} - \Delta)t\}\mu\cos(\omega t) - (\beta_{bb} - \beta_o(b))/T_b \tag{3٣}$$

$$\frac{d\beta_{cc}(t)}{dt} = -\left(\beta_{cc} - \beta_o(c)\right)/T_c \qquad (3٤)$$

The idea with the rotating-wave approximation is now to separate out rapidly oscillating terms of angular frequencies $(\omega + \Omega_{ab})$ and $-(\omega + \Omega_{ab})$ and neglect these terms, compared with more slowly varying terms.[17, 18 and 19]

The second term in Eqs.(3١ and 3٣) are approximated as :

$$\cos(\omega t)\exp i(\Omega_{ab} - \Delta)t = \frac{1}{2}\left(\exp(i\omega t) + \exp-(i\omega t)\right)\exp i\underbrace{(\Omega_{ab} - \Delta)t}_{\omega}$$

$$= \frac{1}{2}\left(1 + \exp(i2\omega t)\right) \Rightarrow \frac{1}{2} \qquad (3٥)$$

The second term in Eq.(3٢) is approximated as :

$$\cos(\omega t)\exp\left[-i(\Omega_{ab} - \Delta)t\right] = \frac{1}{2}\left(\exp(i\omega t) + \exp-(i\omega t)\right)\exp\left[-i\underbrace{(\Omega_{ab} - \Delta)t}_{\omega}\right]$$

$$= \frac{1}{2}\left(1 + \exp(-i2\omega t)\right) \Rightarrow \frac{1}{2}$$

By applying this rotating-wave approximation, the equations of motion (3١,3٢,3٣ and 3٤) hence take the form:

$$\frac{d\beta_{aa}}{dt} = \frac{i}{2}\left\{\sigma_{ab} - \sigma_{ab}\right\}\mu - \left(\beta_{aa} - \beta_o(a)\right)/T_a \qquad (3٦)$$

$$\frac{d\sigma_{ab}}{dt} = i\Delta\sigma_{ab} + \frac{i}{2}\left\{\beta_{bb} - \beta_{aa}\right\}\mu - \left(\sigma_{ab}\right)/T_2 \qquad (3٧)$$

$$\frac{d\beta_{bb}}{dt} = -\frac{i}{2}\left\{\sigma_{ba} - \sigma_{ab}\right\}\mu - \left(\beta_{bb} - \beta_o(b)\right)/T_b \qquad (3٨)$$

$$\frac{d\beta_{cc}}{dt} = -\left(\beta_{cc} - \beta_o(c)\right)/T_c \qquad (3٩)$$

Form equations (3٦ and 3٨) and let T_a=T_b=T_1 we get:

$$\frac{d\left(\beta_{bb} - \beta_{aa}\right)}{dt} = -i\left\{\sigma_{ab} - \sigma_{ab}\right\}\mu - \left((\beta_{bb} - \beta_{aa}) - [\beta_b(o) - \beta_o(a)]\right)/T_1 \qquad (٤٠)$$

By first adding Eq. (3٧) and its complex conjugate and then subtracting them, we obtain:

$$\frac{d\left(\sigma_{ab} + \sigma_{ba}\right)}{dt} = i\Delta\left(\sigma_{ab} - \sigma_{ba}\right) - \left(\sigma_{ab} + \sigma_{ba}\right)/T_2 \qquad (4١)$$

$$\frac{d[i(\sigma_{ab} - \sigma_{ba})]}{dt} = \Delta(\sigma_{ab} + \sigma_{ba}) + \left\{\beta_{bb} - \beta_{aa}\right\}\mu - i\left(\sigma_{ab} - \sigma_{ba}\right)/T_2 \qquad (4٢)$$

Let $\Gamma = \left(\beta_{bb} - \beta_{aa}\right)$, $\zeta = i(\sigma_{ba} - \sigma_{ab})$, $\Gamma_o = (\beta_b(o) - \beta_a(o))$ and $\kappa = \left(\sigma_{ab} + \sigma_{ba}\right)$

Eq.(٤٠)becomes:

$$\frac{d\Gamma}{dt} = -\mu\zeta - \left(\Gamma - \Gamma_o\right)/T_1 \qquad (4٣)$$

Eq.(4١)becomes:

$$\frac{d\kappa}{dt} = -\Delta\zeta - \kappa/T_2 \qquad (4٤)$$

Eq.(4٢)becomes:

$$\frac{d\zeta}{dt} = \Delta\kappa + \Gamma\mu - \zeta/T_2 \qquad (4٥)$$

In these equations, the introduced variable (Γ) describes the population inversion of the three-level system, while ζ and κ are related to the dispersive and absorptive components of the polarization density of the medium.

At steady state Eqs.(41,44 and 45) become:

$$\kappa = i\Delta\zeta T_2 \qquad (4٦)$$

$$\zeta\mu T_1 = \Gamma - \Gamma\circ \qquad (47)$$

$$\Delta\kappa T_2 + \Gamma\mu T_2 = \zeta \qquad (48)$$

Form equations (46,47and 48) we get:

$$\Gamma = \frac{\Gamma_\circ(1+(\Delta T_2)^2)}{1+(\Delta T_2)^2 + \mu^2 T_1 T_2} \qquad (49)$$

where $g = \sigma_S \Gamma$ (50)

where g is the gain realized by pumping the dopants, σ_S is the transition cross section .

Form equations (49 and 50) we get:

$$g = \frac{g_o(1+(\Delta T_2)^2)}{1+(\Delta T_2)^2 + \mu^2 T_1 T_2} \qquad (51)$$

where $g_o = \sigma_s \Gamma_o$ (52)

Using nonlinear Schrödinger equation which describe propagation the pulse through doping fiber[11].

$$\frac{\partial\psi_n}{\partial z} + \frac{i}{2}\left[B_2 + \frac{i}{2} g T_2^2\right]\frac{\partial^2\psi_n}{\partial\tau^2} - \delta\left|\psi_n\right|^2\psi_n = i\,\gamma\left|\psi_n\right|^2\psi_n + \frac{1}{2}.(g-\alpha)\psi_n \qquad (53)$$

α is the optical loss of the host fiber, B_2 is the group velocity dispersion coefficient of the host fiber, γ is the complex parameter accounts for the host nonlinearities responsible for SPM .Fig.(3) describes schematic diagram for theoretical model.

$$\frac{\partial \psi_n}{\partial z} = \frac{-i}{2}\left[B_2 + \frac{ig_\circ(1+(\Delta T_2)^2)T_2^2}{2(1+(\Delta T_2)^2 + \mu^2 T_1 T_2)}\right]\frac{\partial^2 \psi_n}{\partial \tau^2} \tag{54}$$

$$+\left(\frac{g_\circ(1+(\Delta T_2)^2)}{2(1+(\Delta T_2)^2+\mu^2 T_1 T_2)} - \alpha\right)\psi_n + (\delta + i\gamma)|\psi_n|^2 \psi_n$$

$$\frac{\partial \psi_n}{\partial z} = \frac{-i}{2}\left[B_2 + \frac{ig_\circ(1+v)v}{2\Delta(1+v+\xi)}\right]\frac{\partial^2 \psi_n}{\partial \tau^2} + \left(\frac{g_\circ(1+v)}{2(1+v+\xi)} - \alpha\right)\psi_n + (\delta + i\gamma)|\psi_n|^2 \psi_n \tag{55}$$

Where $v = \Delta T_2^{\,2}, \xi = \mu^2 T_1 T_2$.

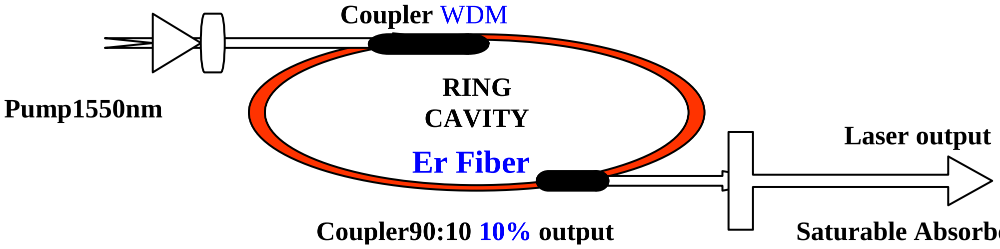

Fig. (3)schematic diagram for theoretical model WDM, wavelength division multiplexer.

The solution is given by:

$$\psi(z,\tau) = a\sec h(\Omega\tau)^n e^{imz} \tag{56}$$

Where n=(1+iq),q is chirp parameter

Substituting Eq.(56)in Eq.(55) we get:

$$\frac{\partial \psi(z,\tau)}{\partial z} = im\,\psi(z,\tau) \tag{57}$$

$$\frac{\partial \psi(z,\tau)}{\partial \tau} = (1+iq)\tanh(\Omega\tau)\psi(z,\tau) \tag{58}$$

$$\frac{\partial^2 \psi.(z,\tau)}{\partial \tau^2} = \Big[\Omega^2\left(1+2iq-q^2\right)\psi(z,\tau) - \Omega^2\left(1+2iq-q^2\right)\sec h^2(\Omega\tau)\psi(z,\tau) \tag{59}$$

$$+\Omega^2 \sec h^2(\Omega\tau)\psi(z,\tau) + iq\Omega^2 \sec h^2(\Omega\tau)\psi(z,\tau)\Big]$$

Substituting Eqs.(57) and (59)in Eq.(55) we get:

$$im = \frac{i}{2}\left[B_2 + \frac{ig_\circ(1+v)v}{2\Delta(1+v+\xi)}\right]\left(1+2iq-q^2\right)\Omega^2 + \left(\frac{ig_\circ(1+v)}{2(1+v+\xi)} - \alpha\right)\Big[\Omega^2\left(1+2iq-q^2\right)$$

$$-\Omega^2\left(1+2iq-q^2\right)\sec h^2(\Omega\tau) + \Omega^2 \sec h^2(\Omega\tau) + iq\Omega^2 \sec h^2(\Omega\tau) + (\delta + i\gamma)\sec h^2(\Omega\tau)\Big] \tag{60}$$

$$m = \Omega^2\left[\frac{ig_\circ(1+v)vq}{2\Delta(1+v+\xi)} + B_2\left(1-q^2\right)\right] \tag{61}$$

$$\Omega = \left[\frac{-ig_{\circ}\Delta(1+v) + \alpha 2\Delta(1+v+\eta)}{ig_{\circ}(1+v)v\left(1-q^{2}\right) + 2qB_{2}\,2\Delta(1+v+\eta)} \right]^{\frac{1}{2}} \tag{62}$$

$$p = |a|^{2} = \frac{\Omega^{2}}{2\gamma}\left[B_{2}q^{2} + \frac{3g_{\circ}(1+v)qv}{2\Delta(1+v+\xi)} - 2B_{2} \right] \tag{63}$$

$$q = \frac{3}{2}\left(\frac{\gamma B_{2}(1+v+\xi) - g_{\circ}(1+v)v\delta}{\delta B_{2}\Delta(1+v+\xi) + g_{\circ}(1+v)v} \right) \pm \left[\frac{9}{4}\left(\frac{\gamma B_{2}\Delta(1+v+\xi) - g_{\circ}(1+v)v\delta}{\delta B_{2}\Delta(1+v+\xi) + g_{\circ}(1+v)v} \right)^{2} + 2 \right]^{1/2} \tag{64}$$

Equations (61 ,62,63 and 64) are the parameters of equation(56) which describe of propagation equations for a three -energy-level laser system and these equation used for continuous wave lasers in the sense that they describe how the pulse parameters m, q ,Ω and p change with different value υ and ζ.

Results and discussion:

1-**Effect of the detuning υ:**To study the effect of the detuning (υ) on the solitary waves, standard parameters used in our formulations are shown in Table 1[20]. Using Matlab5.3 to draw the figures.

Table 1. Numerical Values of the System Parameters.

Saturable absorber parameter δ	0.1
Wavelength λ	1550 nm
Fiber length	4m
Fiber attenuation α	0.1 dB\m^{-1}
Dispersion B_2	-0.01ps^2/m
Nonlinearity γ	0.02Wm^{-1}

The block diagram (1) shows the input parameter and output parameter soliton.

INPUT B_{2} , γ , δ, τ, z, , α , g_{o}

$$q = \frac{3}{2}\left(\frac{\gamma B_2 (1+v+\xi) - g_\circ (1+v) v\delta}{\delta B_2 \Delta(1+v+\xi) + g_\circ (1+v) v}\right) \pm \left[\frac{9}{4}\left(\frac{\gamma B_2 \Delta(1+v+\xi) - g_\circ (1+v) v\delta}{\delta B_2 \Delta(1+v+\xi) + g_\circ (1+v) v}\right)^2 + 2\right]^{1/2}$$

$$\Omega = \left[\frac{-ig_\circ \Delta(1+v) + \alpha\, 2\Delta(1+v+\eta)}{ig_\circ (1+v) v\left(1-q^2\right) + 2qB_2\, 2\Delta(1+v+\eta)}\right]^{\frac{1}{2}}$$

$$p = |a|^2 = \frac{\Omega^2}{2\gamma}\left[B_2 q^2 + \frac{3g_\circ (1+v) qv}{2\Delta(1+v+\xi)} - 2B_2\right]$$

$$m = \Omega^2\left[\frac{ig_\circ (1+v) vq}{2\Delta(1+v+\xi)} + B_2\left(1-q^2\right)\right]$$

OUTPUT $\psi(z,\tau) = a\,\mathrm{sec}h(\Omega\tau)^n\, \mathrm{e}^{\mathrm{imz}}$

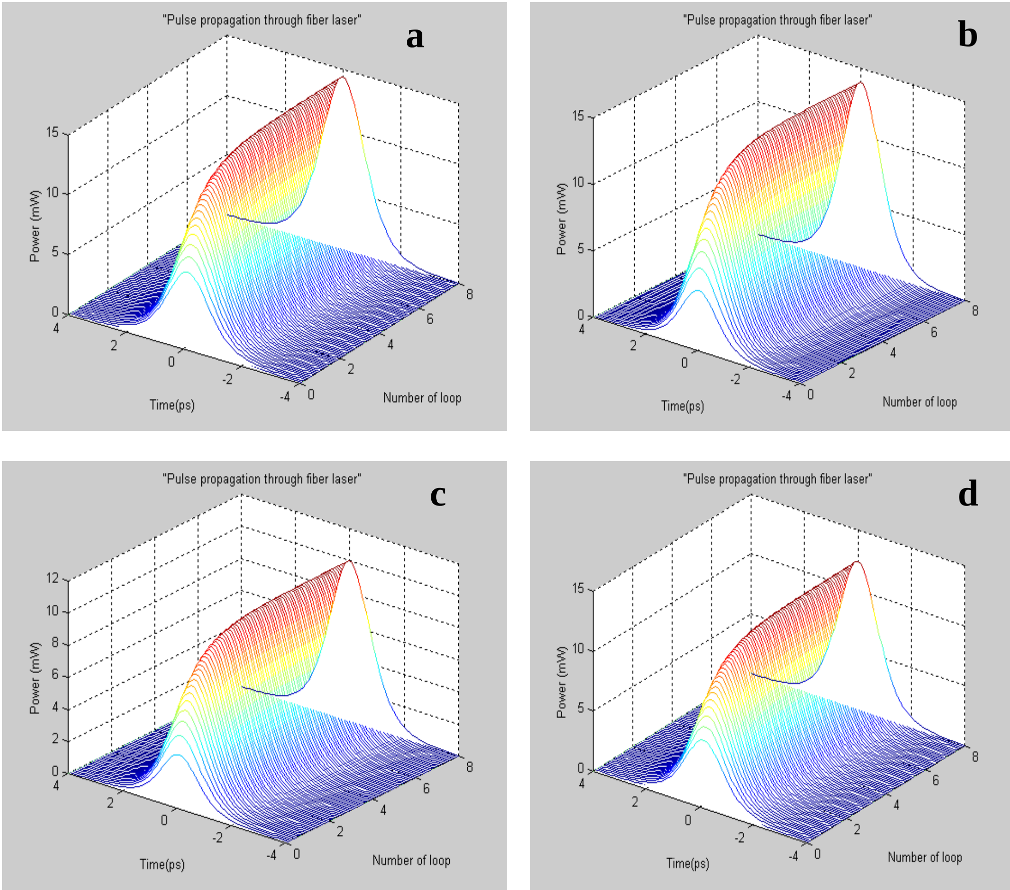

Fig. 4.Evolution toward the steady-state soliton over 4 loops for γ=0.02Wm^{-1} α=0.1dB\m^{-1} δ=0.1, B_2=-0.01ps^2/m, ζ=0.1 and τ =1ps (a) υ =0.1(b) υ =0.2 (c) υ =0.4 (d) υ =0.6.

Fig(4),shows the behavior of the pulse through fiber laser for different value of detuning parameter(0.1,0.2,0.3 and 0.4) with a constant values for ζ =1,κ=0.1 ,We note that the amplitude of the pulse grows exponentially with distance and the power signal increase with decreasing the υ as shows in fig.(5).This is because the frequency of the pump pulse close to the frequency of the transition and leads to the higher net gain which causes pulse compression where the soliton becomes narrower as υ decreases. Fig.(6) shows the pulse width of the soliton varies with the υ. The amount of chirp is increases with decreases υ, the results shown in Fig. 7. The amount of chirp is large at a small value of detuning and then rapidly decreases for anther values . The reason for this behavior is that the chirp increases with increasing the gain .

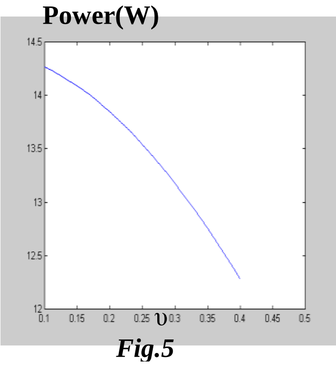

Fig.5

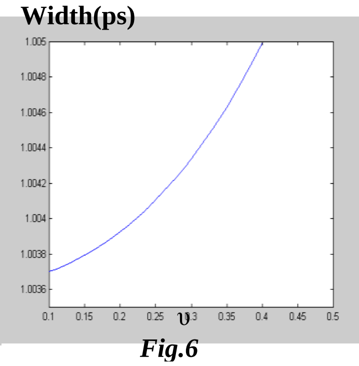

Fig.6

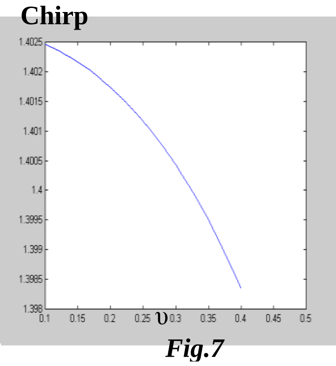

Fig.7

Fig. (5,6and7).The power P , width and chirp of soliton as a function of different values of υ *respectively* .

2-**Effect of ζ:**In this sub section we discuss effect of the parameter ζ on the behavior of the pulse in fiber laser .In Fig. 8. we show how the laser power and width of the soliton varies with the number of loops for different value ζ. First ,we note that also amplitude of the pulse grows exponentially with the number of loops. Second ,the soliton close to steady state over 4 loop. Third , the width of soliton increase with the number of loops.

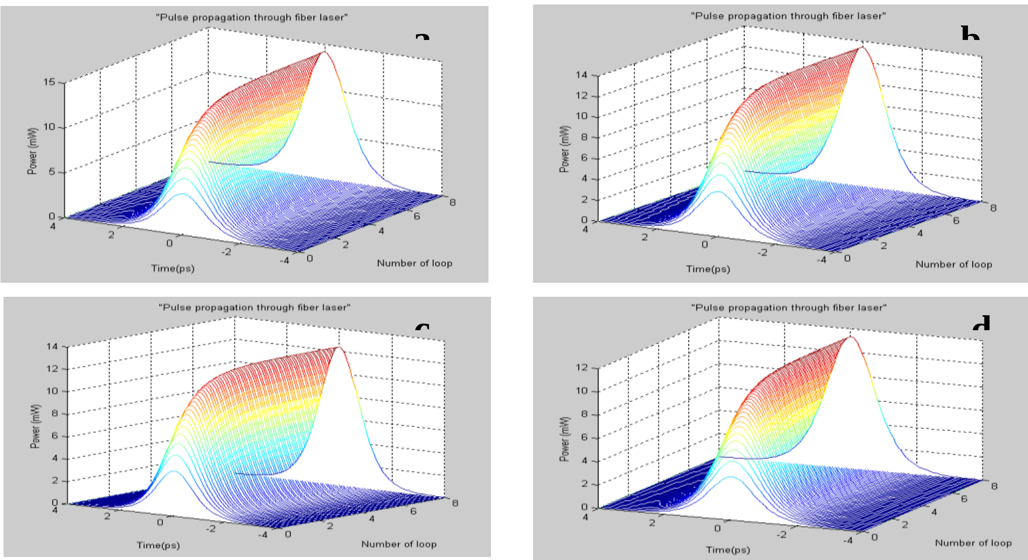

Fig. 8.Evolution toward the steady-state soliton over 4 loops for γ=0.02Wm^{-1} α=0.1db\m^{-1} δ=0.1, B_2=-0.01ps^2/m, υ=0.1 and τ =1ps (a) ζ =0.1(b) ζ =0.2 (c) ζ =0.4 (d) ζ =0.6.

In the anomalous –dispersion region , an increase in the ζ reduces exponentially the power while the width of the soliton increases exponentially as shown in figs.(9 and 10). This is because the dipole momentum μ depends on the frequency of the transition , this phenomenon leads to increase the gain also. the chirp parameter reduce exponentially with increase ζ as shown in Fig.(11) .Comparing Figs.4 and 8 it is found that the power of the soliton increase with increasing the detuning parameter υ and ζ and both solitons have a same behavior .Comparing Figs.(5,6 and 7) with figs.(9, 10 and 11), I observe that the power ,width and chirp of the soliton also have the same behavior.

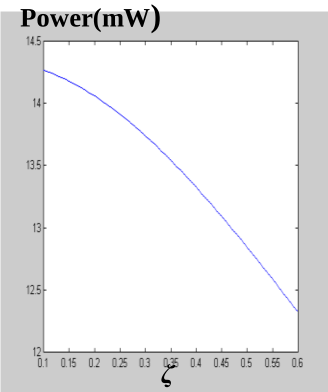

Fig. 9

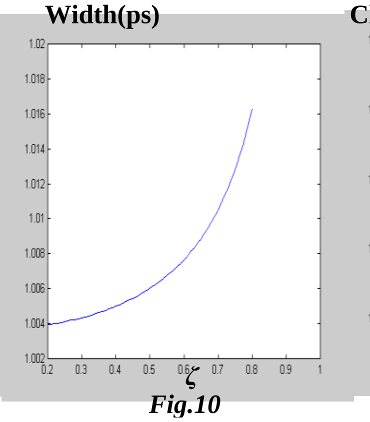

Fig.10

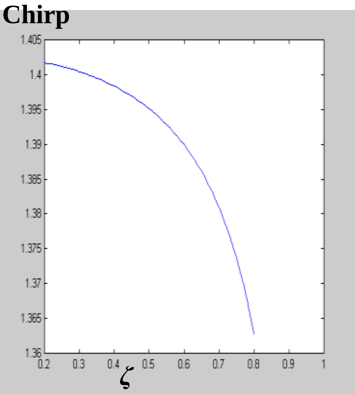

Fig.11

Fig. (9,10and11).The power P , width and chirp of soliton as a function of different values of ζ respectively .

The behavior of soliton is agreed with reference [2 and 5]

Conclusion;

This solution shows that for a pulse to propagate undistorted in an amplifying medium, the soliton must be chirped in addition to satisfying a certain relationship between the peak power and the width of the pulse.The stability of the soliton depends on the amount of the nonlinearity,sign and amount of the GVD, the power of the soliton depend on the amount of the ζ and υ.

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