POROSITY, PERMEABILITY AND TORTUOSITY RELATIONSHIP IN BINARY SPHERICAL PARTICALS.

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العلاقة بين المسامية ،النفاذية والإلتوائية بين الحبيبات الكروية الثنائية

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الخلاصة

العمليات المعقدة الصناعية التي تتضمن تشكيل طبقات حبيبية تمتلك معلومات قليلة ومحددة عن النفاذية التي لها علاقة مباشرة مع مسامية الحشوة و الألتوائية. ولفهم هذه العلاقة في الطبقات المختلطة للجسيمات المختلفة في الحجم ، استعمل نموذج مبسط للوسط المسامي لخليط ثنائي الحبيبات.

وجد إن هناك علاقة ثابتة بين المسامية و الكسر الحجمي للحبيبات الكبيرة في المزيج. .إن الاندماج الحاصل في معادلة المسامية و معادلة Carmen-kozeny التقليدية تعطي نتائج جيدة بين النفاذية المقاسة المتنبأ بها مقابل الكسر الحجمي، بالاعتماد على هذه المعادلة والعلاقة بين المسامية و الكسر الحجمي يمكن استخلاص نموذج للنفاذية والالتوائية الذي يوضح أن الألتوائية مهمة جدا في تغيير النفاذية. إن هذا البحث مهم في تحليل انتقال الظواهر في الطبقات الحبيبية وفي التطبيقات الهندسية ومثال على ذلك الطبقة العميقة المرشحة المستعملة لتصفية مياه الشرب.

Abstract

The complexity of processes in industry, which involved the formation of granular beds, has limited information about permeability (k), which is directly related with packing porosity ε and tortuosity T. For an understanding of the relationship between k, ε and T in mixed beds of particles that are different in size, simplified porous media model of binary mixture of spheres have been. A continuous function relating the

porosity of a mixture (ε) with the volume fraction (x_D) of binary mixture has been established. The incorporation of the proposed porosity model into the conventional Kozeny-Carmen equation gives a good agreement between the measured and predicted permeability vs.x_D. Based on this relation and on the dependence of ε vs.x_D, a model predicted tortuosity and permeability was obtained, having demonstrated that the tortuosity may significantly alter the permeability of a mixed bed. This search is useful for transport phenomena analysis in granular beds as well as in engineering applications, such as deep bed filtration used for drinking water purification.

Introduction

Many models of behavior binary particle beds porosity vs. the volume fraction on the mixture components have been described in many applications [1]. Models of fluid flow or mass transfer in porous media need to establish relationships of packed bed porosity with tortuosity and permeability or diffusivity.

The tortuosity depends on porosity T (ε) [2] and often the correct presentation of T as the function of ε. For granular mixed beds and binary mixtures, in particular both T and ε are related to the volume fraction of different size particles (for binary mixtures, the volume fraction of large particle x_D is often used). Therefore, the relation of porosity as a function of volume fraction and δ must be investigated to analyses mixed bed permeability in all ranges of x_D values (from 0 to 1.0).

Porosity

A model of the binary mixture was used. This model makes it possible to analyze the influence of each particle fraction on the overall porosity in all range of x_D by means of a fractional porosity approach. Let the overall porosity ε as a function of fractional porosity $\varepsilon_D = \varepsilon_D(x_D)$ and $\varepsilon_d = \varepsilon_d(x_D)$ where ε_D is the void fraction of large particles in the total volume of the mixture, ε_d is the specific void fraction of small particles in the remaining void volume of the mixture. Since the overall volume of solids in the mixture, 1-ε, is a sum of volumes of large particles, 1-ε_D, and small particles, $(1-\varepsilon_d).\varepsilon_D$, the porosity of the mixture becomes [1]

$$\varepsilon = \varepsilon_D . \varepsilon_d \quad \text{................} \ (1)$$

$$\varepsilon_d = (\varepsilon^0{}_D + x_D - 1)/(\varepsilon^0{}_D x_D), x_D \in [x_{D\min}, 1] \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\varepsilon_d = \varepsilon^0{}_d, x_D \in [0, x_{D\min}] \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\varepsilon_D = (1 - x_D)/(1 - \varepsilon^0{}_d x_D), x_D \in [0, x_{D\min}] \ldots\ldots\ldots\ldots(4)$$

$$\varepsilon_D = \varepsilon^0{}_D, x_D \in [x_{D\min}, 1] \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

With minimum porosity $\varepsilon_{min}=\varepsilon^0{}_d \cdot \varepsilon^0{}_D$ …………… (6)

The influence of small and large particles arrangement on the shape of the overall porosity dependence was discussed in [2], where it was shown that continuity or discontinuity of ε_D and ε_d dependences are the result of packing effects in the region of $x_{D\ min}$. These effects are caused by small particles wedging between large particles in the skeleton and by disturbance nearby the surface of large particles. The values of porosity have calculated by the following equation:-

$$\varepsilon = 1 - \frac{\rho_b}{\rho_{tmix}} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where

$$\rho_{tmix} = \frac{1}{\frac{x_d}{\rho_{td}} + \frac{x_D}{\rho_{tD}}} \ldots\ldots\ldots\ldots\ldots (8)$$

Tortuosity

Tortuosity is associated with flow and mass transfer characteristics such as permeability, diffusivity, etc. The tortuosity investigations have concentrated on the establishment of a relationship between the overall porosity and tortuosity T [3] theoretical and practical investigations show that the tortuosity of a granular bed depends on fractional content, porosity and particle shape. Tortuosity increases with decreases of d/D, therefore the value of T may vary in a wide range. [4] There is a simple relationship between tortuosity and porosity and it is often used.

$$T = \frac{1}{\left[\varepsilon(x_D)\right]^n} \quad \text{............. (9)}$$

The n value is constant usually between 0.4-0.5[5] and can be determined by this equation :

$$n = 0.5 - 0.2367 x_D{}^2 \quad \text{.................. (10)}$$

This equation is valid when δ>0.1, tortuosity is a rather complex function of several factors; some general conclusions can be put forward: - [6]

1. Tortuosity is evident or latent in all models of mass transfer in porous media
2. Tortuosity is not physically constant and depends first of all on other porous media characteristics like porosity, pore diameter, channel shape, etc.
3. Tortuosity often depends on processes occurring during mass transfer porous media compressing or expanding particles or macromolecules deposition inside pore channel up to pore blocking phenomena, etc
4. Tortuosity depends also on the kind of material being transferred (percolation phenomena). For instance tortuosity determined for gas diffusion and for macromolecule diffusion through the same porous media may be different.

Permeability

For a better understanding of the relationship between porosity, permeability and particle composition may improve mass transfer processes in granular beds.

The flow velocity, u of a fluid through a granular bed under an applied pressure, described by the well-known Kozeny-Carmen equation: - [6]

$$u = \frac{\varepsilon^3 d_p{}^2}{36K(1-\varepsilon)^2} * \frac{\Delta p}{\mu L} \quad \text{.......... (11)}$$

Where L is the bed thickness, μ is the liquid viscosity and permeability k (m^2) defined according to equation (12). For a bed of spherical particles k depends on particle diameter dp, porosity ε, and tortuosity T.

$$k = \frac{\varepsilon^3 d_p{}^2}{36K(1-\varepsilon)^2} = \left[\frac{\varepsilon}{T}\right]^2 \left[\frac{\varepsilon d_p{}^2}{36(1-\varepsilon)^2 K_o}\right] \quad \text{………..….. (12)}$$

Where

$$d_p = \frac{x_d}{D} + \frac{(1-x_D)}{d} \quad \text{…………………………..……. (13)}$$

Moreover, $K=K_oT^2$, the Kozeny's coefficient, for granular beds is K=4.2-5.0; K_o is the shape coefficient (factor) depending on a cross –section capillary pore shape (for cylinder pore K_o = 2.0). Variation of K depends mainly on the packing density and hence, the tortuosity variation [1]

Experimental Work

The packing method and particle composition affect the properties of mixed beds [7]. Different packing methods imposed different packing constraints, which in turn affect the degree of randomness of packing [2] therefore many researchers devoted a lot of effort to develop packing method that could give rise to reproducible packing beds. [8].

Binary mixtures of glass beads were used in all experiments in this research. They are that is (1, 0.8, 0.6, 0.345) cm, the height of column used is (8.4) cm and diameter is (8) cm. The true density values have been computed for each particle by dividing the weight of (10) particles by the volume of one particle then divided by (10). After making the mixing well in column and shaking it in all directions, the weight of particles was determined to compute the bulk density.

Results and Discussion

Figures (1), (2) and (3) shows the relationship between porosity and volume fraction of large particles in different δ for experimental and simulation calculations. In simulation, the equations (1-5) are used to determine the overall porosity. In experiments work, the equation (7) was used. This revealed two types of porosity behavior, which show that the porosity decreases when the volume fraction of large particles increases until it reaches to the minimum. The minimum porosity ranges between (0.35-0.6). After that, the porosity increases when volume fraction increases. This figures was for $\delta>0.1$.

Figures (4),(5) and (6) show the relationship between tortuosity and volume fraction based on the relationship between tortuosity and porosity in simulation data; a linear relation was obtained, but the behavior of experimental values was near to linear, but behavior of tortuosity vs. volume fraction of simulation in δ=0.6. This variation was perhaps because of the effect of packing. The packing was very important in this research so that the packing and computations must be done more than once. Discrepancy between the predicted and measured tortuosity shows that the real average flow pathway is shorter than theoretically expected. The permeability vs. x_D shows in figures (7), (8) and (9) also with different δ. In all figures, the permeability increases when XD increases but in Fig. (8) The experimental work and simulation share the XD=0.35. The effect of packing makes some difference with the simulated data; this gives a good agreement for predicted and experimental data.

Conclusion

Based on experimental data and on a linear mixing model, the relationship of porosity versus volume fraction of large particles for a binary mixture of spherical particles was established as a continuous function. The incorporation of proposed porosity model into the conventional permeability model described the relation between permeability and x_D; there was a good agreement between measured and predicted data. The relationship of tortuosity with a mixture porosity was found to be $T = 1/\varepsilon^n$ .based on this relation and on dependence of ε vs. x_D the model for predicting tortuosity and permeability was obtained.

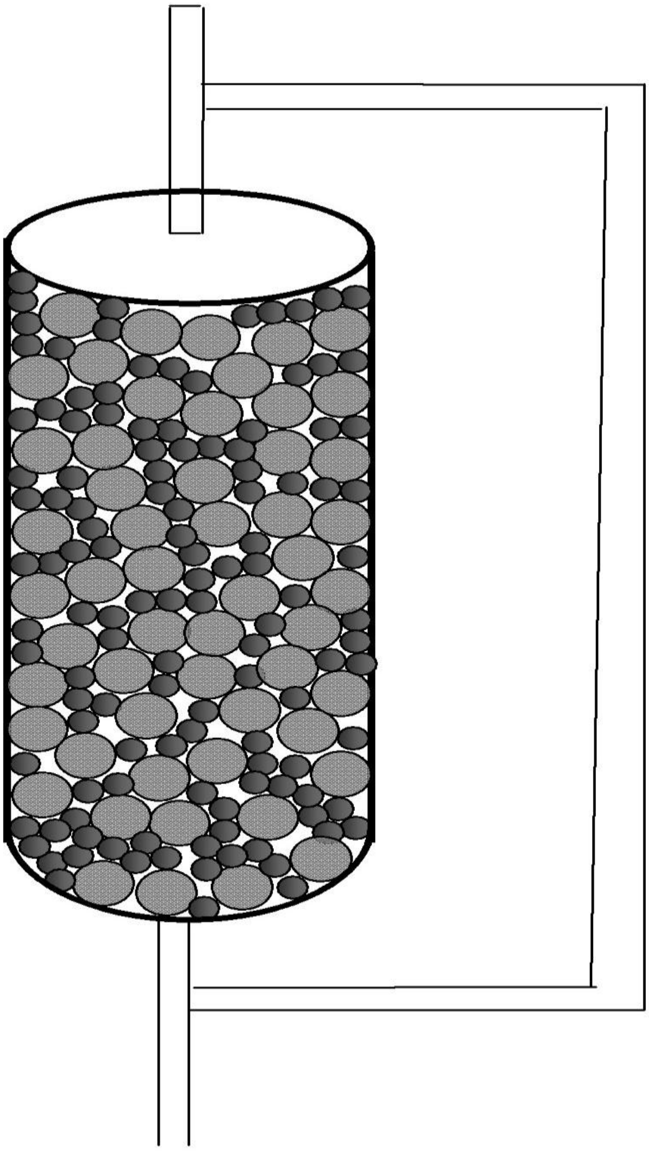


Fig (1-1) packed bed column

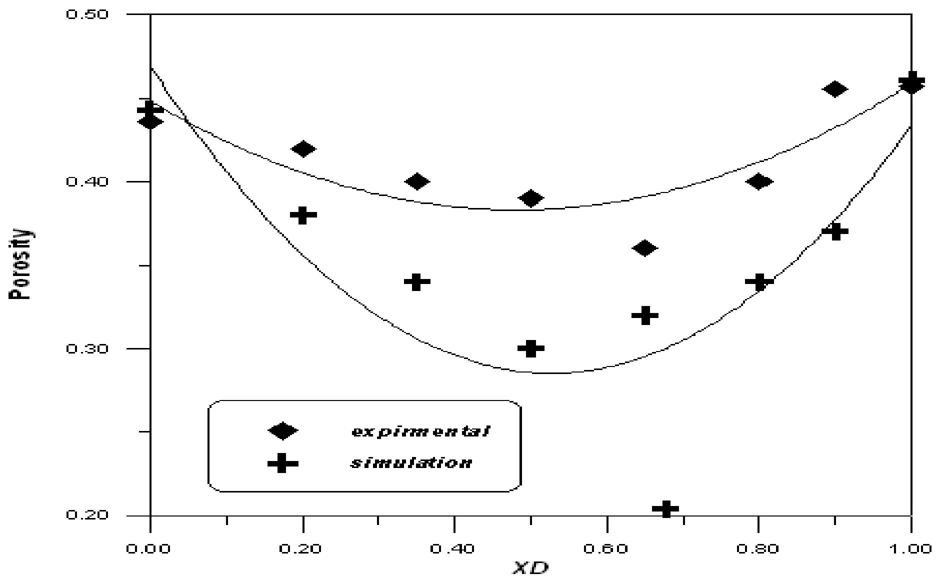
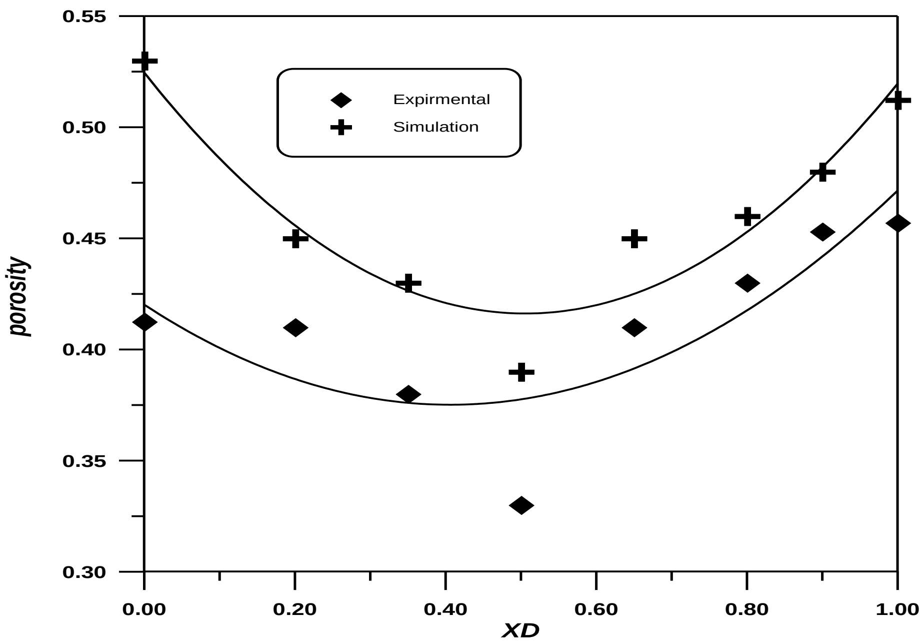
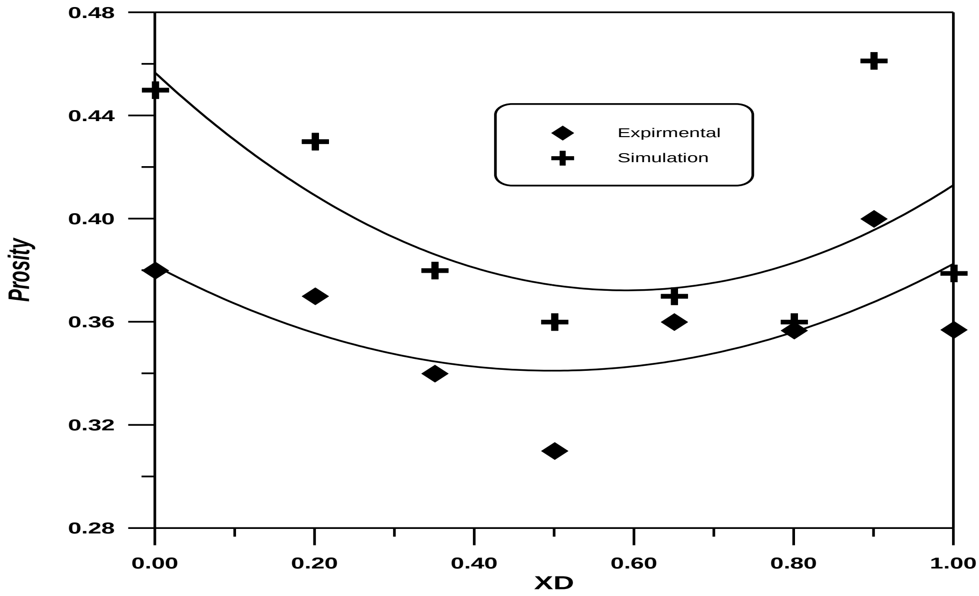


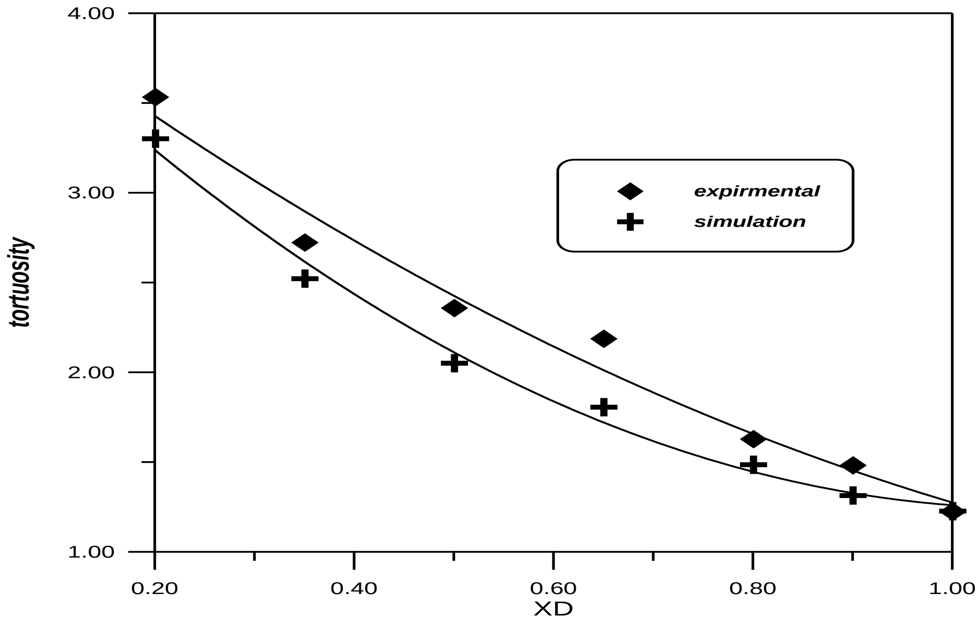
Fig (1) expirmental and simulation porosity at d/D=0.8



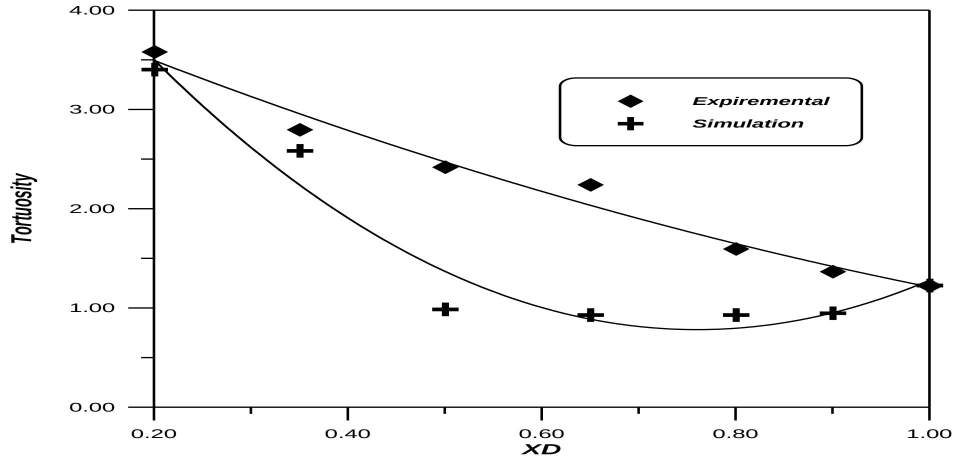
Fig(2) porosity of expirmental and simulation at d/D=0.6



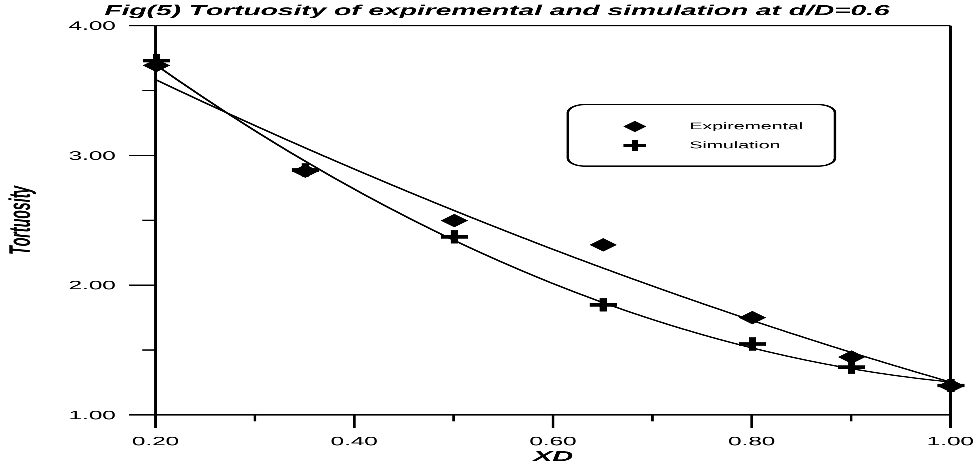
Fig(3) porosity of expirmental and simulation at d/D=0.345



Fig(4) Tortuosity of expiremental and simulation at d/D = 0.8



Fig(5) Tortuosity of expiremental and simulation at d/D=0.6



Fig(6) Tortousity of expiremental and simulation at d/D=0.345

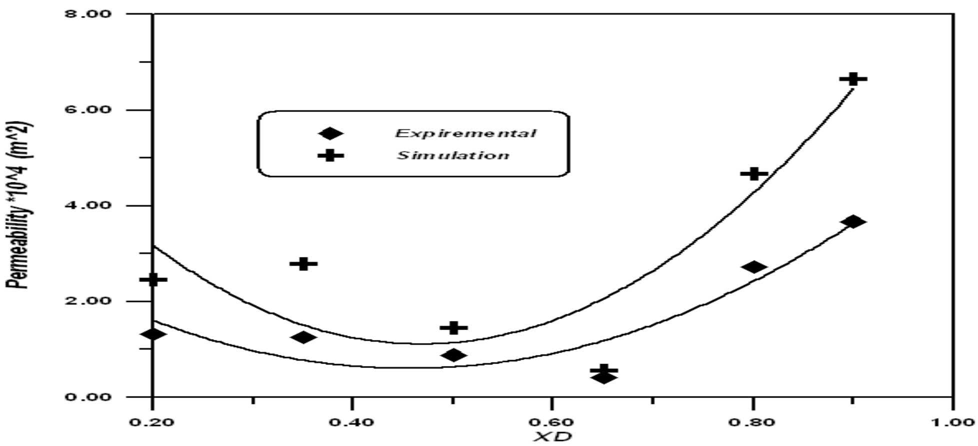


Fig (7) Simulation and expiremental permeability at d/D=0.8

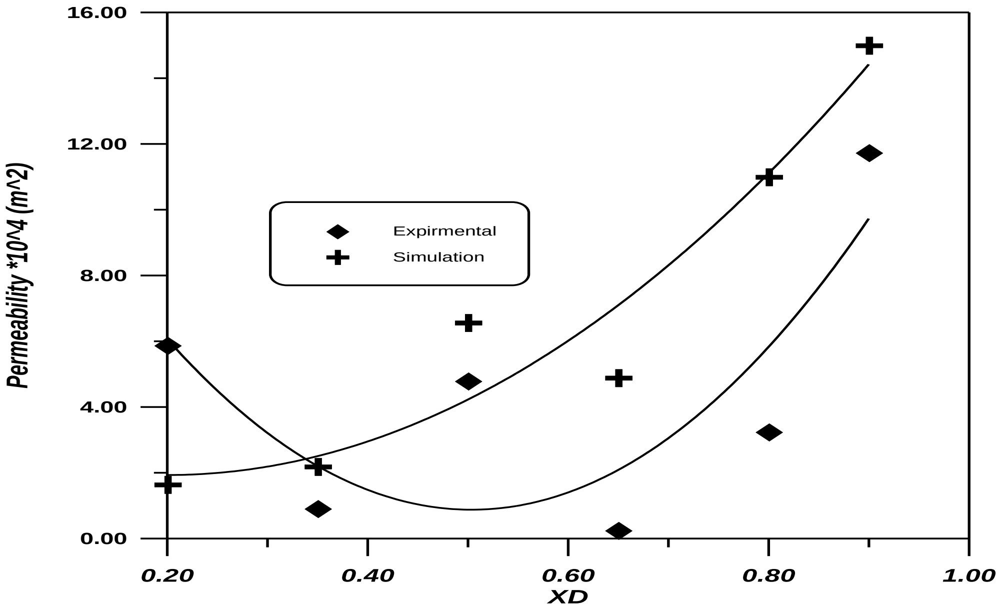


Fig (8) Permeability of experimental and simulation at d/D=0.6

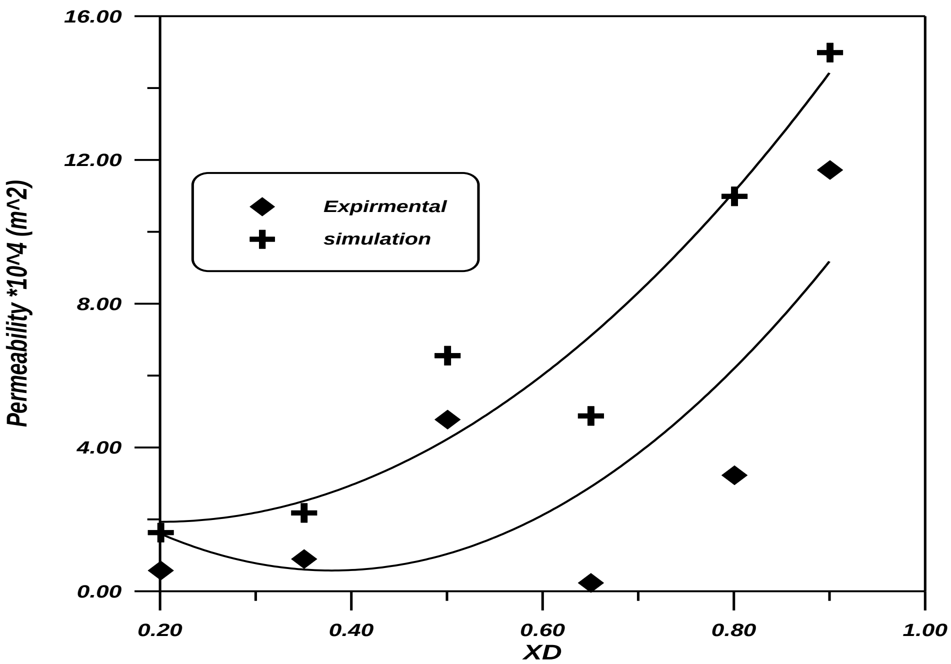


Fig (9) Permeability of expirmental and simulation at d/D=0.345

Nomenclature

d	Diameter of small particles.
D	Diameter of large particles.
T	Tortuosity.
k	Permeability.
K	Kozeny's coefficient.
x_D	Volume fraction of large particles in the total volume of particles in the mixture.
$x_{D\ min}$	Volume fraction of large particles corresponds to the minimum mixture porosity.
x_d	Volume fraction of small particles in the total volume of particles in the mixture.

Greek letters

ε	Overall Porosity of a mixed bed.
ε_D	Fractional porosity of the large size particle fraction.
ε_d	Fractional porosity of the small size particle fraction.
ε_{min}	Minimum porosity of a mixed bed.
ε^o_D	Porosity of a uniform bed of large particles.
ε^o_d	Porosity of a uniform bed of small particles.
δ	Particles size ratio =d/D.
Δp	Pressure drop (pa)

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Recived ………………………………………….……………………. (19/1/2009)
Accepted ……………………………...……...……………... (28/9/2009)