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Classes Locally Conformal Kahler Manifold of W (M) projective Curvature

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Abstract:
This study deals with three novel classes of the L.C.K (Locally Kahler Manifold) of W(M)– projective curvature tensor. The goal of this paper to compute differential– geometrical and topological characteristics closet for new classes W, W and W,. Throughout it, an equivalence relationship was gained between those three classes and one or more of the tensor compounds and the curvature tensor components with adjoint G-structure space. Finally, we discover a relationship between W, W and W with each other.

1. Introduction:
The concept of AH-Almost Hermitian structures states that there is a general rule for classifying AH- AH-structures using the second-order symmetry features of the Riemann-Christoffel tensor's invariants of differential geometry. Based on the theory advanced by A. Gray and developed within several respective works [2] and [4], the importance to understanding the differential-geometrical characteristics of Kahler manifolds is to establish the identities of that satisfy. Gray and Hervella [3] found that the action of the unitary group U(n) on the space for all tensors of kind (3,0) can be decomposed this space into 16 classes. The following are the components that make up the Riemann curvature tensor:

$R_1 : < R(X,Y)Z, W > = < R(X,Y)JZ, JW >;$

$R_2 :< R(X,Y)Z, W > = < R(JX,JY)Z, W > + < R(JX,Y)JZ, W > + < R(JX,Y)Z, JW >$

$R_3 : < R(X,Y)\, Z, W > = < R(JX,JY)\, JZ\, , JW >$

The AH – structures belonging to the class W_i have a tensor R that fulfills the identity W_i . If AH – any subclasses of H – structures is named $\cap\, W_i = 0$, where i = 1, 2 or 3.

In this paper, we will generalize these relationships, definitions and theories related to them for NK-Nearly Kahler manifold of W-projective curvature tensor. In 2018, Ali A. Shihab and Dhabia'a M. Ali where studied classes of almost Hermitian manifold [1]. In the study also concentrates generalized con-harmonic curvature tensor of the Vaisman -Gray manifold.

2. Preliminaries:
Assume that M is a smooth manifold of dimension– 2n; C (M) is an algebra of smooth functions on M; a(M) the smooth vector module fields on M; and that g = <.,.> – Riemannian metrics; Ñ Riemannian connection of the metrics g on M; d - the operator of exterior differentiation. Additionally, all manifolds, Tensor fields, and other objects are assumed to be of class C.So Almost Hermition (is shorter, AH) structure on a manifold M the pair (Q, g), where Q-almost complex structure (Q^2 = id) on M,g=<.,.>(pseudo) Riemannian metric on M. In this case, < Qa, QB >= <a, ß>; α, B E α(M).

3. Classes Locally Conformal Kahler Manifold W (M) projective Curvature Tensor

Definition 3.1:

"The manifold (M, J, g) refers to as manifold of a class:

1. $\overline{M_1}\ if < M(A,B)C,D> = < M(A,B)J\},JD>$.
2. $\overline{M_2}\ if < M(A,B)C,D> = < M(JA,JB)c,D> + < M(JA,B)JC,D> + < M(JA,B)C,JD>$.
3. $\overline{M_3}\ if < M(A,B)C,D> = < M(JA,JB)J,JD>$."

Remark 3.2:

From (3.1) follow that L.C.K–manifold of class

$M_0 = M_3 = M_5 = M_6$

The class M_3 manifold sense of the specified identifications of curvature is most transparent, as revealed in terms of a spectrum M-projective curvature tensor.

Definition 3.3: "The manifold $< W, J, g >$ refers to as manifold of a class:

1. $\overline{W_1}\ if < W(A,B)C,D> = < W(A,B)JC,JD>$
2. $\overline{W_2}\ if < W(A,B)C,D> = < W(JA,JB)C,D> + < W(JA,B)JC,D> + < W(JA,B),C,JD>$.
3. $\overline{W_3}\ if < W(A,B)C,D> = < W(JA,JB)JC,JD>$ "

Theorem 3.4: Let $S = (J, g = <.,.>)$ as a locally conformal Kahler structure. Then, we can say the following statements are be equivalent to :

1. S–structure of a class $(\overline{W_3}, \overline{W_3})$.
2. $(W_0, W_0) = 0$
3. On space of the adjoint G–structure identities $W^a_{bcd}(M^a_{bcd}) = 0$ are fair.

Proof: Let the S-structure of a class$(\overline{W_3}, \overline{W_3})$, apparently is equivalent to the identity of $W(M)(A,B)C + JW(M)(JA,JB)\ JC = 0, A,B,C \in X(M)$.

By definition of a tensor of spectrum W(M) (A, B)C = W_0 (M_0) (A, B)C + $W_1(M_1)$ (A, B)C + W_2 (M_2)(A, B)C + W_3 (M_3)(A, B)C + W_4 (M_4) (A, B)C + W_5 (M_5) (A, B) C+ W_6 (M_6) (A,B)C + W_7 (M_7) (A, B) C A, B, C ∈ X(M).o ooo

$J \circ W(M)(JA,JB)JC = J \circ W_0(M_0)(A,JB)JC + J \circ W_1(M_1)(JA,JB)JC + J \circ W_2(M_2)(JA,JB)JC + J \circ W_3(M_3)(JA,JB)JC + J \circ W_4(M_4)(JA,JB)JC + J \circ W_5(M_5)(JA,JB)JC + J \circ W_6(M_6)(JA,JB)JC + J \circ W_7(M_7)(JA,JB)JC$ = W (M)(A, B)C = W_0 (M_0)(A, B)C – W_1 (M_1)(A, B)C – W_2 (M_2)(A, B)C – W_3 (M_3)(A, B)C – W_4 (M_4)(A, B)C – W_5 (M_5)(A, B)C – W_6 (M_6)(A, B)C – W_7 (M_7)(A, B)C, ∀ A, B, C ∈ X(M).

Putting term by those identities. Will be received $W(M)(A,B)C + JW(M)(JA,JB)JC$ = 2{ W_0 (M_0)(A, B)C + W_3 (M_3)(A, B)C + W_5 (M_5)(A, B)C + $W_6(M_6)$(A, B)C}, with means the identity W(M)(A, B)C + $JW(M)(JA,JB)JC = 0$ is equivalent to

$W(M)(x,y)z + JW(JM)(JA,JB)JC$ = 2{ W_0 (M_0)(A, B)C + W_3 (M_3)(A, B)C + W_5 (M_5)(A, B)C + W_6 (M_6)(A, B)C}=0 and this identity is equivalent to identities

$W_1(M_1) = W_3(M_3) = W_5(M_5) = W_6(M_6) = 0$

Based on a proposition (def.), the received identities on space of adjoint G–structure are equivalent to relation $W^a_{bcd}(M^a_{bcd}) = W^a_{b\hat{c}\hat{d}}(M^a_{b\hat{c}\hat{d}}) = W^a_{\hat{b}c\hat{d}}(M^a_{\hat{b}c\hat{d}}) = W^a_{\hat{b}\hat{c}d}(M^a_{\hat{b}\hat{c}d}) = 0$

Under Materiality tensor W and its properties [5], received relations which can be equivalent to the relations $W^a_{bcd}(M^a_{bcd}) = 0$

The opposite, according to (3.3.3), apparently.

Theorem 3.5: "Let $S = (J, g = <.,.>)$ is locally conformal kahler structure. Then, the following statements are equivalent: "

1. S-structure of class $\overline{W_2}(\overline{M_2})$
2. $W_0(M_0) = W_7(M_7) = 0$
3. In the space of the attached G-structure, identities $W^a_{bcd}(M^a_{bcd}) = W^a_{\hat{b}\hat{c}\hat{d}}(M^a_{\hat{b}\hat{c}\hat{d}}) = 0$ are fair.

Proof: Let class S-structure $\overline{W_2}(\overline{M_2})$. We will copy the identity $\overline{W_2}(\overline{M_2})$ in the following form, with everyone composed, this identity shall be drawn based on the definition of a spectrum tensor

1. W (M)(A, B)C = W_0 (M_0)(A, B)C + W_1 (M_1)(A, B)C + W_2 (M_2)(A, B)C + W_3 (M_3)(A, B)C + W_4 (M_4)(A, B)C + W_5 (M_5)(A, B)C+ W_6 (M_6)(A, B)C + W_7 (M_7)(A, B)C.
2. $W(M)(JA,JB)C$ = W_0 (M_0) $(JA,JB)C$ + W_1 (M_1) $(JA,JB)C$ +W_2 (M_2) $(JA,JB)C$ + W_3 (M_3) $(JA,JB)C$ + W_4 (M_4) $(JA,JB)C$ + W_5 (M_5) $(JA,JB)C$ + W_6 (M_6) $(JA,JB)C$ + W_7 (M_7) $(JA,JB)C$= - W_0 (M_0)(A, B)C + W_1 (M_1)(A, B)C + W_2 (M_2)(A, B)C - W_3 (M_3)(A, B)C - W_4 (M_4)(A, B)C + W_5 (M_5)(A, B)C+ W_6 (M_6)(A, B)C - W_7 (M_7)(A, B)C.

3. $W(M)(JA,B)JC$ = $W_0(M_0)(JA,B)JC + W_1(M_1)(JA,B)JC + W_2(M_2)(JA,B)JC + W_3(M_3)(JA,B)JC + W_4(M_4)(JA,B)JC + W_5(M_5)(JA,B)JC + W_6(M_6)(JA,B)JC + W_7(M_7)(JA,B)JC = -W_0(M_0)(A,B)C - W_1(M_1)(A,B)C + W_2(M_2)(A,B)C + W_3(M_3)(A,B)C + W_4(M_4)(A,B)C + W_5(M_5)(A,B)C - W_6(M_6)(A,B)C - W_7(M_7)(A,B)C$.

4. $JW(JM)(JA,B)C = JW_0(JM_0)(JA,B)C + JW_1(JM_1)(JA,B)C + JW_2(MJ_2)(JA,B)C + JW_3(JM_3)(JA,B)C + JW_4(JM_4)(JA,B)C + JW_5(JM_5)(JA,B)C + JW_6(JM_6)(JA,B)C + JW_7(JM_7)(JA,B)C = -W_0(M_0)(A,B)C - W_1(M_1)(A,B)C + W_2(M_2)(A,B)C + W_3(M_3)(A,B)C - W_4(M_4)(A,B)C - W_5(M_5)(A,B)C + W_6(M_6)(A,B)C + W_7(M_7)(A,B)C$.

Substituting this decomposition in the previous equality, we will receive

$W(M)(A,B)C - W(M)(JA,JB)C + W(M)(JA,B)JC + JW(JM)(JA,B)C = 2\{W_0(M_0)(A,B)C + W_3(M_3)(A,B)C - W_5(M_5)(A,B)C + W_6(M_6)(A,B)C + W_7(M_7)(A,B)C\} = 0$.

This identity is equivalent to that

$W_0(M_0)(A,B)C = W_3(M_3)(A,B)C = W_5(M_5)(A,B)C = W_6(M_6)(A,B)C = W_7(M_7)(A,B)C = 0$

And these identities on space of the adjoint G-structure are equivalent to identities

$$W^a_{bcd}(M^a_{bcd}) = W^a_{b\hat{c}\hat{d}}(M^a_{b\hat{c}\hat{d}}) = W^a_{\hat{b}c\hat{d}}(M^a_{\hat{b}c\hat{d}}) = W^a_{\hat{b}\hat{c}d}(M^a_{\hat{b}\hat{c}d}) = W^a_{\hat{b}\hat{c}\hat{d}}(M^a_{\hat{b}\hat{c}\hat{d}})$$

by virtue of materiality tensor W(M) and his proposition [5] the received relations are equivalent to relations $W^a_{bcd}(M^a_{bcd}) = W^a_{\hat{b}\hat{c}\hat{d}},(M^a_{\hat{b}\hat{c}\hat{d}}) = 0$ i, e . to identities $W_0(M_0)(A,B)C = W_7(M_7)(A,B)C = 0$

Back, let for L. C. K. manifold identities $W_0(M_0)(A,B)C = W_7(M_7)(A,B)C = 0$

Are executed. Then, from (3.3.7) and (3.3.11) have

$$W(M)(A,B)C - W(M)(A,JB)JC - W(M)(JA,B)C - W(M)(JA,JB)C = 0$$

i.e.

$$W(M)(A,B)C = W(M)(A,JB)JC + W(M)(JA,B)JC + W(M)(JA,JB)C$$

In the received identity instead of W(M)(A, JB)JC replace ment $B \to JB$ and $C \to JC$

i.e. $W(M)(A,JB)JC = -JW(JM)(JA,B)C. then, i.e.$

$$W(M)(A,B)C = W(M)(JA,JB)C + W(M)(JA,B)JC - JW(JM)(JA,JB)C$$

$$< W(M)(A,B,C,D) > = < W(M)(JA,JB,C,D) > + < W(M)(JA,B,JC,D) > + < W(M)(JA,JB,C,JD) > \quad ,A,B,C,D \in X(M)$$

Thus, the manifold satisfies to identity $\overline{W_2}(\overline{M_2})$.

Theorem 3.6: Let $S = < J.g = < X,X >$ is locally conformal kahler structure. Then, the following statements are equivalent:

1. S-structure of a class $\overline{W_1}, (\overline{M_1})$.
2. $W_0(M_0) = W_4(M_4) = W_7(M_7) = 0$.
3. On space of the attached G-structure identities $W^a_{bcd}(M^a_{bcd}) = W^a_{\hat{b}cd}(M^a_{\hat{b}cd}) = W^{a(}_{\hat{b}\hat{c}\hat{d}} M^a_{\hat{b}\hat{c}\hat{d}})$ are fair.

Proof: Let S-structure of a class $\overline{W_1}(\overline{M_1})$ obviously, it is equivalent to identity

$< W(M)(A,B,C,D) \geq -< W(M)(A,B,JC,JD)$

$and\ we\ get\ W(M)(A,B)C + JW(JM)(A,B)JC = 0,\ A,B,C \in X(M)$.

By definition of a spectrum tensor

1. $W(M)(A,B)C = W_0(M_0)(A,B)C + W_1(M_1)(A,B)C + W_2(M_2)(A,B)C + W_3(M_3)(A,B)C + W_4(M_4)(A,B)C + W_5(M_5)(A,B)C + W_6(M_6)(A,B)C + W_7(M_7)(A,B)C$. A, B, C $\in$X(M).
2. $J \circ W(M)(A,B)JC = J \circ W_0(M_0)(A,B)JC + J \circ W_1(M_1)(A,B)JC + J \circ W_2(M_2)(A,B)JC + J \circ W_3(M_3)(A,B)JC + J \circ W_4(M_4)(A,B)JC + J \circ W_5(M_5)(A,B)JC + J \circ W_6(M_6)(A,B)JC + J \circ W_7(M_7)(A,B)JC = -W_0(M_0)(A,B)JC - W_1(M_1)(A,B)JC - W_2(M_2)(A,B)JC - W_3(M_3)(A,B)JC + W_4(M_4)(A,B)JC - W(M_5)(A,B)JC - W_6(M_6)(A,B)JC + W_7(M_7)(A,B)JC$; A, B, C $\in$X(M).

Putting (1) and (2) in 9$W(M)(A,B)C + JW(JM)(A,B)JC = 2\{W_0(M_0)(A,B)C + W_4(M_4)(A,B)C + W_7(M_7)(A,B)C\}$ with means this identity $W(M)(A,B)C + JW(JM)(A,B)JC$ is equivalent to that $W_0(M_0)(A,B)C + W_4(M_4)(A,B)C + W_7(M_7)(A,B)C = 0$ and this identity is equivalent to identities $W_0(M_0) = W_4(M_4)W_7(M_7) = 0$According to proposition [5]. The received identities in space of the adjoint G-structure are equivalent to relation $W^a_{bcd}(M^a_{bcd}) = W^a_{\hat{b}cd}(M^a_{\hat{b}cd}) = W^a_{\hat{b}\hat{c}\hat{d}}(M^a_{\hat{b}\hat{c}\hat{d}}) = 0$.

Theorem 3.7: Let $S = (J.g = < X,X >$ is locally kahler structure. Then, the following inclusions of classes $\overline{W_1}(\overline{M_1}) \subset \overline{W_2}(\overline{M_2}) \subset \overline{W_3},(\overline{M_3})$are fair.

Proof: Let S-structure of class $\overline{W_1}(\overline{M_1})$. Obviously, it is equivalent to $\overline{W_0}(\overline{M_0}) = \overline{W_4}(\overline{M_4}) = \overline{W_7}(\overline{M_7}) = 0$ by theorem 3.6.

So, (by theorem 3.5) $\overline{W_0}(\overline{M_0}) = \overline{W_7}(\overline{M_7}) = 0$ is equivalent to class $\overline{W_2}, (\overline{M_2})$ then, $\overline{W_1}(\overline{M_1}) \subset \overline{W_2}(\overline{M_2})$

Also, the class $\overline{W_3}(\overline{M_3})$ is equivalent to W_0 (M_0) = 0. That is clear from theorem 3.4.

Thus, $\overline{W_1}(\overline{M_1}) \subset \overline{W_2}(\overline{M_2}) \subset \overline{W_3}, (\overline{M_3})$.

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