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On Nano Generalized Delta Separation Axioms

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Abstract:
This paper purposes to introduce a novel separation axioms in Nano topological space called N-g-δ-T_i space where I = 0, 1, 2, 3 and define some type of N-g-δ-T_i space such that Ngδ(semi-pre) T_i space where I = 0, 1, 2, 3. And introduce example explain the relation between this type.

Key Words: N-g-δ-T_i space, N-g-δ(semi-pre)-T_i space, N-g-δ-Reg space, N-g-δ(semi-pre) Reg space, N-g-δ –Normal space, N-g-δ(semi-pre) Normal space.

1. Introduction:

Levine Norman in 1970 [5] established the notion of Generalized-closed set. Arya [1] in 1990 define g-semi closed set, after that Maki and Noiri [6] in 1996 introduce definition the g-pre closed set. And in 2000, Dontechev [4] define gδ-closed set. The complement of gδ-closed set is known as gδ-open set. The notion of Nano generalized closed set by Bhuvaneswari[2] in 2014 and the notion of separation axioms (T_0, T_1) introduced by Mccartan[7] in 1968, in the same year Cullen [3] define Regular space, after that Willarad [10] define Normal space in 1970. Sthismohan [9] study (semi – pre) T_i space where i = 0, 1. The definition of the notion of N-g-δ as a closed (open) was set by Saliem and Shihab [8] in 2023.

2. Preliminaries:

Let's introduce the subsequent useful definitions:

Definition 2.1: [5]

"Let (X, T) be a topological space and F $\subseteq$ X, then F is said to be generalized closed set (briefly, g-closed set) if cl(F) $\subseteq$ D where F $\subseteq$ D and D is open set in X. The complement of g-closed set is said to be generalized open set (briefly, g-open set)."

Definition 2.2:

"Let (X, T) be a topological space and F $\subseteq$ X. then, F is said to be:

1- Generalized δ-closed set(briefly, g-δ-closed) if cl(F) $\subseteq$ D where F$\subseteq$ D and D is δ-open set in X. [4]
2- Generalized pre-closed set (briefly, g-pre-closed) if pre cl(F) $\subseteq$ D where F$\subseteq$ D and D is pre-open set in X. [6]
3- Generalized semi-closed set (briefly, g-semi-closed) if semi cl(F) $\subseteq$ D where F$\subseteq$ D and D is semi-open set in X". [1]

The complement for those closed sets is an open set.

Definition 2.3: [2]

"Let (U, T_R(x)) be a Nano topological space, a subset F of (U, T_R(x)) is said to be Nano Generalized closed set (N-g-closed) if Ncl(F) ⊆ D where F⊆ D and D is N-open set in (U, T_R(x))".

Definition 2.4:[8]

"Let (U, T_R(x)) be a Nano topological space, a subset F of (U, T_R(x)) is said to be

1. N-g-δ-closed set if Nclδ (F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x)).
2. N-g-δ-pre-closed set if N-δ-pre-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x)).
3. N-g-δ-semi-closed set if N-δ-semi-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x))."

The complement of these Nano closed sets is considered an Nano open set.

Definition 2.5: [7]

A topological space (X, T) is:

1. T0 if ∀ a ≠ b , ∃ open set G such that a ∈ G, b ∉ G or a ∉ G , b ∈ G.
2. T_1 if ∀ a ≠ b , ∃ open set G and D in which that a ∈ G, b ∉ G or a ∉ D , b ∈ G.

Definition 2.6: [3]

" A topological space (U, T) is said to be Regular space if for any closed set D and a point K such that K ∉ D ∃ a disjoint open sets F_1 and F_2 such that D ∈ F_1 , K ∈ F_2."

Definition 2.7: [10]

" A space X is said to be normal space iff ∀ two disjoint closed sets D_1 and D_2 subset of X, ∃ disjoint open sets W_1 and W_2 such that D_1 ⊆ W_1 and D_2 ⊆ W_2."

Definition 2.8: [9]

The topological space (X, T) is:

1. (Semi–pre) T_0 space i`f ∀ a≠ b ∈ X ∃ (semi - pre) open sets D in which that a ∈ D , b ∉ D or a ∉D , b ∈ D.
2. (Semi – pre) T_1 space if ∀ a≠ b ∈ X ∃ (semi–pre) open sets G and D in which a ∈ G , b ∉ G or a ∉D , b ∈ D.

3. On Nano Generalized Delta Separation Axioms

Definition 3.1:

"A Nano topological space (U, T_R(X)) is said to be:

1. N-g-δ-T_0 if ∀ a ≠ b ∈ U. ∃ N-g-δ-open set D such that a ∈ D , b∉ D or b ∈ D , a ∉ D.
2. N-g-δ-pre-T_0 if ∀ a ≠ b ∈ U. ∃ N-g-δ-pre-open set D such that a ∈ D , b∉ D or b ∈ D , a ∉ D.
3. N-g-δ-semi-T_0 if ∀ a ≠ b ∈ U. ∃ N-g-δ-semi-open set D such that a ∈ D , b∉ D or b ∈ D , a ∉ D."

Remark 3.2: The next diagram explain the relation between N-g-δ-T_0 spaces.

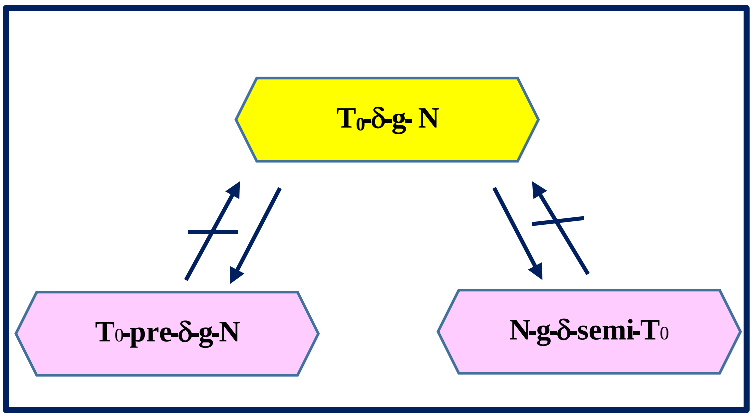


Diagram 3.1: explain the relationship between some types N-g-δ-T_0 spaces

Example 3.3: Let U = { 1, 2, 3, 4} with U/R = {{1}, {2, 4}}, and let X = {1, 2}, so that T_R(x) = { U, ∅, {1}, {2, 4} {1, 2, 4}} Then, N-g-δ is an open set = { U, ∅, {1}, {2}, {4}, {1, 2}, {1, 4},{2, 4}, {1, 2, 4}} N-g-δ-semi-open set = { U, ∅, {1}, {2}, {4}, {1, 2}, {1, 3}, {1, 4}, {1, 2, 3}, {1, 2, 4}, {2, 4}, {1, 3, 4}, {2, 3, 4}}N-g-δ-pre-open set = { U, ∅, {1}, {2}, {4}, {1, 2}, {1, 4},{2, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}}

Example 3.4: From example 3.3, we have:

1. N-g-δ-T_0 Let a = 2 , b = 4 then ∃ N-g-δ an open set that {1, 2} in which 2 ∈ {1, 2} , {4} ∉ {1, 2} or ∃ {1,4} open set in which that 2∉ {1, 4} , 4∈ {1, 4}
2. N-g-δ-pre-T_0 Let a = 1 , b = 4, then ∃ N-g-δ-pre-open set {1, 2, 3} in which 1 ∈ {1, 2, 3}, 4∉ {1, 2, 3}.
3. N-g-δ-semi-T_0

Let a =4, b = 2 then, ∃ N-g-δ-semi-open set {1, 3, 4} in which a ∈ {1, 3, 4}, b ∉ {1, 3, 4}

Remark 3.5:

Based on the former example, we can say that:

1. Every N-g-δ-T0 is N-g-δ-preT0 ,but the converse may not be true.
2. Every N-g-δ-T0 is N-g-δ-semi-T0 ,but the converse is not true.

Definition 3.6:"A Nano topological space (U, $T_R(X)$) is said to be:

1. N-g-δ-T_1 if ∀ K ≠ L ∈ U, ∃ N-g-δ-open set D and F such that K ∈ D , L∉ D and L ∈ F, K ∉ F.
2. N-g-δ-pre-T_1 if ∀ K ≠ L ∈ U, ∃ N-g-δ-pre-open set D and F such that K ∈ D , L∉ D and L ∈ F, K ∉ F.
3. N-g-δ-semi-T_1 if ∀ K ≠ L ∈ U, ∃ N-g-δ-semi-open set D and F such that K ∈ D , L∉ D and L ∈ F, K ∉ F."

Example 3.7: Base on example 3.3, we have

1. K = 2 , L = 4, then 2 ∈ {1,2} , 2∉{1,4} and 4 ∉ {1, 2} , 4∈ {1,4}, then, U is N-g-δ-T_1.
2. U is N-g-δ-pre-T_1 for {1, 2, 3} and {1, 3, 4}.
3. U is N-g-δ-semi-T_1 for {2, 3, 4} and {1, 3, 4}.

Remark 3.8: From the previous example, we have:

1. Every N-g-δ-T_1 is N-g-δ-pre-T_1 but the converse not be true.
2. Every N-g-δ-T_1 is N-g-δ-semi-T_1 but the converse not be true

Remark 3.9: The next diagram explain the relationship between some type of N-g-δ-T_1 spaces.

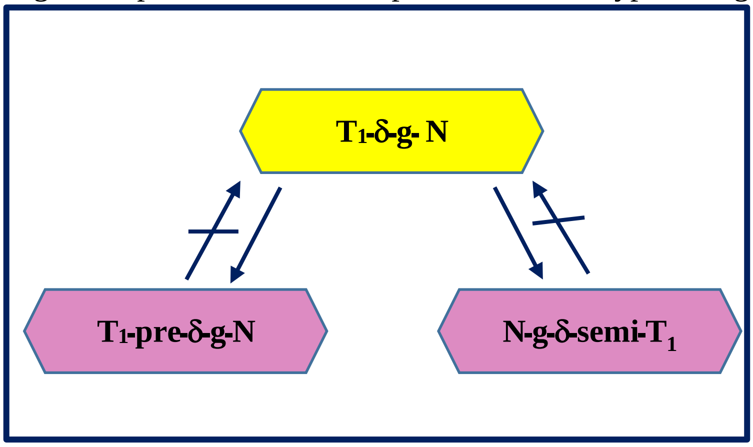


Diagram 3.2: explain the relationship between some types N-g-δ-T_1 spaces

Definition 3.10:"A Nano topological space (U, $T_R(X)$) is said to be:

1. N-g-δ-T_2 if ∀ K ≠ L ∈ U, ∃ a disjoint N-g-δ-open sets D and F such that K ∈ D and L ∈ F.
2. N-g-δ-pre-T_2 if ∀ K ≠ L ∈ U, ∃ a disjoint N-g-δ-pre-open sets D and F such that K ∈ D and L ∈ F.
3. N-g-δ-semi-T_2 if ∀ K ≠ L ∈ U, ∃ a disjoint N-g-δ-semi-open sets D and F such that K ∈ D and L ∈ F."

Example 3.11 Based on example 3.3, we can have

1. U is N-g-δ-T_2 for {1} and {2}
2. U is N-g-δ-pre-T_2 for {4} and {1, 2, 3}
3. U is N-g-δ-semi-T_2 for {2} and {1, 3, 4}

Remark 3.12: Based on the previous example, we can say that:

1. Every N-g-δ-T_2 is N-g-δ-pre-T_2 but the converse is not true.
2. Every N-g-δ-T_2 is N-g-δ-semi-T_2, but the converse is not true.

Remark 3.13: The next diagram explain the relation between some type of N-g-δ-T_2 spaces.

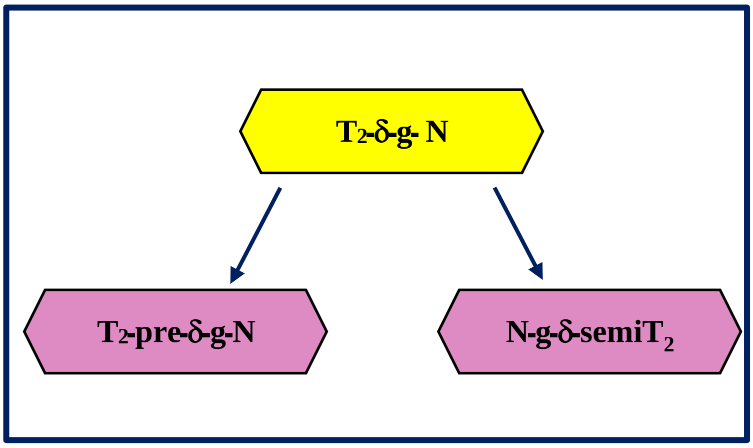


Diagram 3.3: explain the relationship between some types N-g-δ-T_2 spaces

Theorem 3.14: Every N-g-δ-T_2 space is N-g-δ–T_1 space.

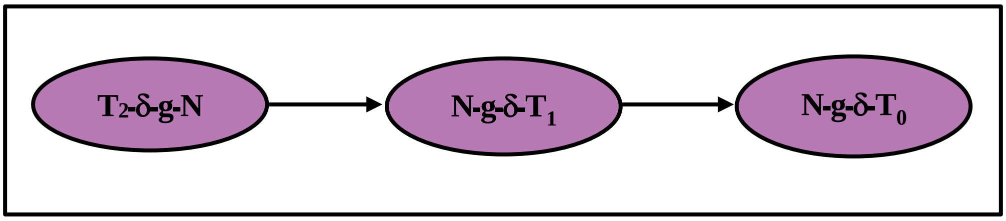
Proof: Let U be N-g-δ-T_2 space and K,L be a two distinct points of U, then ∃ disjoint N-g-δ –open sets D and F in which K ∈ D, thus K∉ F, likewise L ∈ F and L ∉ D, so U is N-g-δ-T_1 space.

Theorem 3.15: Every N-g-δ-T_1 space is N-g-δ-T_0 space.

Proof: Let U be N-g-δ-T_1 space, and K ≠ L ∈ U, ∃ two N-g-δ– open sets D and F in which K ∈ D, then L ∉ D. so, U is N-g-δ}T_0 space.

Remark 3.16: The next diagram explain the relation between N-g-δ-T_i spaces, where i=0, 1, 2

Diagram 3.4:



relationship N-g-δ-T_i, where i=0, 1, 2

Theorem 3.17: Every N-g-δ-pre-T_1(represent N-g-δ-semi-T_1) spaces is N-g-δ-pre-T0 (represent N-g-δ-semi-T_0) spaces.

Proof: Based on Theorem 3.15, Remark 3.5, and Remark 3.8, we could get the results.

Theorem 3.18 Every N-g-δ-pre-T_2(respresent N-g-δ-semi-T_2) spaces is N-g-δ-pre-T_1(respresent N-g-δ-semi-T_1) spaces.

Proof: Based on Theorem 3.14, Remark 3.12, and Remark 3.8, we could get the results.

Remark 3.19: The next diagram explains the relation between N-g-δ-pre-T_i(resp. N-g-δ-semi-T_i) spaces, where i=0, 1, 2.

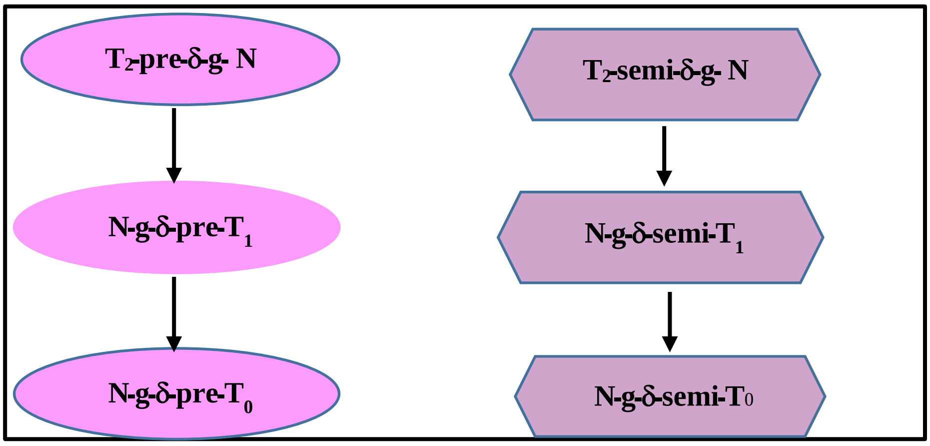


Diagram 3.5: relationship N-g-δ-pre-T_i(resp. N-g-δ-semi-T_i) spaces, where i=0,1,2

Definition 3.20: "A Nano topological space (U, $T_R(X)$) is said to be Nano generalized δ Regular space (briefly, N-g-Reg space) if for any N-g-δ-closed set D and a point K such that K ∉ D there exist a disjoint N-g-δ-open sets F_1 and F_2 such that D ∈ F_1 , K ∈ F_2."

Definition 3.21 " A N-g-δ-Reg space which also N-g-δ-T_1 space is said to be N-g-δ-T_3 space."

Example 3.22: Let U = { k, l, m, n} with U/R = { {k, l}, {m, n}} and X = { k, l, m}. Then, $T_R(X)$ = {∅, U,{k, l},{m, n}}, $T^c{}_R(X)$ = { U, ∅,{m, n}, {k, l}}, therefore, U is N-g-δ-T_1 for {k, l} and {m, n} , and U is N-g-δ– Reg space. So, U is N-g-δ-T_3 is space for {k, l} and {m, n}.

Definition 3.23: "A space U is said to be N-g-δ-normal space iff ∀ two disjoint N-g-δ-closed sets D_1 and D_2 subset of U, ∃ disjoint N-g-δ-open sets W_1 and W_2 such that $D_1 \subseteq W_1$ and $D_2 \subseteq W_2$."

Definition 3.24: "A Nano generalized δ topological space (U, $T_R(X)$) is said to be (N-g-δ-pre and N-g-δ-semi) Normal space if for any two disjoint N-g-δ(pre and semi) closed set D_1 and D_2 , ∃ two disjoint N-g-δ(pre and semi) open sets W_1 , W_2 such that $D_1 \subseteq W_1$ and $D_2 \subseteq W_2$."

Theorem 3.25: If U = $U_R(X)$, $L_R(X) \neq U_R(X)$ and $L_R(X) \neq \emptyset$. Hence, U is N-g-δ is a normal space.

Proof: When U = $U_R(X)$ and $L_R(X) \neq \emptyset$. Then, $T_R(X)$ = {∅, U, $L_R(X)$, $B_R(X)$} and $T^c{}_R(X)$ = { U, ∅,$[B_R(X)]^c$, $[L_R(X)]^c$ }, let $D_1 = [B_R(X)]^c$ and $D_2 = [L_R(X)]^c$, then $D_1 \cap D_2 = \emptyset$, since $D_1 \subset L_R(X)$ and $D_2 \subset B_R(X)$, $L_R(X)$, $B_R(X)$ are disjoint N-g-δ- is an open set. So, U is N-g-δ is a normal space.

Theorem 3.26: ∀ N-g-δ-extremely is disconnected space, the following statements are true:

1. Every N-g-δ-Normal space then N-g-δ-pre is Normal space.
2. Every N-g-δ-Normal space then N-g-δ-semi is Normal space.

Proof:

1. If U be an extremely disconnected space, and U is N-g-δ-Normal(via theorem 3.25) and every N-g-δ-open set is N-g-δ-pre is an open set. Then, U is N-g-δ-pre is Normal space.
2. Based on (1), we could prove that.

Definition 3.27: "A N-g-δ-Normal space which also N-g-δ-T_1 space said to be N-g-δ-T_4 space."

Definition 3.28: "A N-g-δ-pre-Normal space (resp. N-g-δ-semi-Normal space) which also N-g-δ-pre-T_1 space (N-g-δ-semi-T_1 space) said to be N-g-δ-pre-T_4 space(resp. N-g-δ-semi-T_4 space)".

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