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On Certain types of totally disconnected fibers

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Abstract:

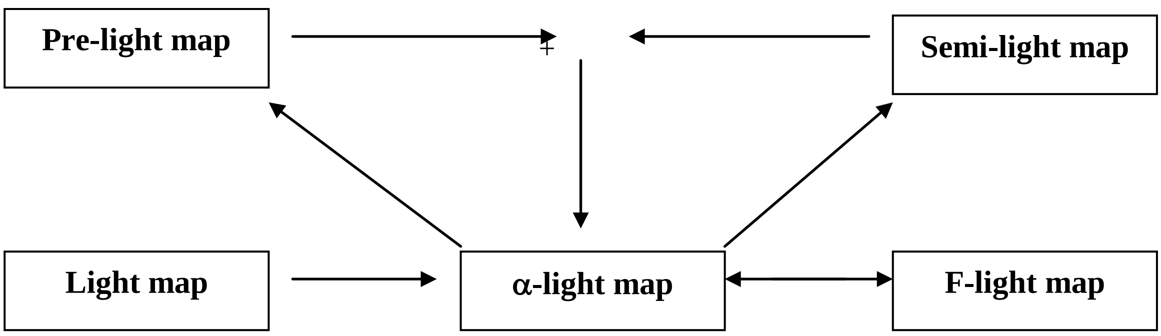
In this work , we give the definitions of certain types of maps which have totally disconnected fibers and the study of the properties of composition and restriction of each type. Also, we study the relation between them. These definitions are novel in the present time at the best of our knowledge.

الخلاصة: في هذا البحث تم تقديم تعاريف جديدة لبعض الانواع من التطبيقات التي تمتلك الالياف غير المتصلة كلياوتم دراسة بعض الخصائص كالتركيب والقصرلكل نوع. كذلك درسنا العلاقة بين هذه التطبيقات وتم وضع بعض الشروط على التطبيق او على الفضاء لتحقيق العكس .

Introduction:

Levine, N. in [6] give the definition of generalized open set(semi-open set) and study the properties of it and Maximilian, G. in [7] give the definition of pre-open set and study the properties of it .A subset A of X is called α-open set if $A \subseteq (\overline{A^{\circ}})^{\circ}$ [7], and a subset A of X is called Feebly open set (F-open) if there exists an open subset U of X , such that $U \subseteq A \subseteq U^{-s}$, where –s is the semi-closure of U.Let $f:X\rightarrow Y$ be a map from X onto Y ,then $f^{-1}(y)$is called the fiber for all $y\in Y$.A map $f:X\rightarrow Y$ is called light map if for all $y\in Y$ the fiber $f^{-1}(y)$ is totally disconnected set[10] and a map $f:X\rightarrow Y$ is called semi-light map if for all $y\in Y$ the fiber $f^{-1}(y)$ is totally semi- disconnected set [2].

In this work , we give the definition of pre-light map , α-light map and F-light map and we study some properties of these maps. Also,we give the relation between types and from of proposition (II.1,V.1,V.2,V.3,V.4,and V.6) we have the following diagram:



The converse of arrows in this diagram is not true in general .We give conditions either on map or on a space to satisfy the converse. Throughout this work, (X,τ) simply space X always means topological space and map $f:X\rightarrow Y$ means a continuous mapping from a topological space X onto a topological space Y .

I .Basic definitions and Notations:

The following definitions are given in [2]

I.1 Definition: Let X be a space and let A,B are non empty open sets in X then A/B is said to be a disconnection to X iff $A\cup B=X$ and $A\cap B=\phi$.

I.2 Definition: Let X be a space and $G\subseteq X$, A,B are non empty open sets in X then A/B is said to be a disconnection to G in X iff

1- $A\cap G\neq\phi$ and $B\cap G\neq\phi$.

2- $(A\cap G)\cup(B\cap G)=G$

3- $(A\cap G)\cap(B\cap G)=\phi$

I.3Definition: A space X is called totally disconnected space if every pair of point $a,b\in X$ there exist a disconnection A/B to X such that $a\in A$ and $b\in B$.

The following example is given in [4].

I.4Examle: The set Q of rational numbers with the usual topology is totally disconnected set.

Now we introduce the following definitions:-

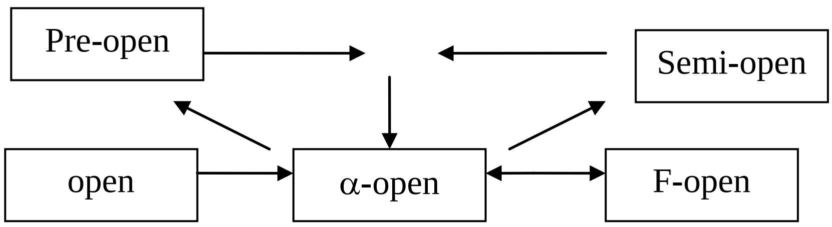
I.5Definitions:

1- Let X be a space and let A,B are non empty α- open sets in X then we say that A/B is α-disconnection to X if $A\cup B=X$ and $A\cap B=\phi$.

2- Let X be a space and let A,B are non empty pre- open sets in X then we say that A/B is pre-disconnection to X if $A\cup B=X$ and $A\cap B=\phi$.

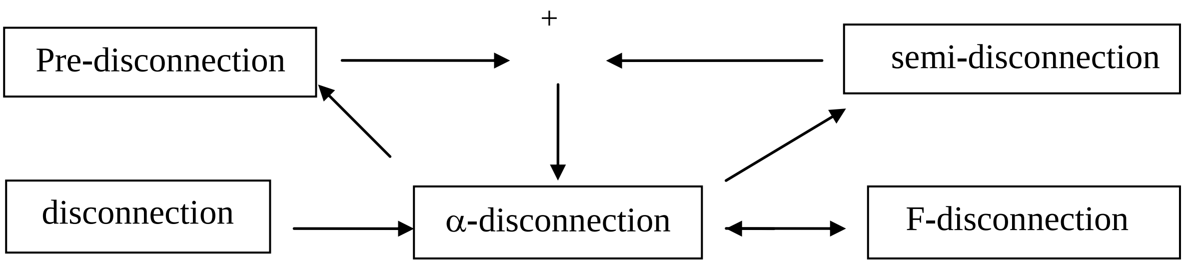
3- Let X be a space and let A,B are non empty F- open sets in X then we say that A/B is F-disconnection to X if $A\cup B=X$ and $A\cap B=\phi$.

Now the following diagram is given in [8] shows the relations among the different types of open sets.



(I)

And from the above diagram we get the following diagram shows the relations among different types of disconnection:-



(II)

Now we introduce the following definitions:-

I.6Definitions:

1- A space X is said to be totally α-disconnected if for each pair of points $a,b\in X$ there is a α-disconnection A/B of X such that $a\in A$ and $b\in B$.
2- A space X is said to be totally pre-disconnected if for each pair of points $a,b\in X$ there is a pre-disconnection A/B of X such that $a\in A$ and $b\in B$.
3- A space X is said to be totally F-disconnected if for each pair of points $a,b\in X$ there is a F-disconnection A/B of X such that $a\in A$ and $b\in B$.

I.7Remark: Each totally disconnected space is totally α-disconnected space and totally pre-disconnected space and totally F-disconnected space. But the converse is not true for example let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\},\{b,c\}\}$.It is clear that X is totally α-disconnected space and hence X is totally pre-disconnected space and totally F-disconnected space but X is not totally disconnected space because there is no exists a disconnection of a,b $\in$X.

***I.8Example*:** The set Q of rational numbers with the usual topology is totally α-disconnected set(since Q is totally disconnected set with the usual topology (I.4))

The following definition is given in [5].

I.9Definition: A mapping $f:X\rightarrow Y$ is said to be α- homeomorphism if

1- f is bijective mapping.
2- f is continuous mapping.
3- f is α-open (α-closed)mapping.

The following definition is given in [9].

I.10Definition: A mapping $f:X\rightarrow Y$ is said to be pre- homeomorphism if

1- f is bijective mapping.
2- f is continuous mapping.
3- f is pre-open (pre-closed)mapping.

The following definition is given in [8].

I.11Definition: A mapping $f:X\rightarrow Y$ is said to be F- homeomorphism if

1- f is bijective mapping.
2- f is continuous mapping.
3- f is F-open (F-closed)mapping.

As a consequence of definitions(I.9,I.1o,I.11) we have the totally α-disconnected space , totally pre-disconnected space and totally F-disconnected is topological property respectively.

I.12Theorem: Let X and Y be two spaces and

a- let $f:X\rightarrow Y$ be a α-homeomorphism .If X or Y is a totally α-disconnected so is the other.
b- let $f:X\rightarrow Y$ be a pre-homeomorphism .If X or Y is a totally pre-disconnected so is the other.
c- let $f:X\rightarrow Y$ be a F-homeomorphism .If X or Y is a totally F-disconnected so is the other.

Proof: Now to proof the part a and b and the part c is given in [8]

a- Suppose that X is totally α-disconnected and let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ .since f is bijective mapping then there exists two points $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1)=y_1$, and $f(x_2)=y_2$ since X is totally α-disconnected space , then there is a α-disconnection U/V such that $x_1 \in U, x_2 \in V$ since f is α-homeomorphism then each of $f(U)$ and $f(V)$ are α-open sets in Y but $f(U)\bigcup f(V) = f(U \bigcup V) = f(X) = Y$ and since f is one-to-one then $f(U)\bigcap f(V) = f(U \bigcap V) = f(\phi) = \phi, \in X$ and $y_1 \in f(U), y_2 \in f(V)$ and hence Y is totally α-disconnected space .

b- Suppose that X is totally pre-disconnected and let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ .since f is bijective mapping then there exists two points $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1)=y_1$, and $f(x_2)=y_2$ since X is totally pre-disconnected space , then there is a pre-disconnection U/V such that $x_1 \in U, x_2 \in V$ since f is pre-homeomorphism then each of $f(U)$ and $f(V)$ are pre-open sets in Y but $f(U)\bigcup f(V) = f(U \bigcup V) = f(X) = Y$ and since f is one-to-one then $f(U)\bigcap f(V) = f(U \bigcap V) = f(\phi) = \phi, \in X$ and $y_1 \in f(U), y_2 \in f(V)$ and hence Y is totally pre-disconnected space

II. Certain types of totally disconnected Fibers:

The following proposition is given in [2].

II.1proposetion: Every Light map is semi-light map.

Now we introduce the following definitions:-

II.2Definition: A map $f:X\to Y$ is called α- light map if for all y∈Y the fiber $f^{-1}(y)$ is totally α- disconnected set in X.

II.3Definition: A map $f:X\to Y$ is called pre- light map if for all y∈Y the fiber $f^{-1}(y)$ is totally pre-disconnected set in X.

II.4Definition: A map $f:X\to Y$ is called F-light map if for all y∈Y the fiber $f^{-1}(y)$ is totally F-disconnected set in X.

II.5Definition: A map $f:X\to Y$ is said to be totally α-disconnected map if each totally α-disconnected set U in X, $f(U)$ is totally α-disconnected set in Y.

II.6Definition: A map $f:X\to Y$ is said to be totally pre-disconnected map if each totally pre-disconnected set U in X, $f(U)$ is totally pre-disconnected set in Y.

II.7Definition: A map $f:X\to Y$ is said to be totally F-disconnected map if each totally F-disconnected set U in X, $f(U)$ is totally F-disconnected set in Y.

As a consequence of definition of (II.5,II.6,II.7) we give the following results

II.8proposetion: Every α-disconnected map is pre-disconnected map.

Proof: let $f:X\to Y$ is α-disconnected map then $f(U)$ is totally α-disconnected set in Y for all U α-disconnected set in X and since each totally α-disconnected set is totally pre-disconnected (diagram II),then $f(U)$ is totally pre-disconnected set in Y for all U pre-disconnected set in X and hence f is pre-disconnected map.

II.9proposetion: A map $f:X\to Y$ is α-disconnected map iff f is F-disconnected map.

Proof: since $f(U)$ is totally α-disconnected set iff $f(U)$ is totally F-disconnected set (diagram II), then $f:X\to Y$ is α-disconnected map iff f is F-disconnected map.

II.10proposetion: Every F-disconnected map is pre-disconnected map.

***Proof*:** It is easy from (II.8) and (II.9).

III. Composition of Certain types of maps with totally disconnected Fibers:

Now we give the following results

III.1proposetion: let $f:X\to Y$ be the composition $f = f_2 o f_1$ of two map f_1:X→Y$'$ and f_2:Y$'$→Y , then
1- If f_2 is an bijective mapping and f_1 is a α-Light map then f is a α-Light map.
2- If f is a α-Light map and f_2 is an injective mapping then f_1 is a α-Light map.
3- If f is a α-Light map and f_1 is a totally α-disconnected maps then f_2 is a α-Light map.

Proof: 1- let $y\in Y$, since f_2 is a bijective mapping then there exists one point $y'\in Y$ such that $f_2(y')=y$ and since $f^{-1}(y)=(f_2 o f_1)^{-1}(y)=f_1^{-1}(f_2^{-1}(y))=f_1^{-1}(f_2^{-1}(f_2(y')))=f_1^{-1}(y')$

and f_1 is α-light map then $f_1^{-1}(y')$ is totally α-disconnected set in X, so $f_1^{-1}(y)$ is totally α-disconnected set in X and hence f is α-light map.

2- let $y' \in Y'$, then $f_2(y') \in Y$ and since f is α-light then $f^{-1}(f_2(y'))$ is a totally α-disconnected set in X but $f^{-1}(f_2(y')) = f_1^{-1}(f_2^{-1}(f_2(y')) = f_1^{-1}(y')$, then f_1 α- light map.

3- let $y \in Y$,since f is a α-light , then $f^{-1}(y)$ is a totally α-disconnected set in X and since f_1 is totally α-disconnected map then $f_1(f^{-1}(y))$ is a totally α-disconnected set in Y', but $f_1(f^{-1}(y)) = f_1((f_2 o f_1)^{-1}(y)) = f_1(f_1^{-1}(f_2^{-1}(y))) = f_2^{-1}(y)$. then $f_2^{-1}(y)$ is a totally α-disconnected set in Y', so f_2 is α-light map.

III.2proposetion: let $f:X\rightarrow Y$ be the composition $f = f_2 o f_1$ of two map f_1:X→Y′ and f_2:Y′→Y , then
1- If f_2 is an bijective mapping and f_1 is a pre-Light map then f is a pre-Light map.
2- If f is a pre-Light map and f_2 is an injective mapping then f_1 is a pre-Light map.
3- If f is a pre-Light map and f_1 is a totally pre-disconnected maps then f_2 is a pre-Light map.

Proof: 1- let $y \in Y$, since f_2 is a bijective mapping then there exists one point $y' \in Y$ such that $f_2(y')=y$ and since $f^{-1}(y) = (f_2 o f_1)^{-1}(y) = f_1^{-1}(f_2^{-1}(y)) = f_1^{-1}(f_2^{-1}(f_2(y')) = f_1^{-1}(y')$
and f_1 is pre-light map then $f_1^{-1}(y')$ is totally pre-disconnected set in X, so $f_1^{-1}(y)$ is totally pre-disconnected set in X and hence f is pre-light map.

2- let $y' \in Y'$, then $f_2(y') \in Y$ and since f is pre-light then $f^{-1}(f_2(y'))$ is a totally pre-disconnected set in X but $f^{-1}(f_2(y')) = f_1^{-1}(f_2^{-1}(f_2(y')) = f_1^{-1}(y')$, then f_1 is a pre- light map.

3- let $y \in Y$,since f is a pre-light , then $f^{-1}(y)$ is a pre-disconnected set in X and since f_1 is totally pre-disconnected map then $f_1(f^{-1}(y))$ totally is a totally pre-disconnected set in Y', but $f_1(f^{-1}(y)) = f_1((f_2 o f_1)^{-1}(y)) = f_1(f_1^{-1}(f_2^{-1}(y))) = f_2^{-1}(y)$. then $f_2^{-1}(y)$ is a totally pre-disconnected set in Y', so f_2 is pre-light map.

III.3proposition: let $f:X\rightarrow Y$ be the composition $f = f_2 o f_1$ of two map f_1:X→Y′ and f_2:Y′→Y , then
1- If f_2 is an bijective mapping and f_1 is a F-Light map then f is a F-Light map.
2- If f is a F-Light map and f_2 is an injective mapping then f_1 is a F-Light map.
3- If f is a F-Light map and f_1 is a totally F-disconnected maps then f_2 is a F-Light map.

Proof: 1- let $y \in Y$, since f_2 is a bijective mapping then there exists one point $y' \in Y$ such that $f_2(y')=y$ and since $f^{-1}(y) = (f_2 o f_1)^{-1}(y) = f_1^{-1}(f_2^{-1}(y)) = f_1^{-1}(f_2^{-1}(f_2(y')) = f_1^{-1}(y')$
and f_1 is F-light map then $f_1^{-1}(y')$ is totally F-disconnected set in X, so $f_1^{-1}(y)$ is totally F-disconnected set in X and hence f is F-light map.

2- let $y' \in Y'$, then $f_2(y') \in Y$ and since f is F-light then $f^{-1}(f_2(y'))$ is a totally F-disconnected set in X but $f^{-1}(f_2(y')) = f_1^{-1}(f_2^{-1}(f_2(y')) = f_1^{-1}(y')$, then f_1 is a F- light map.

3- let $y \in Y$,since f is a F-light , then $f^{-1}(y)$ is a totally F-disconnected set in X and since f_1 is totally F-disconnected map then $f_1(f^{-1}(y))$ is a totally F-disconnected set in Y', but

$f_1(f^{-1}(y)) = f_1((f_2 o f_1)^{-1}(y)) = f_1(f_1^{-1}(f_2^{-1}(y))) = f_2^{-1}(y)$. then $f_2^{-1}(y)$ is a totally F-disconnected set in Y', so f_2 is F-light map.

IV. Restriction of Certain types of maps with totally disconnected Fibers:

Now we give the following results:

***IV.1proposetion*:** let $f:X\to Y$ be α-Light map and let $G \subset X$, then the restriction map $f|_G:G\to f(G)$, is also α-Light map.

Proof: to show that for all $y \in f(G)$, $f^{-1}(y)\cap G$ is totally α-disconnected set in G. let a,b $\in f^{-1}(y)\cap G$ then a,b $\in f^{-1}(y)$ and since f is α-Light map then for all $y \in Y$, $f^{-1}(y)$ is totally α-disconnected set in X (by II.2) , that is there exists a α-disconnection A/B such that $(A\cap f^{-1}(y))\cup(B\cap f^{-1}(y)) = f^{-1}(y)$ and
$(A\cap f^{-1}(y))\cap(B\cap f^{-1}(y)) = \varphi$
Such that A,B are disjoint α-open subsets of X, and $a \in A, b \in B$ now to show that A/B is α-disconnection to $f^{-1}(y)\cap G$ also .since
$((G\cap f^{-1}(y))\cap A)\cup((G\cap f^{-1}(y))\cap B) = (G\cap((f^{-1}(y)\cap A))\cup(G\cap(f^{-1}(y)\cap B))$
$= G\cap[(f^{-1}(y)\cap A)\cup(f^{-1}(y)\cap B)] = G\cap f^{-1}(y)$ and
$((G\cap f^{-1}(y))\cap A)\cap((G\cap f^{-1}(y))\cap B) = (G\cap(f^{-1}(y)\cap A))\cap(G\cap(f^{-1}(y)\cap B))$
$= G\cap[(f^{-1}(y)\cap A)\cap(f^{-1}(y)\cap B)] = G\cap\varphi = \varphi$
Such that $(G\cap f^{-1}(y))\cap A, (G\cap f^{-1}(y))\cap B$ are two disjoint α-open sets , hence $f^{-1}(y)\cap G$ is totally α-disconnected set , $f|_G$ is α-Light map.

IV.2proposition:let $f:X\to Y$ be a pre-Light map and let $U\subset X$, then the restriction map $f|_U:U\to f(U)$, is also pre-Light map.

Proof: to show that for all $y \in f(U)$, $f^{-1}(y)\cap U$ is totally pre-disconnected set in U. let a,b $\in f^{-1}(y)\cap U$ then a,b $\in f^{-1}(y)$ and since f is pre-Light map then by (II.3) for all $y \in Y$, $f^{-1}(y)$ is totally pre-disconnected set in X , that is there exists a pre-disconnection A/B such that $(A\cap f^{-1}(y))\cup(B\cap f^{-1}(y)) = f^{-1}(y)$ and
$(A\cap f^{-1}(y))\cap(B\cap f^{-1}(y)) = \varphi$
Such that A,B are disjoint pre-open subsets of X, and $a \in A, b \in B$ now to show that A/B is α-disconnection to $f^{-1}(y)\cap U$ also .since
$((U\cap f^{-1}(y))\cap A)\cup((U\cap f^{-1}(y))\cap B) = (U\cap((f^{-1}(y)\cap A))\cup(U\cap(f^{-1}(y)\cap B))$
$= U\cap[(f^{-1}(y)\cap A)\cup(f^{-1}(y)\cap B)] = U\cap f^{-1}(y)$ and
$((U\cap f^{-1}(y))\cap A)\cap((U\cap f^{-1}(y))\cap B) = (U\cap(f^{-1}(y)\cap A))\cap(U\cap(f^{-1}(y)\cap B))$
$= U\cap[(f^{-1}(y)\cap A)\cap(f^{-1}(y)\cap B)] = U\cap\varphi = \varphi$
Such that $(U\cap f^{-1}(y))\cap A, (U\cap f^{-1}(y))\cap B$ are two disjoint pre-open sets , hence $f^{-1}(y)\cap U$ is totally pre-disconnected set , $f|_G$ is pre-Light map.

IV.3proposetin:let $f:X\to Y$ be a F-Light map and let $U\subset X$, then the restriction map $f|_U:U\to f(U)$, is also F-Light map.

Proof: to show that for all $y \in f(U)$, $f^{-1}(y)\cap U$ is totally pre-disconnected set in U. let a,b

$\in f^{-1}(y)\cap U$ then a,b $\in f^{-1}(y)$ and since *f* is pre-Light map then by (II.4) for all $y\in Y, f^{-1}(y)$ is totally F-disconnected set in X , that is there exists a F-disconnection *A/B* such that $(A\cap f^{-1}(y))\cup(B\cap f^{-1}(y))=f^{-1}(y)$ and

$(A\cap f^{-1}(y))\cap(B\cap f^{-1}(y))=\varphi$

Such that *A*,*B* are disjoint F-open subsets of X, and $a\in A, b\in B$ now to show that A/B is F-disconnection to $f^{-1}(y)\cap U$ also .since

$((U\cap f^{-1}(y))\cap A)\cup((U\cap f^{-1}(y))\cap B)=(U\cap((f^{-1}(y)\cap A))\cup(U\cap(f^{-1}(y)\cap B))$

$=U\cap[(f^{-1}(y)\cap A)\cup(f^{-1}(y)\cap B)]=U\cap f^{-1}(y)$ and

$((U\cap f^{-1}(y))\cap A)\cap((U\cap f^{-1}(y))\cap B)=(U\cap(f^{-1}(y)\cap A))\cap(U\cap(f^{-1}(y)\cap B))$

$=U\cap[(f^{-1}(y)\cap A)\cap(f^{-1}(y)\cap B)]=U\cap\varphi=\varphi$

Such that $(U\cap f^{-1}(y))\cap A,(U\cap f^{-1}(y))\cap B$ are two disjoint F-open sets, hence $f^{-1}(y)\cap U$ is totally F-disconnected set , $f|_G$ is F-Light map.

V. Relation between types of Light maps:

V.1proposetion:Every light map is α-light map.

Proof: let *f:X→Y* be Light map then for all y∈Y , the fiber $f^{-1}(y)$ is totally disconnected set in X and from(I.7) , that $f^{-1}(y)$ is totally α- disconnected set in X and hence *f:X→Y* be α-Light map.

V.2proposetion:Every light map is pre-light map.

Proof: let *f:X→Y* be Light map then for all y∈Y , the fiber $f^{1}(y)$ is totally disconnected set in X and from(I.7) , that $f^{1}(y)$ is totally pre- disconnected set in X and hence *f:X→Y* be pre-Light map.

V.3proposetion:Every light map is F-light map.

Proof: let *f:X→Y* be Light map then for all y∈Y , the fiber $f^{-1}(y)$ is totally disconnected set in X and from(I.7) , that $f^{-1}(y)$ is totally F- disconnected set in X and hence *f:X→Y* be F-Light map.

V.4proposetion: let *f:X→Y* be map , then *f* is α- light map iff *f* is F- light map.

***Proof*:** since $f^{1}(y)$ is totally α-disconnected set iff $f^{-1}(y)$ is totally F-disconnected set in X for all y∈Y (diagram II).

V.5Remarak: The converse of the propositions (V.1,V.2,V.3) is not necessarily true.

Now we shill put some conditions either on the space or on the maps to get the converse of preceding propositions.

V.6proposetion: let *f:X→Y* be semi-light and pre-light map, then *f* is α- light map.

Proof: Since *f* is semi – and pre- light map, then the fiber $f^{-1}(y)$ is totally semi-disconnected set and totally pre-disconnected set and hence $f^{-1}(y)$ is totally α-disconnected set ,then *f* is α- light map.

The following definition is given in [9].

V.7Definition: A space X is submaximal if every dense subset of X is open in X.

The following theorem is given in [3].

V.8Theorem: A space X is submaximal iff every pre-open set of X is open in X.

Now we give the following propositions .

V.9proposition: let $f:X\rightarrow Y$ be pre-light map and X is submaximal space .Then f is light map.

Proof: Since f is pre-light map the fiber $f^{-1}(y)$ is totally pre-disconnected set in X for all $y\in Y$ then there exist a pre-disconnection U/V of $f^{-1}(y)$ in X where U and V are pre-open set in X and since X is submaximal space then U and V are open set in X and U/V be a disconnection of $f^{-1}(y)$ in X and hence $f^{1}(y)$ is totally disconnected set in X and hence f is light map.

V.10poposition: let $f:X\rightarrow Y$ be pre-light map and X is submaximal space .Then f is α-light map.

Proof: It easy from (V.10) and (V.1).

The following definition is given in [9].

V.11Definition: A space X is called extremely disconnected if the closure of each open set of X is open in X .

The following theorem is given in [1].

V.12Theorem: In a submaximal extremely disconnected space X all semi- open sets are open.

Now we give the following propositions.

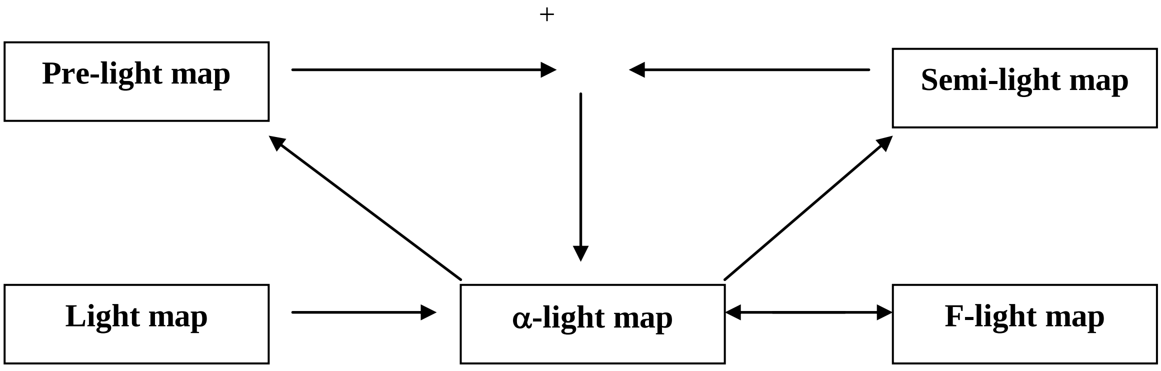
V.13proposetion: let $f:X\rightarrow Y$ be semi -light map and X is subaximal extremely space. Then f is light map.

Proof: Since f is semi-light map then $f^{-1}(y)$ is totally pre-disconnected set in X for all $y\in Y$ then there exist a semi-disconnection U/V of $f^{-1}(y)$ in X where U and V are semi -open sets in X , and since X is submaximal extremely disconnected space then U and V are open sets in X and hence U/V be a disconnection to $f^{-1}(y)$ in X then $f^{-1}(y)$ is totally disconnected set in X .then f is light map.

V.14proposition: let $f:X\rightarrow Y$ be semi -light map and X is submaximal extremely space. Then f is α- light map.

Proof: It easy from (V.15) and (V.1).

Now as a consequence of propositions (II.1,V.1,V.2,V.3,V.4,and V.6) we have the following diagram:



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