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GEOMETRIC IMAGE SCALING USING FRACTAL TECHNIQUE

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الخلاصة:-

اقترحنا في هذا البحث طريقة لتغيير حجوم الصور باستخدام تقنية الكسوريات بدلا من الطريقة التقليدية لكي نحصل على خوارزميات أكثر عمومية و بساطة و سرعة حيث طورنا إجراء تنفيذي سهل و سريع باستخدام نظام الدوال التكرارية و الذي يستخدم بصورة واسعة في ضغط بيانات الصور و أظهرت التجارب أن النموذج المقترح قد أعطى نتائج مقبولة مقارنة بطريقة الاستكمال القياسية.

Abstract:-

In this paper, we propose a method to re-size images using fractal technique instead of the traditional one in order to obtain faster, simpler, and more general algorithms. We develop a simple and practical implementation procedure that uses the iteration function system (***IFS***) which widely used in image compression. Experiments shows that the proposed algorithm gives an acceptable results compared with the standard interpolation method.

Introduction:-

Image scaling (magnification and reduction) is an important operation in digital image processing. For example, resolution conversion is required on a routine basis for medical imaging, multimedia, and digital hotography. The standard interpolation approach is to fit the digital image with a continuous model and resample this function on a new sampling grid [1]. Nearest neighbor and bilinear interpolation are simplest and fastest, but they produces images, which are either blocky or over-smoothed. However, these interpolation methods are sub-optimal since they are not designed to minimize information loss. In this paper, we will adopt new approach of image scaling using fractal technique approach by generating the iteration function system (***IFS***) codes from the data of the image and from the ***(IFS***); codes can generate any desired size of the image.

1.1 The Iteration Function System (*IFS*):-

Barnsly and Demo [2] have introduced iterated function systems as a Unified way for generating and classifying a broad class of fractals. This method replaced the traditional process, which was based on implementing successive Microscopic refinements. The ***IFS*** theory, introduced in [3], provide a new ruler through which we can describe any complicated geometrical shape in terms of another simple one; e.g., an architect can describe a cloud simply as a house, which may Defined. The ***IFS***-theory, in fact, concerns deterministic Geometry, which is an extension to the classical geometry. It uses classical Geometrical entities to express relation between parts of generalized geometrical Objects [4].

Generally, iterated function system (***IFS***), which is consists of a collection of contractive transformations {i.e. $W_i : R^2 \rightarrow R^2 | i = 1........, n|$ } which map the plan R^2 to itself. This collection of transformations defines a map

$$W(\bullet) = \bigcup_{i=1}^{n} W_i(\bullet) \quad .....(1)$$

The map *W* is not applied to the plan , but to sets which are collections of points In the plan; e.g. given an input set S, w(S) can be computed for each *i* (corresponds for making reduced copy of the input image S). Taking the union of these sets (this corresponds to assembling the reduced copies), and get a new set *W(S)* (the output of the copier). So *W* is a map on its space of subsets of the plan, the subset of the plane can be called an image , because the set defines an image when the points in the set are drawn in black, and because later we will want to use the same notation for graphs of the functions representing actual images or pictures.
There are two important facts should mentioned, these are;

- When the *w* is contractive in the plane, then *W* is contractive in the space of (closed and bounded) subset of the plane.
- If we are given a contractive map *W* in a space of images, then there is a Special image, called attractor and dented by x_w with the following Properties;

When the copy machine process is applied to the attractor, the output is equal to the: the image is fixed, and the attractor ***x*** is called the fixed point of ***W***, that is

$$W(x_w) = x_w = w_1(x_w) \bigcup w_2(x_w) \bigcup ... \bigcup w_n(x_w) \quad ....(2)$$

Given an input image so, by running the copy machine process once to get

$S_1 = w(S_0)$, twice to get $S_2 = w(S_1) = w(w(S_0)) = w^{02}(S_0)$ and so on, the superscript "0" indicates iterations not exponent; i.e. w^{02} is output of the second iteration. The attractor, which is the result of running the coping the coping machine in a feedback loop is limit set.

$$x_w = S_\infty = \lim_{n\rightarrow\infty} W^{0n}(S_0) \quad ....(3)$$

This is not dependent on the choice of the initial image.

3. x_w is unique; if *S* is any set and an image transformation *W* satisfies *W(S)=S*. in this case *S* is the attractor of *W*:i.e. *S=W(S)* This means that Only one set will satisfy the fixed point in property 1 above.

These three properties, however, are known as ***The Contractive Mapping Fixed Point Theorem*** [4].

1.2*IFS*-Code:-

Mathematical form of the affine transformation can be represented by:

$$w\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}a_{12} \\ a_{21}a_{22} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \text{... (4)}$$

Where the coefficients a_{11}, a_{12}, a_{21}, a_{22}, b_1, and b_2 are real numbers; each of its coefficients can skew, Stretch, rotate, scale and translate an input image [5].

An affine transformation $w : R^2 \rightarrow R^2$ in two-Dimensional space R^2 , where a_{ij}` s and $b_{ij}$` s are real constants and called the *IFS* code. In matrix form , if *A* denotes matrix { a_{ij} },*B* denotes (b_1,b_2) , and X Denotes the vector (x_1 ,x_2), then in matrix form:

$$W(x) = AX+B \quad \ldots(5)$$

Once a positive number *s* is defined, if an affine transformation satisfies;

$$d((w(x),w(y)) \leq s\, d(x,y) \quad \ldots(6)$$

Note that

$$d(x) = \sqrt{x_1^2 + x_2^2} \quad \ldots.(7)$$

The smallest number of *s* satisfying the formula (6) called Lipschitz Constant for *W* . The affine transformation can be called contractive; if s < 1 , symmetry; if s = 1 and expansive ; if s > 1 . in fact, the *IFS* code is a set of *N* affine transformations given by;

$$\boldsymbol{IFS} = \{w_1, w_2, ..., w_N\}$$

1.3 Fractal Functions:-

Let us consider functions defined on particular sub- attractors of iterated function systems. In the graphical case, these sub- attractors will correspond to pixels. Let K be a complete normal vector space and $W_1 \ldots\ldots\ldots W_N$ be an ***IFS*** of order *N* with attractor $A \subset K$ let m be nonnegative integer , termed resolution . Therefore, the following set can be considered;

$$P_m = \{ A_{k1\ldots\ldots km} : K_1 \ldots K_m = 1 \ldots N \} \quad \ldots (8)$$

Where $A_{k1\ldots\ldots km}$ denotes W_{k1} o…… W $_{k\,m}$ (A), and "A" is any gray – scale function given as $f : p_m \rightarrow R$.

However , the space of all gray – scale functions , at resolution "m" can be denoted by F_m . Moreover , the fractal function can be defined in a similar way to IFS, by introducing a contraction mapping on F_m , using mapping of the form

$$Mf\left(A_{k1\ldots km}\right) = \left(c_{k1}x + t_{k1}\right)dx + s_{k1}\sum_{i=1}^{N} f\left(A_{k2\ldots kmi}\right)\ldots(9)$$

Where , s_k and t_k represent the parameters of the mapping , which are real constants . c_k Is an element of K (i.e. of the vector space).

Graphically, eq. (5) may be interpreted as to be a new pixel 's value depending on the current values of the pixels mapped by the IFS and their positions . It should be noted that " M" is affine in f . The least – square approximation , with respect to the parameters c_k, s_k and t_k , can be defined , in the Euclidean metric , on F_m , as follows ;

Definition 1.1

Let W_1 …….. W_N , A , m , F_m as above , the Euclidean metric resolution " m " is defined by

$$d(f,g) = \sqrt{\sum_{k1\ldots.km}^{N}\left[\left(f - g\left(A_{k1\ldots.km}\right)\right)\right]^2}$$, the two grey scale function f.g ∈ F_m

Theorem 1.1

Let M be a mapping on F_m , of the above form , defined for *IFS* $w_1 \ldots\ldots w_N$

If $\sum_{k=1}^{N} |s_k| \langle 1$, then M is eventually contractive in the Euclidian metric at any resolution

A gray – scale mapping that is eventually contractive in all Euclidean metrics and at all resolutions is termed attractive . Practically , it has been found that ; the condition on M , described above , is sufficient to make the codes obtained for image blocks , almost always attractive . By the ***Contractive Mapping Fixed – point Theorem***, there is a unique invariant function for each M, and the recursive definition of this function implies a fractal characters are similar to that for the attractor of an *IFS* . In fact , under a more restrictive contractivity condition , the parameters defining a fractal function may be used to construct an *IFS* in a higher – dimensional space , whose attractor can be represented as a fractal " graph " The Collage Theorem [3] may now be applied ;

$$d_m(f,g) \le (1-s)^{-1} \; d_m(g, Mg) \quad ,...(10)$$

Where g is a given scale – function , and f is the fixed point of the mapping M . This theorem can now be applied to encode an image block [10].

1.4 Least – Squares Approximation :-

Here we shall describe how fractal code for a given gray – scale function is obtained , before applying the method to the image blocks . If a function $g \in F_m$ is given on the set P_m of attractor A , then we can assume that ; an IFS W is known; whose attractor is A. The problem is then how to find the gray – scale mapping M whose invariant function f best approximates g. This can be performed by applying the least – squares approach at resolution m , so that M is sought to minimize $d_m(f, g)$.

From Collage Theorem, a fractal approximated g may be found by minimizing $d_m(g,Mg)$ with respect to the parameters of M . This does not guarantee the best possible fractal match of the upper bound for $d_m(f, g)$ is minimized , and one assumes that the contractivity factor for M (i.e."s") does not significantly affect the bound . Furthermore, if M is only eventually contractive , then it can, also , assume that M will minimize $d_m(g, M''g)$ for suitably large n , which is not necessary true .

In this case we have K = R and the parameter c_k in the grey – scale mapping is a Cartesian pair (a_k, b_k) . We can now minimize $d_m(g, Mg)$ by minimizing $d_m(Mg, g)^2$, using standard calculus [7] .

$$d_m(Mg,g)^2 = \sum_{k1...km}^{N} \left[(Mg-g)(A_{k1...km})\right]^2 \quad ...(11)$$

$$= \sum_{k1\ldots km}^{N} \left[\int a_{k1}x + b_{k1} + t_{k1} dxdy + s_{k1} \sum_{i=1}^{N} g(A_{k2\ldots kmi}) - g(A_{k2\ldots mi}) \right]^2$$

We can economize on by letting $v(p) = \sum_{i=1}^{N} g(A_{k2\ldots kmi})$ and $k= k_1$ When $p = A_{k1\ldots\ldots km}$

Then

$$d_m(Mg, g)^2 = \sum [\int p\,(a_k x - + b_k y + t_k)\,dxdy + s_k v(p) - g(p)]^2$$

$$= \sum_{K=1}^{N} \sum_{p \in w_k(p_m - 1)} [\int p(a_k x + b_k y + t_k) dxdy + s_k v(p) - g(p)]^2$$

This may be minimized by minimizing

$$E = \sum_{p \in w_k(p_m - 1)} [\int p(a_k x + b_k y + t_k)\, dxdy + s_k v(p) - g(p)]^2 \quad \ldots(12)$$

This will result in

$$\frac{\partial E}{\partial a_k} = 0$$

$$= \Sigma_p 2\left[\int_p (a_k x + b_k y + t_k) dxdy + s_k v(p) - g(p)\right] \int_p x = 0$$

$$= \left[(\Sigma_p a_k (\int_p x)^2 + \Sigma_p b_k \int_p x \int_p y + \Sigma_p t_k \int_p x \int_p 1) dxdy + \Sigma_p s_k v(p) \int_p x \right] = g(p) \int_p x$$

Similarly, the Same above optimization method can be carried out for b_k , t_k, and s_k, (i. e. for each k) , because the sums are independent for a non-overlapping attractor, we shall have a system in the following matrix form ;

$$\begin{bmatrix} \Sigma_p (\int_p x)^2 & \Sigma_p \int_p x \int_p y & \Sigma_p \int_p x \int_p 1 & \Sigma_p v(p) \int_p x \\ \Sigma_p \int_p x \int_p y & \Sigma_p (\int_p y)^2 & \Sigma_p \int_p y \int_p 1 & \Sigma_p v(p) \int_p y \\ \Sigma_p \int_p x \int_p 1 & \Sigma_p \int_p y \int_p 1 & \Sigma_p (\int_p 1)^2 & \Sigma_p v(p) \int_p 1 \\ \Sigma_p v(p) \int_p x & \Sigma_p v(p) \int_p y & \Sigma_p v(p) \int_p 1 & \Sigma_p (v(p))^2 \end{bmatrix} \begin{bmatrix} a_k \\ b_k \\ t_k \\ s_k \end{bmatrix} = \begin{bmatrix} \Sigma_p g(p) \int_p x \\ \Sigma_p g(p) \int_p y \\ \Sigma_p g(p) \int_p 1 \\ \Sigma_p g(p) v(p) \end{bmatrix} \quad \ldots(13)$$

This system can be solved (for individual k) to obtain the approximating gray – scale . Note; all the sums are for P ∈ W_k (P_{m-1}). If we , now, consider a square images block of side R pixels , where R is a power of 2 , and the coordinates of the pixels in the block are normalized so that the block is represented by a square centered at the origin , with a corner at (R/2, R/2) . Then A is a square ; each $A_k = W_k(A)$ matches a quadrant of the block and $A_{k1 \ldots km}$ matches an individual pixel , where $m = \log_2(R)$. In fact, if M is minimized at resolution $1og_2(R)$, this means that we have performed a least square optimization at the sampling resolution(i. e. a pixel size) [3].

Where: a_k, b_k, t_k and s_k are the *IFS*-codes

$\Sigma_p (\int_p x)^2$ is the sum of the square of x-coordinate of each 2x2 sub squares of the image block.
$\Sigma_p (\int_p y)^2$ is the sum of the square of y-coordinate of each 2x2 sub squares of the image block.
$\Sigma_p (\int_p 1)^2$ is the sum of the square of the pixel in each quadrant of the image block .
$\Sigma_p v(p)$ is the sum of the pixels in each 2x2 sub squares of the image block .
$\Sigma_p g(p)$ is the sum of the pixels of the image block.

2. Image Scaling using Fractal Technique:-

To rescale an image using fractal technique you can follow this algorithm that based on the ideas mentioned in this paper

1. ***partition the original image into square blocks***
2. ***choose variance tolerance e_t***
3. ***compute the variance e of each block***
4. ***if e <= e_t then use a flag-code (i.e. 0-value) refers that the next value is mean of the coded block else; using equation (13) find the IFS –codes i.e.(a_k,b_k,t_k and s_k) for each block by using lower-upper method.***
5. ***Choose if you want to reduce or enlarge the size of the image.(i.e.) If you want to double the size of it then open a matrix of size twice the size of the original image and vice versa when reducing the size of the image to one-half of the original size .***
6. ***making use of the IFS-codes fill the new size of the image by the decoding data***

3. Results:-

we use Lenna image of size 256x256 pixels that illustrated in figure to examine the approach of image scaling i.e. as in figure (2) which enlarge the original image to twice of the original size with coding block size of 4x4 pixels and reducing the image to one-half of its original size i.e. 128x128 pixels as in figure (4) while in figure (3) use coding block of size 8x8

Figure (1): original image of size 256x256 pixels

Figure (2): Resized image twice 512x512 with coding block 4x4 pixels

Figure (3): Resized image twice 512x512 with coding block size 8x8 pixels

Figure (4): Reducing image to 128x128 with coding block size 8x8

4. Conclusion:-

The objective of this paper is to introduce a new technique for image scaling using fractal approach. In fact there are several methods used in image scaling. Our presented method use the least square approximation approach to compute the IFS-codes.

It can be observed that when the block size is small the results is relatively better than when we choose larger block size to calculate the IFS-codes using least square approximation method because the small block can be covered better than large ones because contiguous pixels in an image tend to be highly correlated and accumulated errors arises when the size of the block is increases that is the blockness appear obviously at size 8x8 sub squares blocks.

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