ON CENTRALIZERS ON SOME GAMMA RING

Rajaa C .Shaheen
Department of Mathematics, College of Education,
University of Al-Qadisiya, Al-Qadisiya, Iraq.

الخلاصة العربية: قدمنا في هذا البحث دراسة حول تطبيق جوردان المركزي على بعض الحلقات

ABSTRACT

Let M be a 2-torsion free Γ-ring satisfies the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$. In section one ,we prove if M be a completely prime Γ-ring and $T:M\to M$ an additive mapping such that $T(a\alpha a)=T(a)\alpha a$ (resp., $T(a\alpha a)=a\alpha T(a)$)holds for all $a\in M, \alpha\in\Gamma$.Then T is a left centralizer or M is commutative (res.,a right centralizer or M is commutative) and so every Jordan centralizer on completely prime Γ-ring M is a centralizer .In section two ,we prove this problem but by another way. In section three we prove that every Jordan left centralizer(resp., every Jordan right centralizer) on Γ-ring has a commutator right non-zero divisor(resp., on Γ-ring has a commutator left non-zero divisor)is a left centralizer(resp., is a right centralizer) and so we prove that every Jordan centralizer on Γ-ring has a commutator non –zero divisor is a centralizer .

Key wards : Γ-ring, prime Γ-ring,semi-prime Γ-ring , left centralizer, Right centralizer, centralizer, Jordan centralizer.

1-INTRODUCTION

Throughout this paper,M will represent Γ-ring with center Z .In [7] B.Zalar proved that any left (resp.,right)Jordan centralizer on a 2-torsion free semi-prime ring is a left (resp.,right)Centralizer.In [3] authors prove the same question on the condition that R has a commutator right (resp., left) non- zero divisor .And J.Vukman in [6] proved that if R is2-torsion free semi-prime ring and $T:R\to R$ be an additive

mapping such that $2T(x^2)=T(x)x+xT(x)$ holds for all $x,y\in R$ .Then T is left and right centralizer.In this paper we define Jordan centralizer on Γ-ring and we show that the existence of a non-zero Jordan centralizer T on a 2-torsion free completely prime Γ-ring M which satisfies the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$ implies either T is centralizer or M is commutative Γ-ring.

Let M and Γ be additive abelian groups, M is called a Γ-ring if for any $x,y,z\in M$ and α, $\beta\in\Gamma$,the following conditions are satisfied

(1)$x\alpha y\in M$

(2)$(x+y)\alpha z=x\alpha z+y\alpha z$

$x(\alpha+\beta)z=x\alpha z+x\beta z$

$x\alpha(y+z)=x\alpha y+x\alpha z$

(3)$(x\alpha y)\beta z=x\alpha(y\beta z)$

The notion of Γ-ring was introduced by Nobusawa[5] and generalized by Barnes[1],many properties of Γ-ring were obtained by many research such as [2]

Let A,B be subsets of a Γ-ringM and Λ a subset of Γ we denote $A\Lambda B$ the subset of M consisting of all finite sum of the form $\sum a_i\lambda_i b_i$ where $a_i\in A, b_i\in B$ and $\lambda_i\in\Lambda$.Aright ideal(resp.,left ideal) of a Γ-ring M is an additive subgroup I of M such that $I\Gamma M\subset I$(resp.,$M\Gamma I\subset I$).If I is a right and left ideal inM,then we say that I is an ideal .M is called a 2-torsion free if 2x=0 implies x=0 for all $x\in M$.A Γ-ringM is called prime if $a\Gamma M\Gamma b=0$ implies a=0 or b=0 and M is called completely prime if $a\Gamma b=0$ implies a=0 or b=0($a,b\in M$),Since $a\Gamma b\Gamma a\Gamma b\subset a\Gamma M\Gamma b$,then every completely prime Γ-ring is prime.A Γ-ring M is called semi-prime if $a\Gamma M\Gamma a=0$ implies a=0and M is called completely semi-prime if $a\Gamma a=0$ implies a=0($a\in M$)

Let R be a ring,an additive mapping $D:R\rightarrow R$ is called derivation if $D(xy)=D(x)y+xD(y)$ holds for all $x,y\in R$.A left(right) centralizer of R is an additive mapping $T:R\rightarrow R$ which satisfies $T(xy)=T(x)y$ ($T(xy)=xT(y)$) ***for all $x,y\in R$.A Jordan centralizer be an additive mapping T which satisfies $T(x\circ y)=T(x)\circ y=x\circ T(y)$.***

A Centralizer of R is an additive which is both left and right centralizer.An easy computation shows that every centralizer is also a Jordan centralizer.Many Papers work about the problem every Jordan centralizer be centralizer such as in[7] .In this paper ,we work this problem on some kind of Γ-ring.

Now ,we shall give the following definition which are basic in this paper.

<u>Definition1.1</u>:-Let M be a Γ-ring and let D:M → M be an additive map,D is called a Derivation if for any a,b $\in$M and $\alpha \in \Gamma$,if the following condition satisfy

$D(a\alpha b)=D(a)\alpha b+a\alpha D(b)$

<u>Definition1.2</u>:- Let M be a Γ-ring and let T:M → M be an additive map ,T is called <u>Left centralizer</u> of M,if for any a,b $\in$M and $\alpha \in \Gamma$, the following condition satisfy $T(a\alpha b)=T(a)\alpha b$,

<u>Right centralizer</u> of M,if for any a,b $\in$M and $\alpha \in \Gamma$, the following condition satisfy

$T(a\alpha b)=a\alpha T(b)$,

<u>Jordan left centralizer</u> if for all a$\in$M and $\alpha \in \Gamma$, the following condition satisfy

$T(a\alpha a)=T(a)\alpha a$

<u>Jordan Right centralizer</u> if for all a$\in$M and $\alpha \in \Gamma$, the following condition satisfy

$T(a\alpha a)= a\alpha T(a)$

<u>Jordan centralizer</u> of M,if for any a,b $\in$M and $\alpha \in \Gamma$, the following condition satisfy $T(a\alpha b+b\alpha a)=T(a)\alpha b+b\alpha T(a)=a\alpha T(b)+T(b)\alpha a$

A centralizer of M is an additive mapping which is both left and right centralizer.An easy computation shows that every centralizer is also a Jordan centralizer but the converse is not true .In this paper we prove this problem when M is 2-torsion free completely prime Γ-ring.Now we shall prove the following Lemmas which are necessarily to prove our main result in this paper.

<u>Lemma 1.3</u>:-Let M be a 2-torsion free Γ-ring and let T:M → M be an additive mapping which satisfies $T(a\alpha a)=T(a)$

α a,(resp., T(a α a)= a α T(a)) for all a $\in$ M and $\alpha \in \Gamma$,then the following statement holds for all a,b,c $\in$ M and α, $\beta \in \Gamma$,

(i) $T(a\alpha b+b\alpha a)=T(a)\alpha b+T(b)\alpha a$
(resp., $T(a\alpha b+b\alpha a)=a\alpha T(b)+b\alpha T(a)$)

(ii) Especially if M is 2-torsion free and $a\alpha b\beta c=a\beta b\alpha c$ for all a,b,c $\in$ M and $\alpha,\beta \in \Gamma$ then
$T(a\alpha b\beta a)=T(a)\alpha b\beta a$ (resp., $T(a\alpha b\beta a)=a\alpha b\beta T(a)$)

(iii) $T(a\alpha b\beta c+c\alpha b\beta a)=T(a)\alpha b\beta c+T(c)\alpha b\beta a$.
(resp., $T(a\alpha b\beta c+c\alpha b\beta a)=a\alpha b\beta T(c)+c\alpha b\beta T(a)$)

<u>Proof</u>:-(i) Since $T(a\alpha a)=T(a)\alpha a$ for all a $\in$ M and $\alpha \in \Gamma$,…….(1)

Replace a by a+b in (1),we get

$T(a\alpha b+b\alpha a)=T(a)\alpha b+T(b)\alpha a$……………………………….(2)

(ii) by replacing b by $a\beta b+b\beta a$, $\beta \in \Gamma$

$$W=T(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)$$
$$=T(a)\alpha(a\beta b+b\beta a)+T(a\beta b+b\beta a)\alpha a$$
$$=T(a)\alpha(a\beta b)+T(a)\alpha(b\beta a)+(T(a)\beta b+T(b)\beta a)\alpha a$$
$$=T(a)\alpha(a\beta b)+T(a)\alpha(b\beta a)+T(a)\beta b\alpha a+T(b)\beta a\alpha a$$

Since $a\alpha b\beta c=a\beta b\alpha c$,then

$$W=T(a)\alpha(a\beta b)+2T(a)\alpha(b\beta a)+T(b)\beta a\alpha a$$

On the other hand

$$W=T(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)$$
$$=T(a\alpha(a\beta b)+a\alpha(b\beta a)+(a\beta b)\alpha a+(b\beta a)\alpha a$$
$$=(a\alpha a\beta b+b\beta a\alpha a)+2T(a\alpha b\beta a)$$

By comparing these two expression of W, we get

$$2T(a\alpha b\beta a)=2T(a)\alpha b\beta a$$

Since M is 2-torsion free ,then

$T(a\alpha b\beta a)=T(a)\alpha b\beta a$……………………………….(3)

(iii)In (3) replace a by a+c,to get

$T(a\alpha b\beta c+c\alpha b\beta a)=T(a)\alpha b\beta c+T(c)\alpha b\beta a$………………(4)

<u>Theorem 1.4:</u>- Let M be a 2-torsion free completely prime Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all

$x,y,z\in M$, α, $\beta\in\Gamma$, and let $T:M\rightarrow M$ be an additive mapping which satisfies $T(a\alpha a)=T(a)\alpha a$,for all $a\in M$ and $\alpha\in\Gamma$,then $T(a\alpha b)=T(a)\alpha b$, for all $a,b\in M$ and $\alpha\in\Gamma$ or M is commutative Γ-ring.

<u>Proof</u>:-By [Lemma 1.3,iii] ,we have

$$T(a\alpha b\beta c+c\alpha b\beta a)=T(a)\alpha b\beta c+T(c)\alpha b\beta a$$

Replace c by $a\alpha b$

$$W=T(a\alpha b\beta(a\alpha b)+(a\alpha b)\alpha b\beta a)$$
$$=T(a)\alpha b\beta a\alpha b+T(a\alpha b)\alpha b\beta a$$

On the other hand

$$W=T((a\alpha b)\beta(a\alpha b)+a\alpha(b\alpha b)\beta a)$$
$$=T(a\alpha b)\beta a\alpha b+T(a)\alpha b\alpha b\beta a$$

By comparing these two expression of W, we get

$$T(a\alpha b)\beta(a\alpha b-b\alpha a)+T(a)\alpha b\beta(b\alpha a-a\alpha b)=0$$
$$T(a\alpha b)\beta(a\alpha b-b\alpha a)-T(a)\alpha b\beta(a\alpha b-b\alpha a)=0$$
$$(T(a\alpha b)-T(a)\alpha b)\beta(a\alpha b-b\alpha a)=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Since M is completely prime Γ-ring ,then either $T(a\alpha b)-T(a)\alpha b=0$ or $a\alpha b-b\alpha a=0$ if $T(a\alpha b)-T(a)\alpha b=0$then $T(a\alpha b)=T(a)\alpha b$ and if $a\alpha b-b\alpha a=0$ for all a, $b\in M$ and $\alpha\in\Gamma$,then M is commutative Γ-ring◙

<u>Theorem 1.5:</u>- Let M be a 2-torsion free completely prime Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, α, $\beta\in\Gamma$, and and let $T:M\rightarrow M$ be an additive mapping which satisfies $T(a\alpha a)=a\alpha T(a)$for all $a\in M$ and $\alpha\in\Gamma$,then $T(a\alpha b)=a\alpha T(b)$ for all $a,b\in M$ and $\alpha\in\Gamma$ or M is commutative Γ-ring.

<u>Proof</u>:- From[Lemma 1.3,iii],we have for all $a,b,c\in M$ and α, $\beta\in\Gamma$,

$$T(a\alpha b\beta c+c\alpha b\beta a)=a\alpha b\beta T(c)+c\alpha b\beta T(a)\ldots\ldots\ldots\ldots\ldots(6)$$

In (6) replace c by $b\alpha a$, then

$$W=T(a\alpha b\beta(b\alpha a)+(b\alpha a)\alpha b\beta a)$$
$$=a\alpha b\beta T(b\alpha a)+b\alpha a\beta b\alpha T(a)$$

on the other hand

$$W=T(a\alpha(b\beta b)\alpha a+(b\alpha a)\alpha(b\beta a))$$
$$=a\alpha b\beta b\alpha T(a)+b\alpha a\beta T(b\alpha a)$$

by comparing these two expression of W ,we get
$a\alpha b\beta(T(b\alpha a)-b\alpha T(a)) - b\alpha a\beta(T(b\alpha a)-b\alpha T(a))=0$
$(a\alpha b-b\alpha a)\beta(T(b\alpha a)-$
$b\alpha T(a))=0$……………………………..(7)
since M is completely prime Γ-ring,then
either $(T(b\alpha a)-b\alpha T(a))=0 \Rightarrow T(b\alpha a)=b\alpha T(a)$
or $a\alpha b-b\alpha a=0 \Rightarrow a\alpha b=b\alpha a \Rightarrow$M is commutative Γ-ring◙
<u>Corrolary 1.6</u>:- Every Jordan centralizer of 2-torsion free completely prime Γ-ring M which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, $\alpha, \beta\in\Gamma$, is a centralizer on M.

<u>2-The second result</u>

In this section we again divided the proof in few lemmas.
<u>Lemma2.1</u>:- Let M be a semi-prime Γ-ring and D a derivation of M and $a\in M$ some fixed element.
(i)$D(x)\alpha D(y)=0$ for all $x,y\in M$, $\alpha\in\Gamma$ implies that D=0 on M
(ii) $a\alpha x-x\alpha a\in Z$, for all $x\in M$, $\alpha\in\Gamma$ implies that $a\in Z$.
<u>Proof</u>:-
(i) since $D(x)\alpha D(y)=0$ for all $x,y\in M$, $\alpha\in\Gamma$.
and $D(y\alpha x)=D(y)\alpha x+y\alpha D(x)$
and so $D(x)\alpha D(y\alpha x)=0$,then
$D(x)\alpha D(y)\alpha x+D(x)\alpha y\alpha D(x)=0$
since $D(x)\alpha D(y)=0$,then
$D(x)\alpha y\alpha D(x)=0$ for all $x,y\in M$, $\alpha\in\Gamma$
And since M be a semi-prime Γ-ring ,then
$D(x)=0$ for all $x\in M$.
(ii)define $D(x)=a\alpha x-x\alpha a$
it is easy to see that D is derivation on M
since $D(x)\in Z$ for all $x\in M$,we have
$D(y)\alpha x=x\alpha D(y)$…………………………………..(8)
Replace y by $y\alpha z$ in (8)
$D(y\alpha z)\alpha x=x\alpha D(y\alpha z)$
$D(y)\alpha z\alpha x+y\alpha D(z)\alpha x=x\alpha D(y)\alpha z+x\alpha y\alpha D(z)$
$D(y)\alpha(z\alpha x-x\alpha z)=D(z)\alpha(x\alpha y-y\alpha x)$
Now, take z=a, then it is easy to see that D(a)=0,so
$D(y)\alpha(a\alpha x-x\alpha a)=0$
$D(y)\alpha D(x)=0$,then from (i),we get D=0 and hence $a\in Z$◙

***Lemma 2.2**:- Let M be a semi-prime Γ-ring and $a \in M$ some fixed element.*
If $T(x)= a \alpha x + x \alpha a$, for all $x \in M, \alpha \in \Gamma$ is a Jordan centralizer ,then $a \in Z$

***Proof**:-from [definition 1.2]*

$T(x \alpha y+y \alpha x)=T(x) \alpha y+y \alpha T(x)$

Gives us

$T(x \alpha y)+T(y \alpha x)=T(x) \alpha y+y \alpha T(x)$

$a \alpha x \alpha y + a \alpha y \alpha x+x \alpha y \alpha a+y \alpha x \alpha a=$

$=(a \alpha x +x \alpha a) \alpha y+y \alpha (a \alpha x +x \alpha a)$

$= a \alpha x \alpha y +x \alpha a \alpha y+ y \alpha a \alpha x +y \alpha x \alpha a$

Then

$a \alpha y \alpha x-x \alpha a \alpha y+x \alpha y \alpha a-y \alpha a \alpha x =0$

$(a \alpha y-y \alpha a) \alpha x-x \alpha (a \alpha y-y \alpha a)=0$ *for all* $x,y \in M, \alpha \in \Gamma$

Then $a \alpha y-y \alpha a \in Z$ and so by [Lemma 2.1,ii],we get $a \in Z$.

***Lemma 2.3**:- Let M be a semi-prime Γ-ring ,then every Jordan centralizers of M maps from Z into Z.*

***Proof**:-take any $c \in Z$ and denote $a=t(c)$*

$2T(c \alpha x)=T(c \alpha x+x \alpha c)$

$= T(c) \alpha x+x \alpha T(c) =a \alpha x +x \alpha a$

Then $S(x)=2T(c \alpha x)$ is also a Jordan centralizer ,by[lemma 2.2],we get $a \in Z$.

Then $T(c) \in Z$◙

***Lemma 2.4**:- Let M be a semi-prime Γ-ring and $a ,b \in M$ two fixed elements.*
If $a \alpha x=x \alpha b$ for all $x \in M, \alpha \in \Gamma$ then $a=b \in Z$.

***Proof**:-Since $x \alpha b= a \alpha x$*

Replace x by $x \alpha y$

$x \alpha y \alpha b=a \alpha x \alpha y$

$x \alpha y \alpha b=x \alpha b \alpha y$

$x \alpha (y \alpha b-b \alpha y)=0$*,and so*

$(y \alpha b-b \alpha y)x \alpha (y \alpha b-b \alpha y)=0$

Since M is semi-prime Γ-ring, then

$(y \alpha b-b \alpha y)=0$

$y \alpha b=b \alpha y$ *for all* $y \in M$*, then* $b \in Z$

since $a \alpha x=x \alpha b=b \alpha x$

it is easy to see that

$(a-b)\alpha x=0$ for all $x\in M$

and $(a-b)\alpha x\alpha(a-b)=0$ for all $x\in M$

again since M is semi-prime Γ-ring then $a-b=0\Rightarrow a=b\in Z$◙

<u>Proposition 2.5</u>:-everyJordan centeralizerof 2-torsion free completely prime Γ-ringM is a centralizer.

<u>Proof</u>:-Let T be a Jordan centeralizer,i.e

$T(x\alpha y+y\alpha x)=T(x)\alpha y+y\alpha T(x)=x\alpha T(y)+T(y)\alpha x$

If we replace y by $x\alpha y+y\alpha x$, then the left side

$W= T(x\alpha(x\alpha y+y\alpha x) +(x\alpha y+y\alpha x)\alpha x)$

$=T(x)\alpha(x\alpha y+y\alpha x)+(x\alpha y+y\alpha x)\alpha T(x)$

$= T(x)\alpha(x\alpha y)+ T(x)\alpha y\alpha x+x\alpha y\alpha T(x)+ y\alpha x\alpha T(x)$

and the right side

$W=x\alpha T(x\alpha y+y\alpha x)+T(x\alpha y+y\alpha x)\alpha x$

$=x\alpha T(x)\alpha y+x\alpha y\alpha T(x)+T(x)\alpha y\alpha x+y\alpha T(x)\alpha x$

Then

$T(x)\alpha x\alpha y+ y\alpha x\alpha T(x) -x\alpha T(x)\alpha y-y\alpha T(x)\alpha x=0$

$(T(x)\alpha x -x\alpha T(x))\alpha y +y\alpha(x\alpha T(x)- T(x)\alpha x)=0$

Then

$(T(x)\alpha x -x\alpha T(x))\alpha y=y\alpha(T(x)\alpha x -x\alpha T(x))$ ***for all*** $x,y\in M,\ \alpha\in\Gamma$.

And so $(T(x)\alpha x -x\alpha T(x))\in Z$

then we must prove that

$T(x)\alpha x -x\alpha T(x)=0$

Take any $c\in Z$

$2T(c\alpha x)=T(c\alpha x+x\alpha c)$

$= T(c)\alpha x+x\alpha T(c)$

$=2T(x)\alpha c$

Using[Lemma 2.3]and since M is 2-torsion free Γ- ring

$T(c\alpha x)= T(x)\alpha c= T(c)\alpha x$

$(T(x)\alpha x-x\alpha T(x))\alpha c= T(x)\alpha x\alpha c-x\alpha T(x)\alpha c$

$=T(c)\alpha x\alpha x-x\alpha T(c)\alpha x=0$

then $(T(x)\alpha x-x\alpha T(x))\alpha c\alpha(T(x)\alpha x-x\alpha T(x)) =0$

since M is semi-prime Γ- ring ,then $T(x)\alpha x-x\alpha T(x) =0$

$2T(x\alpha x)=T(x\alpha x+x\alpha x) =T(x)\alpha x+x\alpha T(x)$

$=2T(x)\alpha x =2x\alpha T(x)$

Since M is 2-torsion free ,then

$T(x\alpha x)= T(x)\alpha x=x\alpha T(x)$

And so by [Theorem 1.4,Theorem1.5],we get the result.

3-JORDAN CENTRALIZERS ON SOME GAMMA RING

Theorem 3.1:- Let M be a 2-torsion free Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, $\alpha, \beta\in\Gamma$ and has a commutator right non-zero divisor and let $T:M\to M$ be an additive mapping which satisfies
$T(a\alpha a)= T(a)\alpha a$ for all $a\in M$ and $\alpha\in\Gamma$, then $T(a\alpha b)= T(a)\alpha b$ for all $a,b\in M$ and $\alpha\in\Gamma$.

Proof:- from (5),we have
$(T(a\alpha b)- T(a)\alpha b)\beta(a\alpha b-b\alpha a)=0$
if we suppose that
$\delta(a,b)= T(a\alpha b)- T(a)\alpha b$ and $[a,b]= a\alpha b-b\alpha a$
then $\delta(a,b)\beta[a,b]=0$ ***for all*** $a,b\in M$ ***and*** $\alpha,\beta\in\Gamma$ ……………(9)
Since M has a commutator right non-zero divisor ,then
$\exists x,y\in M$, $\alpha\in\Gamma$ ***such that if for every*** $c\in M$, $\beta\in\Gamma$
$c\beta[x,y]=0\Rightarrow c=0$
by (9),we have $\delta(x,y)\beta[x,y]=0$ ***and so***
$\delta(x,y)=0$………………(10)
replace a by a+x
$\delta(a+x,b)\beta[a+x,b]=0$ ***and so by (9) and (10)***
$\delta(x,b)\beta[a,b]+\delta(a,b)\beta[x,b]=0$……………..(11)
Now replace b by b+y
$\delta(x,b+y)\beta[a,b+y]+\delta(a,b+y)\beta[x,b+y]=0$
and so by (10) and (11),we get
$\delta(x,b)\beta[a,y]+\delta(a,y)\beta[x,b]+\delta(a,b)\beta[x,y]+\delta(a,y)\beta[x,y]=0$
$\delta(a,b)\beta[x,y]+\delta(a,y)\beta[x,y]=0$
by (11),we get
$\delta(a,b)\beta[x,y]-\delta(x,y)\beta[a,y]=0$
then
$\delta(a,b)\beta[x,y]=0$, ***and so*** $\delta(a,b)=0$ ***for all*** $a,b\in M$ ***and*** $\alpha\in\Gamma$
$T(a\alpha b)= T(a)\alpha b\Rightarrow T$ ***is left centralizer of M.***

Theorem 3.2:- Let M be a 2-torsion free Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, $\alpha, \beta\in\Gamma$ and has a commutator left non-zero divisor and let $T:M\to M$ be an additive mapping which satisfies

$T(a\alpha a)= a\ \alpha\ T(a)$ for all $a\in M$ and $\alpha\in\Gamma$, then $T(a\alpha b)= a\ \alpha\ T(b)$ for all $a,b\in M$ and $\alpha\in\Gamma$.

<u>Proof</u>:- From[Lemma 1.3,iii],we have

$T(a\ \alpha b\ \beta c+c\ \alpha b\ \beta a)=a\ \alpha b\ \beta T(c)+c\ \alpha b\ \beta T(a)$……………(12)

In (12) replace c by $b\ \alpha a$,then

$W= T(a\ \alpha b\ \beta (b\ \alpha a) +(b\ \alpha a)\ \ \alpha b\ \beta a)$

$=a\alpha b\beta T(b\ \alpha a)+ b\ \alpha a\ \beta b\ \alpha T(a)$

on the other hand

$W= T(a\ \alpha (b\ \beta b\ )\alpha a +(b\ \alpha a)\ \ \alpha (b\ \beta a))$

$=a\alpha b\beta b\alpha T(a)+b\alpha a\beta T(b\alpha a)$

by comparing these two expression of W ,we get

$a\alpha b\beta (T(b\alpha a)-b\alpha T(a)) -b\alpha a\beta (T(b\alpha a)-b\alpha T(a))=0$

then if we suppose $B(b,a)= (T(b\alpha a)-b\alpha T(a))$

$[a,b]\ \beta\ B(b,a)= [a,b]\beta B(a,b)=0$ for all $a,b\in M$, α, $\beta\in\Gamma$ ……………………..(13)

Since M has a commutator left non-zero divisor then $\exists x,y\in M$, $\alpha\in\Gamma$ such that if for every $c\in M$, $\beta\in\Gamma$, $[x,y]\ \beta c=0\Rightarrow c=0$

then by (13),we have

$[x,y]\ \beta\ B(x,y)=0\Rightarrow B(x,y)=0$…………………(14)

in (13) replace a by a+x

$[a+x,b]\ \beta\ B(a+x,b)=0$

then by (13)

$[x,y]\ \beta\ B(a,b)+[a,b]\ \beta\ B(x,b)=0$…………………(15)

Now replace b by b+y

$[x,b+y]\ \beta\ B(a,b+y)+[a,b+y]\ \beta\ B(x,b+y)=0$

then by using (14) and (15),we get

$[x,y]\ \beta B(a,b)=0$

and since [x,y] is a commutator left non-zero divisor then $B(a,b)=0\Rightarrow T(a\alpha b)=a\alpha T(b)$ which is mean that T is right centralizer

<u>Corrolary3.7</u>:- Let M be a 2-torsion free Γ-ring which satisfy the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$, α, $\beta\in\Gamma$, has a commutator non-zero divisor and let $T:M\rightarrow M$ be a Jordan centralizer then T is centralizer◙

***Acknowledgment**:-the authors grateful to the referee for several suggestions that helped to improved the final version of this paper and especially Prof.Haetinger .*

References

[1]W.E.Barnes"On the Γ-ring of Nabusawa",PacificJ.Math.,18(1966),411-422.

[2]Y.Ceven,"Jordan left derivations on completely prime Gamma rings",Fenbilimleri Dergisi(2002)cilt 23say12.

[3]W.Cortes and C.Haetinger"On Lie ideal and Jordan left centralizers of 2-torsion free rings",URL:http://ensino.univates.br/~Chaet.

[4]I.N.Herstein"Topics in ring theory "University of Chicago press,1969.

[5]N. Nabusawa"On a generalization of the ring theory",OsakaJ.Math.,1(1964).

[6] J.Vukam"An identity related to centralizers in semi-prime rings", Comment .Math.Univ.Carolinae,40,3(1999)447-456.

[7]B.Zalar"On centralizers of semi-prime rings" Comment.Math.Univ.Carolinae 32(1991)609-614.