

MODELING OF THE FORCES AND AREA MILLING OF THE PLASTICS IN RECYCLING

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الخلاصة:

في هذه الدراسة النظرية تمت دراسة مجموعة من المحددات أو المتغيرات للتقريز (التقطيع) وذلك عن طريق أقرص متعددة ذات تركيبة جيدة لغرض تحديد نموذج ومعادلة مهمة لتمثيل قوى ومساحات التقطيع لتحليل قوى شبه القص وخصوصاً في البلاستيك (القياسي والمعاد).

في هذا البحث تمت معالجة الأمور الحسابية بواسطة البرنامج (Solid Work) وكذلك تم استخدام برنامج لشبه القص لصناعة نماذج من البلاستيك (البوليمر) القياسي والمعاد لإيجاد القوى والانفعالات في عملية التقريز الحقيقية. كذلك التشغيل المستمر والانهيار للمواد البلاستيكية المعادة تم اختبارها.

هذا البحث يسمح للباحثين أن يقرروا الكفاءة والقدرة المطلوبة من أجل خلق المثالية لعملية تقريز البلاستيك.

ABSTRACT

An example is presented in this study, the characteristic of multi-disc mill for a good construction has an important role modeling of forces and area milling of the plastics in recycling specifically analysing loads quasi-shearing. In this theoretical study advanced software such as Solid Works are preformed. The authors used a program of quasi-shearing for plastics specimens made of primary and recycled polymer in order to determine strains and loads in actual mill. Continuous machine crumbling of recycled plastic materials is also examined. This allows the authors to determine efficiency and power requirements, in direction of optimization.

1. Introduction:

The loads in grinding unit are most frequently examined and analysed in the context of the existing databases of other solutions, otherwise one is bound to create original databases [1, 2, 6]. Great amount of data must be collected and

stored to provide information on diverse conditions of machining (type of process, geometry and material of the tool, work piece material, cutting speed, etc.). Much attention has been given to develop as models of cutting. One of the most useful is the model of quasi-shearing¹. The quantities derived from the model, on the basis of measured quasi-shearing loads, can be used to estimate the actual tool-disc geometry and texture of the designed surface. The most important problems in the examination of quasi-shearing are specification of the shear plane, the rake face, the cut surface and the normal plane. The later is useful for defining directions of velocity and force vectors of quasi-shearing in actual machining conditions [2.3].

2. Model of Loads in Multi-Disc Assembly:

In the milling process of quasi-shearing material that has the form of grains (small pieces of pipes from thermoplastic material), there exists two typical kinds of stresses shown in Fig. (1):

shearing τ ,

$$\tau = \frac{P_w}{S}$$

(2.1a)

and normal σ ,

$$\sigma = \frac{P_k}{S}$$

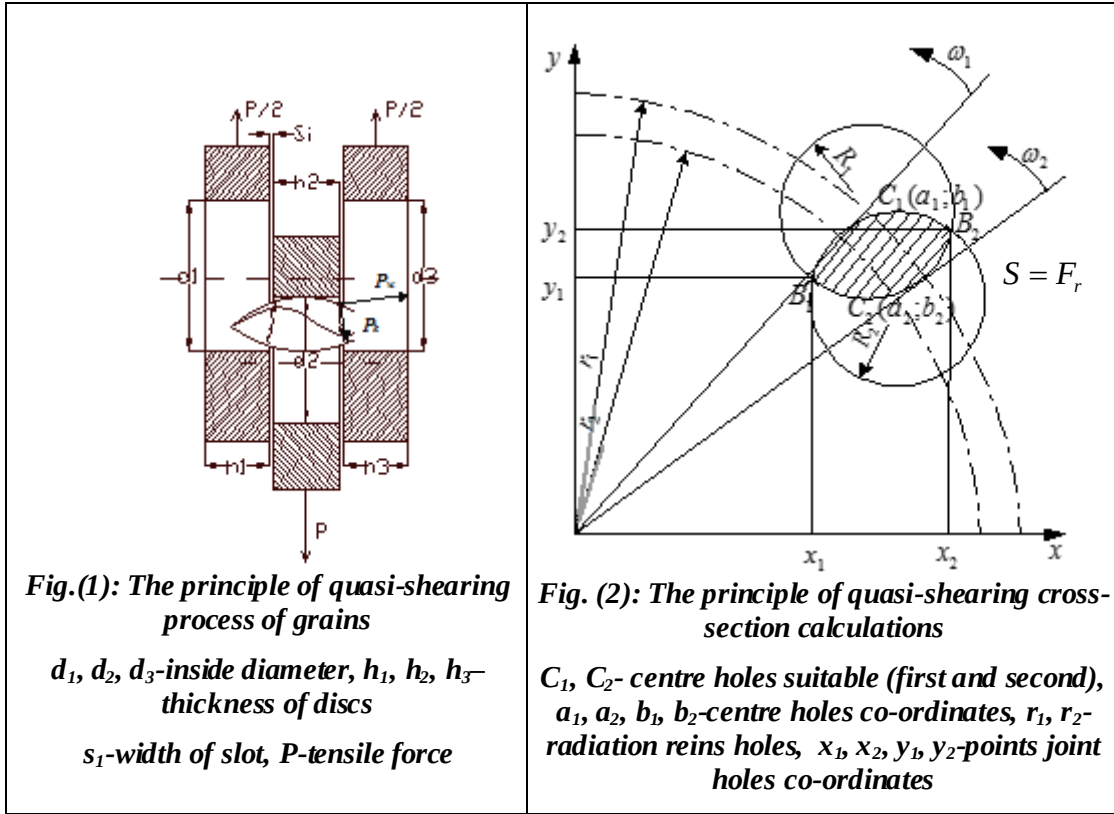
(2.1b)

Where:

P_w - transverse forces,

P_k - longitudinal force,

S - cross-section area, $S = F_r$



As it is known, resistance ($P_{q-s} = P_w + P_k$) of heterogeneous material subjected to shredding depends on characteristics of the machine, properties of the material and parameters of the cutting process, and in the multi-disc assembly it can be determined as the so called resistance to quasi-shearing [1, 3, 6].

Taking into account geometrical structure of the assembly shown in Figures (1 and 2) and the influence of friction, one can propose a model of instantaneous loads on the edges of cutting discs for the case when the machine is used for cutting pieces of pipes from thermoplastic material. The influence of material properties is expressed by the following relation of forces: load related to material cohesion P_s equals the product of material compression strength in the plane perpendicular to pipe axis R_{tr} and the area of ring surface (area of pipe cross-section F_r).

$$P_s = R_{tr} \cdot F_r; \quad F_r = \pi \frac{(d_{rz}^2 - d_{rw}^2)}{4} \quad (2.2) \quad \text{Where:}$$

P_s - force, internal load concentrated in comminuted material,

R_{ts} - instantaneous strengths in transverse section of comminuted plastic pipe,

F_r - area of transverse section of comminuted pipe,

d_{rz}^2 - outside diameter of pipe,

d_{rw}^2 - inside diameter of pipe.

Ali Abdul-Emir and Raid Salman Fahad

At the same time, one must take into account normal force N multiplied by the coefficient of friction that yields the friction force μN . The symbol μ denotes coefficient of friction between the comminuted material and the tool- which in this case is the cutting disc.

Effect of bed-knife should replace axial component force N_{wr} and contiguous component force T_{wr} equals friction force completely: $\mu_r N_{wr}$, when μ_r – coefficient of friction as in this bed-knife.

Effect of knife on plastic materials brings to the following forces Fig.(3):

- force S which affects, the knife for causing cutting off recycle of plastics,
- axial forces P_1 and P_2 , which holes edges effect on recycle of plastics,
- **limiting friction force T_1 and T_2 on frontal and back surface of knife, $T_1 = \mu P_1$, $T_2 = \mu P_2$.**

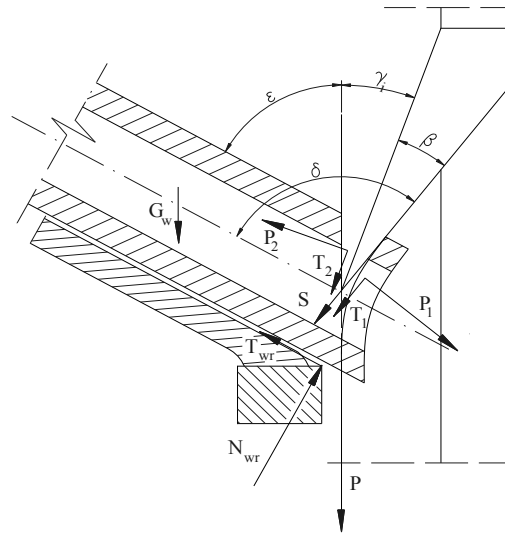


Fig.(3) Forces system have an effect on pipe-recycle in process of crumbling

Equation of equilibrium forces system as given, Figures (1 and 2) and Fig. (3) have the form:

$$\begin{aligned} \Sigma P_{\parallel} = G_w \cdot \cos \varepsilon + P_1 \cdot \sin \delta + \mu \cdot P_1 \cdot \cos \delta - P_2 \sin(\varepsilon + \gamma_i) + \\ + \mu P_2 \cdot \cos(\varepsilon + \gamma_i) - \mu_r \cdot N_{wr} = 0 \end{aligned} \quad (2.3)$$

$$\begin{aligned} \Sigma P_{\perp} = G_w \cdot \sin \varepsilon + \mu \cdot P_1 \cdot \sin \delta - P_1 \cdot \cos \delta + P_2 \cos(\varepsilon + \gamma_i) + \\ + \mu \cdot P_2 \cdot \sin(\varepsilon + \gamma_i) - N_{wr} + S = 0 \end{aligned} \quad (2.4)$$

With index: $\parallel$ – marked direction axis, $\perp$ - direction radial to oblong line crumbled pipes; μ_r - coefficient of friction plastic materials bed-knife (counter-friction).

Accepting $S=0$, skipping force from mass pipes G_w and eliminating reaction N_{wr} one receives after transformation:

$$P_2 = P_1 \cdot \frac{(1 - \mu \cdot \mu_r) \cdot \sin \delta + (\mu + \mu_r) \cdot \cos \delta}{(1 + \mu \cdot \mu_r) \cdot \sin(\varepsilon + \gamma_i) + (\mu_r - \mu) \cdot \cos(\varepsilon + \gamma_i)} \quad (2.5)$$

If we introduce the following:

$$e = \frac{(1 - \mu \cdot \mu_r) \cdot \sin \delta + (\mu + \mu_r) \cdot \cos \delta}{(1 + \mu \cdot \mu_r) \cdot \sin(\varepsilon + \gamma_i) + (\mu_r - \mu) \cdot \cos(\varepsilon + \gamma_i)} \quad (2.6)$$

Then will have:

$$P_2 = P_1 \cdot e \quad (2.7)$$

In order to define the force P_o of interaction between disc and knife, the system of forces acting on knife must be examined according to Fig. (4).

$$\Sigma P = P_o - P_1 \cdot \sin(\beta + \gamma_i) - \mu \cdot P_1 \cdot \cos(\beta + \gamma_i) - \mu \cdot P_2 \cos \gamma_i + P_2 \sin \gamma_i = 0 \quad (2.8)$$

From here, it follows that:

$$P_o = P_1 [\sin(\beta + \gamma_i) + \mu \cdot \cos(\beta + \gamma_i)] + \mu \cdot P_2 \quad (2.9)$$

Substituting equation (2.7) into equation (2.9) one obtains:

$$P_o = P_1 [\sin(\beta + \gamma_i) + \mu \cdot \cos(\beta + \gamma_i) + \mu \cdot e] \quad (2.10)$$

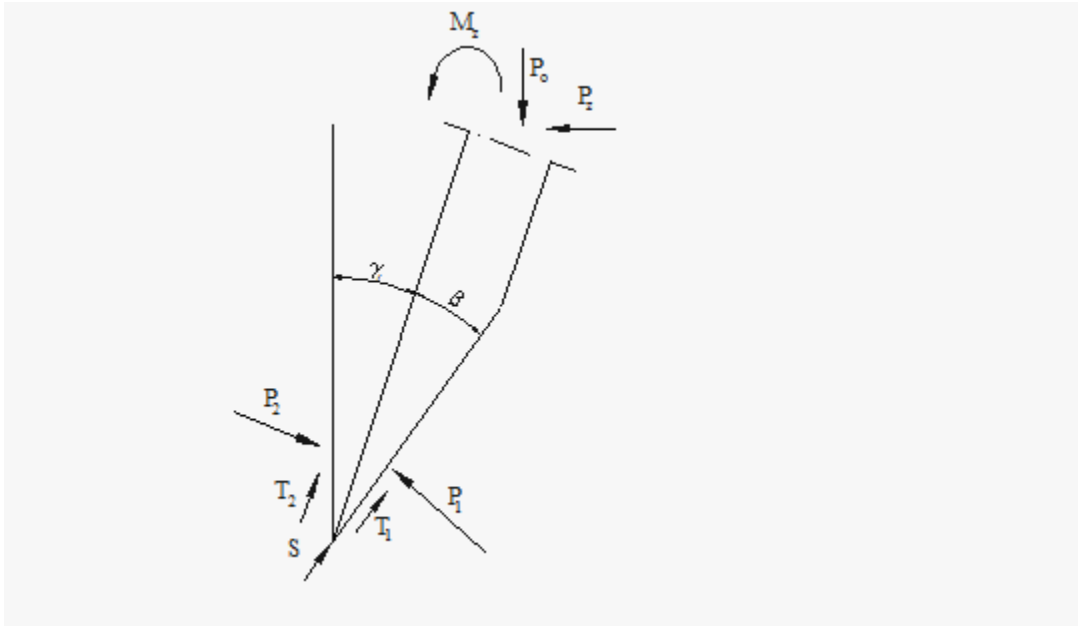


Fig. (4) Condition of force System balance tool in multi-knives crumbling

The reason for developing the model is to recognise and describe, in an analytical way, essential relations and mechanisms deciding the machine power requirements. One should also find out which variables decide the state of loads, and whether it would be possible to find an alternative design solution that would reduce the existing loads. It must be noticed that shredding, including cutting pieces of plastic pipes, is associated with a complex state of loads that arise in the area of quasi-shearing until the comminution product is deposited out of this area. Cutting loads are superimposed on those related to pressure between slats, but the other depending on the resulting friction force N and associated with the heat produced by friction.

Inadequate knowledge of complex heat that affects the multi-disc shredding assembly is the reason why one often applies simplified mathematical models to many parameters determined experimentally. In creating the presented model, the authors additionally considered the condition of limiting temperature of working parts and comminuted material that cannot be exceeded during long-term machine operation. This temperature is assumed arbitrarily as:

$\theta = 36^{\circ}\text{C}$ - for biological materials, reasons on destructive proteins, that perhaps to exceed temperature.

$\theta = 65-95^{\circ}\text{C}$ - for thermoplastic polymers, reasons on plasticizing plastics.

The heat balance is calculated for each section of the assembly.

1. The power is effectively consumed P_{q-s} and the total power are lost in the shredding assembly P_{vi} depending on power of friction between discs, P_{vzi} , losses of disc wading, P_{voi} , and friction in bearings, P_{vLPi} , as follows:

$$(2.11) \quad P_{vi} = P_{q-s} + P_{vzi} + P_{voi} + P_{vLPi},$$

where:

$$(2.12) \quad P_{q-s} = P_j + P_r + P_d = \left(k_j \cdot v_r + \sigma_{\max} \cdot F_r + \varepsilon \cdot F_r' \cdot v_r^2 \right) \cdot v_r$$

P_j - power lost motion, P_r - power on crumbling, P_d - power dynamics effect,

k_i - coefficient of idle run resistance (only the disc assembly), [kg sec⁻¹],

v_r - linear velocity of shredding, [m sec⁻¹],

$\sigma_{\max}$ - maximal stress in shredding area, [N m⁻²],

F_r, F_r' - area of milling section, primary (on edges of holes) and secondary (between discs), respectively, m²,

ε - proportionality coefficient of dynamic loads (relating phenomenal between discs), N sec²m⁻⁴;

- $P_{vzi} = Ka_i P_i \mu_{zi} [\eta_{zi} \cos \gamma_m \cos \alpha_n (\sin \gamma_m + \mu_{zi} \cos \gamma_m)]^{-1}$ - losses of power due to friction of material between discs (kW), where:

$\mu_{zi} = \mu_{zoi} Y_w V R$ coefficient of friction in disc assembly (kind of comminuted material, v_{gmi}) - the model friction coefficient,

v_{gmi} - velocity of slip, $Y_w = Y_w$ (material of rotating and stationary disc, respectively) - coefficient relating parameters of actual material and sample material,

V - calculated coefficient of slip speed, $V=1.6432$ for flat disc, $V=1.8432$ for discs of complex form (cone and plate),

$R = (R_a/3)^{0.25}$ - roughness-factor surfaces to roughness samples,

$\eta_{zi} = \operatorname{tg} \gamma_m [\operatorname{tg}(\gamma_m + \rho_{zi})]^{-2}$ - efficiency of quasi-shearing in operating conditions, where $\rho_{zi} = \operatorname{arc} \operatorname{tg} \mu_{zi}$ - friction angle;

- $P_{voi} = 4,26 a n_i^{1,333} (v_{50} + 90) \cdot 10^{-10}$ - losses due to disc wading,

$v_{50} = v_{50}(v_g)$ - viscosity of the shredding medium; approximate values of the variables are as follows:

$$\begin{aligned} v_{50} &= 75 + 26,5(10 - v_g) & \text{for } 0 < v_g \leq 10, \\ v_{50} &= 50 + 5(15 - v_g) & \text{for } 10 < v_g \leq 15, \\ v_{50} &= 50 & \text{for } 15 < v_g, \end{aligned}$$

$v_{50} = 50$ - kinematics stickiness oil

and the average slip rate

$$v_g = 5,24 \cdot 10^{-5} d_{m1} (\cos \gamma_m)^{-1} \sum_{i=1}^n n_i t_i (m/sec); \quad (2.13)$$

- $P_{VLP} = CKa_i P_i \eta_{zi}^{-1}$ - mechanical losses, in bearings, in which
 $C = 0.0075$ - pertains to multi-disc assembly with roll bearings,
 $C = 0.025$ - pertains to slide bearings.

2. Dissipated energy, lost as heat, Q_{ab} [kW]. Inadequate knowledge of thermal conditions and heat conversion in the shredding process are the reasons why the models applied in the design of the machine are often unduly simplified. The common assumption taken in these models is that the heat is dissipated only by conduction to environment, that is:

$$Q_{ab} = \mathcal{G}_u A_{ca} k_{ca}, \quad (2.14)$$

Where:

$\mathcal{G}_u = \left[(\mathcal{G}_{gr} - \mathcal{G}_a) (1,03 + 3,162 \cdot 10^{-3} n_i^{0,5})^{-1} \right]$ - is the formula for the calculation of temperature difference between inner and outer surface of side walls of the assembly body, in which (taken from my research [1]): ν_a - ambient temperature ($^{\circ}\text{C}$), $A_{ca} = 9 \cdot 10^{-5} a^m$ - area of heat abstraction (m^2), m - exponent depending on area of cooling surface, equal to 1.80 for body without cooling fins, and $m=1.85$ for body with cooling fins [3, 6];

$k_{ca} = C \cdot 10^{-3} (1 + D \cdot 10^{-2} n_i^{0,75})$ - coefficient of heat penetration ($\text{KW} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$); it is responsible for the shape of the body, kind of drive unit, method of evacuating the material from the shredding chamber, and position of the disc assembly (taken from my research [1]): $C=5.28$ for vertical discs, $C=6.6$ for horizontal discs. Coefficient parameters also depend on conditions of internal circulation of cooling air and intake of air: $D=1.07$ for natural in flow, $D=1.86$ for forced intake, e.g. by application of a fan on the shaft of disc assembly.

In order to evaluate the heat balance, one takes into account the ratio of powers of the produced heat to the evacuated heat, assumed as:

$$S_T = Q_{ab} / P_v \geq 1 \quad (2.15)$$

the so called cooling condition factor. In literature, S_T is also called the thermal factor [1,3, 5].

In the case when worm gear is used in the drive unit, the bending stress at the base of the worm wheel tooth is given by the formula:

$$\sigma_g = K_A F_{t2} (mb_2)^{-1} \leq \sigma_{gdop}, \quad (2.16)$$

and the inequality expresses its relation with allowable stress σ_{gdop} refers to unrestricted fatigue strength.

It is then accepted that the bending strength of the driving gear element is determined due to the i^{th} interval of shredding load spectrum of maximum value σ_g , irrespective of the number of load cycles.

Circumferential forces on working discs (N) are:

$$F_{tm2} = 19,1 \cdot 10^6 K_A P [\eta_z n d_{m1} \operatorname{tg}(\gamma_m + \rho_z)]^{-1}, \quad (2.17)$$

where b_2 is the disc thickness (mm).

Safety factor in bending is defined, as follows:

$$S_F = \sigma_{gdop} / \sigma_g. \quad (2.18)$$

By comparing maximum deflection of the shaft of disc assembly $f_{\max}$ with its admissible deflection f_{dop} , and assuming that

$$f_{dop} / f_{\max} \geq 0,5 \div 1, \quad (2.19)$$

one can derive the formula for maximum distance of bearings of the worm shaft support

$$l_{\max} \leq A (m d_{m1}^4)^{0,333} (F_{rm1}^2 + F_{trm1}^2)^{-0,1667} \quad (2.20)$$

The following are admissible deflections of shaft:

hardened steel - $f_{dop}=0.004m$,

toughened steel or cast iron - $f_{dop}=0.001m$.

According to the accepted model of shaft, consisting of a beam and two symmetrical supports loaded with radial (F_{rmi}) and circumferential (F_{tmi}) components of quasi-shearing forces, the deflection is:

$$f_{\max} = (f_r^2 + f_t^2)^{0,5} = l^3 (48EI)^{-1} (F_{rm1}^2 + F_{tm1}^2)^{0,5}. \quad (2.21)$$

Assuming $E=2.1 \times 10^5$ MPa for steel, and $E=10^5$ MPa for cast iron, one calculates the values of coefficient A for the shaft [2, 3, 6]: hardened steel - $A=15$, toughened steel - $A=21$, cast iron - $A=16$.

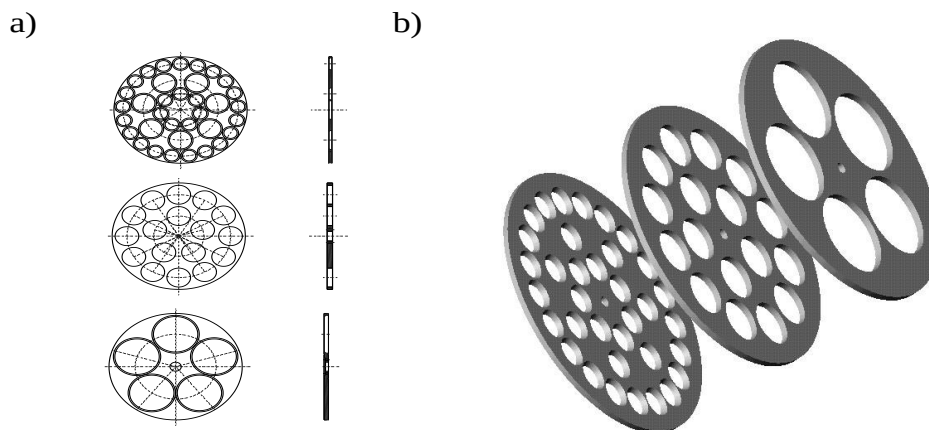


Fig. (5) View of Discs with Holes to quasi-shearing; a) Auto-CAD notation, b) Solid-Works notation.

3. The Obtained Results:

The theoretical study and simulation programs (Solid Work and Test-4) for some crumbling (cutting) properties done by the multi-hole disks show the effect of number of disks and types of holes (diameter, cutting angle, distance between disks) as shown in figure (6-a). Other variables were also studied via the simulation program including the use of five different polymers $Pr(t_{1,...,5})$, as shown in figure (6-b).

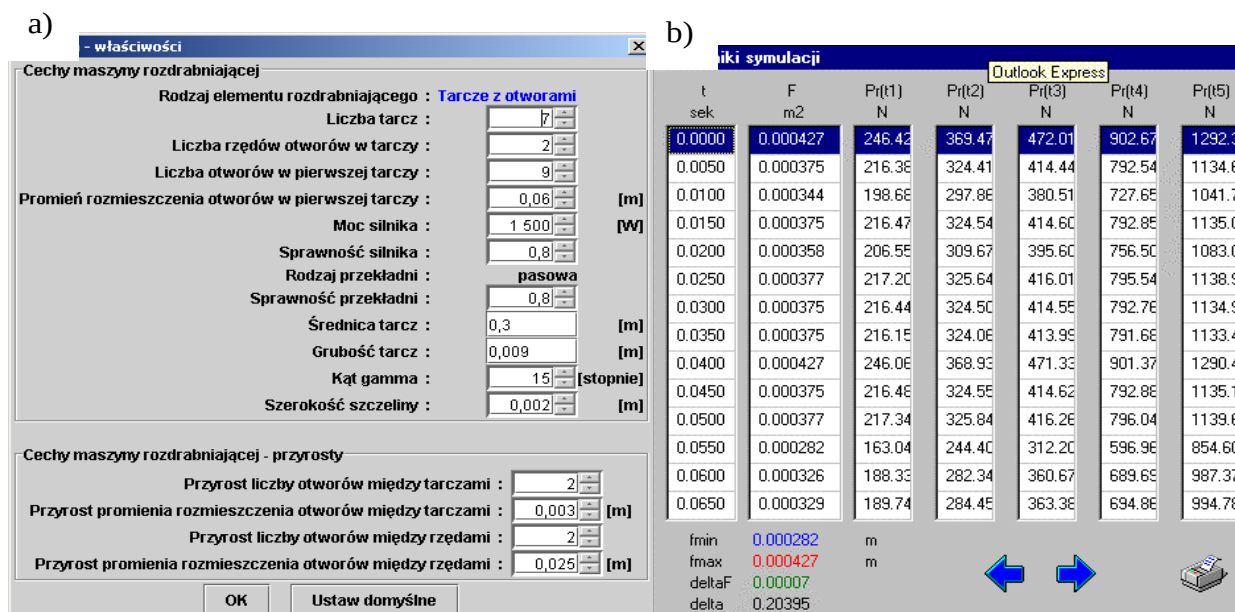


Fig. (6) Machine data (a) and the results of the calculations (b) the sections F and forces of crumbling for five different kinds of plastic materials $Pr(t1-t5)$; lower of board (b) the value minimum, maximum of section F and the characteristic of irregularity Δ).

The results show the effect of quasi-shearing process which sometimes decreasing the efficiency of crumbling. This theoretical model and the simulation program enable the optimum design of the crumbling machine

according to the influencing variables (gamma angle, direction of disks rotation and the applied loads) as shown in tables (1 and 2) below.

Table (1): The Properties of Crumbling Machine:

Types of Crumbling Elements	Perforated Disks
Number of Disks	7
Number of Holes Level in Disks	2
Number of Holes in the First Level	9
Radius of the Hole in the First Disk	0.06 (m)
Engine Power	1500 (W)
Engine Efficiency	0.8
Gear Engagement Type	Transmission Belts
Efficiency of Gear System	0.8
Diameter of Disk	0.3 (m)
Thickness of Disk	0.009 (m)
Gamma	15 (degree)
Distance between Disks	0.002 (m)
Tolerances of Holes Number between Disks	2
Tolerances of Radius for One Hole between Disks	0.003 (m)
Tolerances of the Holes Number in the Levels	2

Table (2):Simulation

t (s)	F (m^2)	$Pr(t_1)$ (N)	$Pr(t_2)$ (N)	$Pr(t_3)$ (N)	$Pr(t_4)$ (N)	$Pr(t_5)$ (N)
0.0000	0.000427	246.43	369.47	472.01	902.67	1292.3
0.0050	0.000375	216.36	324.41	414.44	792.54	1134.8
0.0100	0.000344	198.68	297.86	380.51	727.65	1041.7
0.0150	0.000375	216.47	324.54	414.60	792.85	1135.0
0.0200	0.000358	206.55	309.67	395.60	756.50	1083.0
0.0250	0.000377	217.20	325.64	416.01	795.54	1138.5
0.0300	0.000375	216.44	324.50	414.55	792.76	1134.9
0.0350	0.000375	216.15	324.06	413.95	791.66	1133.4
0.0400	0.000427	246.06	368.93	471.33	901.37	1290.4
0.0450	0.000375	216.48	324.55	414.62	792.88	1135.1
0.0500	0.000377	217.34	325.84	416.26	796.04	1139.6
0.0550	0.000282	163.04	244.40	312.20	596.96	854.60
0.0600	0.000326	188.33	282.34	360.67	689.69	987.37
0.0650	0.000329	189.74	284.45	363.36	694.86	994.78
$f_{\min} = 0.000282(m)$		$\Delta F = 0.00007$				
$f_{\max} = 0.000427(m)$		$\Delta = 0.20395$				

4. Conclusion:

Modeling of forces and sections, quasi-shearing materials from mechanical point of view, the process of quasi-shearing of plastic pipes, with cutting direction perpendicular to the pipe axis, is difficult to describe analytically. This is due to a complicated state of stress in the comminuted plastic materials, geometry of pipe is very simple and material properties are inhomogeneous.

The analysis of quasi-shearing of plastic pipes, concerning loads associated with operation of multi-disc working assembly, shows that the shape, dimensions and constructional tolerances of the tool (multi-disc assembly) can be optimised. It results in a better shredding susceptibility of the material.

The angle of the cutting edge of the hole and its linear velocity play the most important role in the mechanism of cutting chips of the plastics in recycling process (the form of shredding product). These parameters then strongly affect the construction of the multi-disc assembly, and on quality of milling product (plastics chips) efficiency of the shredding process. The correlation between quasi-shearing forces and area milling loads and non-dilatational strains, disc velocity, and geometrical form of the chip is significant, as it can be confirmed by means of multiple regression equations [1, 6] with very high determination coefficient.

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