

BUCKLING AND STRESS ANALYSIS OF STIFFENED CYLINDRICAL SHELL STRUCTURE UNDER HYDROSTATIC PRESSURE

Dr. Qasim H. Bader¹ and Israa S. Abdul Zuhra²

1 Department of mechanical Engineering, College of Engineering, University of Babylon, <u>drqasimbader@yahoo.com</u>

2 Department of mechanical Engineering, College of Engineering, University of Babylon, <u>mec.enga@yahoo.com</u>

ABSTRACT

The present work investigates stress analysis and buckling analysis of cylindrical shell. It had been wide applications in the industry especially in mechanical and nuclear applications.. etc. The most common type of failure which may be happen in this type of structure was the buckling when was subjected to external hydrostatic pressure which represents the aim of this work. The buckling happen due to transfer major amount of strain energy into bending energy which leads to sudden failure in the structure. To avoid this type of failure the cylindrical shell must be stiffened by adding stiffeners rings along the shell from internal or external surface. In this work two types of stiffeners existed, longitudinal and circumferential stiffeners based on the direction of the installation on the shell surface. In this work the critical pressure due to buckling was calculated numerically by using ANSYS15 for both stiffened and un-stiffened cylinder for various location and installing types, strengthening of the cylinder causes a significant increase in buckling pressures than non reinforced cylinder. Also the deflection for stiffened cylinder is function to mode shape and increasing with increasing the mode number because when the mode is increased, the strength of cylinder is increased and this leads to lower deflection. Also the stress analysis due to buckling for different depths of sea water was done to find the critical depth which may be the stresses exceeded the elastic limit. And from stress analysis results, it is observed that there is not a huge difference in stresses between stiffened shell and un-stiffened shell in similar boundary and loading condition so the value of stresses in stiffened shell is a little lower compared to un-stiffened shell.

KEYWORDS: Stiffened Shell, Stress Analysis, Buckling, Constraint, Finite element analysis.

تحليل الاجهاد والانبعاج للهياكل الاسطوانية المقواة تحت تأثير الضغط الهيدر وستاتيكي

الخلاصة

في هذا البحث يتم دراسة تحليل الاجهاد للاسطوانات المقواة والتي اعتمدت على نطاق واسع في الهياكل المتطورة وذلك بسبب خصائصه الميكانيكية المميزة. الانبعاج هو واحد من اهم انواع الفشل الذي يصيب الاسطوانات والتي تدعونا الى اهتمام اكثر. في هذا البحث يتم تحليل ومعايرة الاسطوانة بدون وجود حلقات تقوية داخلية او خارجية ودراسة تحليل الاجهاد والانبعاج في حالة الاسطوانة بوجود حلقات تقوية محيطية داخلية او خارجية ودراسة تحليل طولية داخلية وخارجية ودراسة تحليل الاجهاد عند الانبعاج وعلى اعماق مختلفة من المحيط لمعرفة عند اي عمق سوف يتجاوز منطقة المرونه ويدخل بمرحلة التشوه اللدن وقد قمنا بدراسة الانبعاج للاسطوانات المقواة بطريقتين باستخدام المعادلات او باستعمال برنامج (ANSYS) وعمل مقارنة بين الطريقتين وقد تحققت نتائج مقاربة .

1. INTRODUCTION

Ring stiffened cylindrical shells are important configurations widely used in modern structures such as pressure vessels, submarine hulls, aircrafts, and launch vehicles. Structural weight is one of the most important parameters for designers. Most of these shells are required to operate in a dynamical environment. Therefore, it is very critical to investigate the dynamic characteristics of these shells to develop a strategy for controlling their modal vibration on specific operating conditions and determination of their structural integrity and fatigue life [8]. Stress Analysis and design formulae of local and global mode ultimate strength currently available are reviewed and evaluated based on the results of finite element analysis, considering their capabilities of accounting for effects of initial hull deflection and variation of hull thickness, [5] concentrates on buckling and ultimate strength assessment of ring stiffened shells and stringer stiffened shells involving various modes of buckling and under various loading like axial compression, radial pressure and combined loading. Comparisons are made with screened test data, which have realistic imperfections and various radiuses to thickness ratio values in the range generally used in offshore structures. [2] investigated steel cylindrical shells and their stiffening with the use of ring stiffeners. The more complete the stiffening, the more closely the shell will act to beam theory, and the calculations will be much easier. However, this would make realization of the structure more expensive and more laborious. The target of the study is to find the limits of ring stiffeners for cylindrical shell. Buckling analysis of a submarine with hull imperfections the design of submarines for deep sea exploration has many challenges. The greatest challenge is its buckling strength against the crushing pressures of the ocean floor. The problem lies in the fact that there are no theoretical solutions for such complex geometry [3]. The elastic buckling of stiffened cylindrical shells by rings and stringers made of functionally graded materials subjected to axial compression loading. The shell properties are assumed to vary continuously through the thickness direction [7]. The paper reports on the buckling of thin walled geometrically imperfect tubes, which were tested to destruction under uniform external hydrostatic pressure. The paper also reports on other similar tests to destruction, carried out on quite a large number of geometrically imperfect tubes. Theoretical studies were also carried out with well-known analytical solutions, together with a numerical solution using the famous finite element computer package, namely ANSYS. Whereas the theoretical analyses agreed with each other, they did not agree with the experimental data for the shorter tubes; this was because the shorter tubes collapsed by inelastic instability due to initial geometrical imperfections of the tubes [6]. Elastic stability of ring and stringer-stiffened

cylindrical shells under axial, internal and external pressures is studied using Ritz method. The stiffeners are rings, stringers and their different arrangements at the inner and outer surfaces of the shell. Critical buckling loads are obtained using Ritz method. It has been found that the cylindrical shells with outside rings are more stable than those with inside rings under axial compressive loading. The critical buckling load for inside rings is reducing by increasing the eccentricity of the rings, while for outside ring stiffeners the magnitude of eccentricity does not affect the critical buckling load. It has also been found that the shells with inside stringers are more stable than those with outside one. Moreover, the stability of cylindrical shells under internal and external pressures is almost the same for inside and outside arrangements of stringers [1].

2. BUCKLING THEORY

Buckling is a mathematical instability, leading to a failure mode. Theoretically, buckling is caused by a bifurcation in the solution to the equations of static equilibrium. At a certain stage under an increasing load, further load is able to be sustained in one of two states of equilibrium and un-deformed state or a laterally deformed state.

In practice, buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. For example, during earthquakes, reinforced concrete members may experience lateral deformation of the longitudinal reinforcing bars. This mode of failure is also described as failure due to elastic instability. When load is constantly being applied on a member, such as column, it will ultimately become large enough to cause the member to become unstable. Further load will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of load-carrying capacity. The member is said to have buckled, to have deformed. There are two types of buckling analysis:-

2.1. Nonlinear buckling analysis

Nonlinear buckling analysis is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. This technique employs a nonlinear static analysis with gradually increasing loads to seek the load level at which your structure becomes unstable. as depicted in Fig. 1-a using the nonlinear technique, your model can include features such as initial imperfections, plastic behavior, gaps, and large-deflection response. In addition, using deflection-controlled loading, you can even track the postbuckled performance of your structure (which can be useful in cases where the structure buckles into a stable configuration, such as "snap-through" buckling of a shallow dome).



Fig. 1. a Nonlinear load-deflection curve[5].

2.2. Eigen value buckling analysis

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Eigen value buckling analysis predicts the theoretical buckling strength (the bifurcation point) of an ideal linear elastic structure See Fig. 1-b. For instance, an Eigen value buckling analysis of a cylinder will match the classical Euler solution. However, imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength. Thus, Eigen value buckling analysis often yields un conservative results, and should generally not be used in actual day-to-day engineering analyses.



Fig. 1. b Linear(Eigen-value) buckling curve[5].

3. DESIGN VARIABLES

Selection of appropriate design variables is one of the most important decisions in creating a design model. Factors to be considered are:

1-Each design variables has appreciable influence on the objective and/or the constraint functions.

2-Each design variable is directly related to physically significant quantity such as dimension of part. So that designer can modify the drawing or the hardware based on the responded design.

3-In the most method used, the total number of design variables may be limited only by the computational resources (i.e. memory size, secondary storage size, etc.) The optimization process is efficient for reduced number of design variables that are showed by [10].

In many practical problems, the design variables cannot be chosen arbitrarily rather, have to satisfy certain specified functional and other requirements. The restrictions that must be satisfied to produce an acceptable design are collectively called design constraints .Constraints that represent limitations on the behavior or performance of the system are termed behavior or functional constraints that were noticed by [4].

4. FINITE ELEMENT MODELING:

A. Material Properties :The mechanical and physical properties of steel 4340 are given below:

(Young 's modulus : 210 Gpa -Yield stress : 1030Mpa - Poisson 's ratio : 0.3) **B- Element Type:** The most commonly used element type in stiffened shell cylinder is (solid 72) this type of element is well suited to model irregular meshes .And the element of stiffener is beam188 .This type of element can be used with six degree of freedom

C-Boundary conditions: Simply supported at each end.

D- Shell and stiffeners dimensions

Length of cylinder = 1.2mRadius of cylinder = 0.25mThickness of cylinder =0.0015mWidth of stiffener = 0.02mHeight of stiffener = 0.01m

5. RESULTS

The FEA model, shown was set up in the global cylindrical coordinate system and an external reference pressure of (82.7MPa) was applied. An Eigen value buckling analysis was then conducted with several iterations of mesh refinement until the solution converged to the theoretical solution with an error of 4%. The results are shown in Table 1 and confirming [13] theoretical equation. As a result, the FEA model of the cylindrical section has been calibrated.

In the second stage, the buckling analysis was done for cylinder alone (CA) and for stiffened cylinder with both internal and external stiffeners. In this stage the critical buckling pressure of each model was calculated as a function to mode number and also the deformation shape for all structure type was calculated. In this stage the analysis had done get a clear idea about the strength of structure and also the effect of stiffener type and location on the strength of structure.

The third stage is to analyze the stresses at the critical buckling pressure to determine if they have exceeded the yield strength of the material.

E size	FEA(MPa)	Flugge's (MPa)	Error %
4	31.73	28.25	12%
3	30.34	28.25	7.4%
2	29.9	28.25	5.8%
1	27.11	28.25	4%

Table 1. Cylindrical Section Eigen value Buckling Results.

5.1. Buckling Results Of Stiffened and Un Stiffened Cylinder.

The critical pressure as a function to mode shape for both stiffened and un stiffened cylinder is done. The critical pressures is estimated by applying hydrostatic pressure of 1MPa and solve the model by Eigen value buckling analysis to get the critical pressure versus mode number for each case. Fig. 2 shows the results of critical pressure versus mode number for cylinder alone without any stiffener (CA), from this figure the results show the critical pressure for CA in the first mode is (25.87MPa) this value for FEM and theoretical according to (equ.1) this value is(24.4MPa) for nodal diameter is 2, the percentage of error between FEM and theoretical is 6% .The value of critical pressure for CI show in Fig. 3 is (36.56MPa) this value by FEM but for theoretical method this value is (35.5MPa) according to (eq.2) the percentage of error between FEM and theoretical is 3%.The value of critical pressure for CEL is (24.2MPa) show in Fig. 6 this value by FEM but for theoretical method this value is (24.7MPa) according to (eq.3) [12] the percentage of error between FEM and theoretical is 5.7%.

$$p_{cr} = \frac{\mathrm{Et}}{\mathrm{r}(1-\mathrm{v}^2)} \left\{ \frac{\mathrm{A}+\mathrm{k}[\mathrm{B}-\mathrm{C}+\mathrm{D}]}{\mathrm{E}} \right\}$$
(1)

$$A=(1 - v^{2})\lambda^{4}$$

$$B=(\lambda^{2} + m^{2})^{4}$$

$$C=2(v\lambda^{6} + 3\lambda^{4}m^{2} + (4 - v)\lambda^{2}m^{4} + m^{6})$$

$$D=2(2 - v)\lambda^{2}m^{2} + m^{4}$$

$$E=m^{2}(\lambda^{2} + m^{2})^{2} - m^{2}(3\lambda^{2} + m^{2})$$

$$E = Modulus of Elasticity$$

$$r = Mean cylinder radius$$

$$t = thickness of cylinder$$

$$v = Poisson's ratio$$

m = nodal diameter

Buckling equation of stiffened cylindrical shell(circumferential stiffeners),

$$p_{cr} = \frac{E(t/r)}{1-\nu^2} \left[\frac{(1-\nu^2)}{(m^2-1)[\frac{m^2L^2}{m^2r^2}+1]} \right] + \left[\frac{E(t/r)^3}{12(1-\nu^2)} \right] \left[m^2 - 1 + \frac{2m^2-1-\nu}{\frac{m^2L^2}{\pi^2r^2}-1} \right]$$
(2)

Buckling equation of stiffened cylindrical shell(longitudinal stiffeners)

$$\operatorname{pcr} = \frac{1}{Rn^2 \lambda^2 L^2} \left(\mathbf{D} + \frac{B^2}{A} \right) = \frac{1}{\left(\frac{R}{h}\right)^3 n^2 \lambda^4} \left(\overline{\mathbf{D}} + \frac{\overline{B}^2}{\overline{A}} \right)$$
(3)

To get an idea about the effect of stiffeners on the value of critical pressure as a function to mode number, the same analysis was done for all types of stiffened cylinder. Fig. 3 shows the critical pressure versus mode number for stiffened cylinder with internal circumferential stiffener CI. Fig. 4 shows the critical pressure versus mode number for external circumferential stiffener CE. Fig. 5 shows the critical pressure versus mode number for internal longitudinal stiffener CIL. Fig. 6 shows the critical pressure versus mode number for external longitudinal stiffener CEL. Table 2 will summarize the results of critical pressure as a function for mode shape for Fig. 2 to Fig. 6.

From the above buckling theory, strengthening of the ring causes a significant increase and slightly buckling pressures than non-reinforced cylinder. With the added enhancement of the ring, the buckling pressure increases. Also the effect of ring stiffening was to increase the buckling resistances. There are good agreements between the numerical and experimental results. The number of stiffeners was varied around the circumference and along the length of the cylinder ,and the stiffness values of these supports were varied .As a result of the analysis it was shown that there was a maximum level of improvement that can be obtained for a given number of stiffeners around the circumference of the shell. Furthermore increasing the number of stiffeners around the circumference of the shell in the middle of the shell length increases the critical buckling greatly.

Mode shape	CA (MPa)	CI(MPa)	CE(MPa)	CIL(MPa)	CEL(MPa)
Mode 1	25.87	36.56	35.2	25.9	24.2
Mode 2	26.7	37.6	36	26.85	27.8
Mode 3	27.1	38.4	39	27.2	30.6
Mode 4	27.3	38.6	40.4	27.5	33.1

 Table 2. critical pressure of structure versus mode number

Mode shape	CA(FEM)MPa	CA(Theoretical)MPa	Nodal Diameter (n)
Mode 1	25.87	24.4	2
Mode 2	26.7	25.9	2
Mode 3	27.1	26.7	4
Mode 4	27.3	26.5	4

Table 3. critical pressure of CA.

Mode shape	CA(FEM)MPa	CA(Theoretical)MPa	Nodal Diameter (n)
Mode 1	36.56	35.5	2
Mode 2	37.6	36.9	2
Mode 3	38.4	37.4	4
Mode 4	38.6	38.1	4

Table 4. Critical pressure of CI.

Generally when inspect the values of critical pressure in Table 2, it is clear that the values of this pressure was increased with existing the stiffeners. But when it is wanted to make a comparison between the stiffened shell, it is clear that the circumferential stiffeners for both internal and external type (CI and CE) is better than longitudinal stiffeners (CIL and CEL). Also when comparing the result between internal stiffeners only, this is a function of mode shape and change according to mode number as shown in Table 3.



Fig. 2. Critical pressure for un-stiffened cylinder (CA)



Fig. 3. Critical pressure for external stiffened cylinder (CE)





Fig. 4. Critical pressure for internal stiffened cylinder (CI).

Fig. 5. Critical pressure for internal longitudinal stiffened cylinder (CIL).



Fig. 6. Critical pressure for internal longitudinal stiffened cylinder (CEL).

The result for six mode shape for all model of cylinder with stiffener and without stiffeners is summarized in Table 5. From this table, the deflection mode for cylinder is increased with increasing the value of critical pressure when comparing the result of cylinder alone (CA) with the result of all type of stiffened cylinder.

Mode shape	CA	CI	CE	CIL	CEL
Mode 1	0.119	0.12	0.13	0.115	0.121
Mode 2	0.121	0.126	0.14	0.114	0.12
Mode 3	0.12	0.137	0.128	0.124	0.116
Mode 4	0.113	0.129	0.128	0.124	0.114
Mode 5	0.04	0.066	0.07	0.064	0.062
Mode 6	0.064	0.071	0.067	0.065	0.064

Table 5. Maximum deflection versus mode shapes for all structure types

Figs. 7 to 12 show the contour result of six mode shape for cylinder alone while in figures



Fig. 7. First mode shape for (CA).



Fig. 9. Third mode shape for (CA).



Fig. 11. Fifth mode shape for (CA).



Fig. 8. Second mode shape for (CA).



Fig. 10. Fourth mode shape for (CA).



Fig. 12. Sixth mode shape for (CA).

To illustrate the buckling deformation type and counting method of the number of buckling waves, the buckling deformation are demonstrated in Figs 7 to 12, The critical pressure increases with the increase of stiffener's stiffness. there are two buckling patterns existing in local bucking was confirmed through the numerical analysis, One of buckling patterns is that only buckled at the end of stiffened pipe while the other happens in overall stiffened pipe. The deformed location varies with the spacing of stiffeners; generally for large spacing the buckling deforms all inter-stiffener shells of overall stiffened pipe, while inter-stiffener shell at the end of stiffened pipe deforms for narrow spacing.

5.2. Result of stress analysis

This stage is to analyze the stresses at the critical buckling pressure to determine if they have exceeded the yield strength of the material. For this work under different hydrostatic pressure and simply supported boundary condition the buckling will be nonlinear buckling and two methods are available in ANSYS to solved it .These methods are Newton-Raphson and Arc-Length Method [3] obtain the value of pressure on the different depths of ocean to explain at any depth the stress of the cylinder will be behave as a plastic stress. Table 6 explains the value of pressure at each depth of ocean.

Pressure (MPa)	Depth (miles)
78.533	4.9
68.572	4.3
55.65	3.5
48.65	3

Table 6.	Hydrostatic	pressure under	different	depths [3]
		r		

From results in Table 7 under depth equal to (4.9mile) the maximum von Misses stress for CA is (1380MPa) and the yield stress of this type of material is (1030Mpa) thus, under this value of pressure the cylinder will deform as plastic deformation because it will exceed the yield stress. when we added stiffeners to the cylinder the maximum von Misses stress will become elastic for (CI) this value of stress is (693MPa), For (CE,CIL,CEL) the maximum von Misses stress not exceeding the yield stress.

Table 7. Maximum stresses under	pressure equal	to (78.533MPa).
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Type of cylinder	Depth (Mile)	Yield Stress (MPa)	Von Mices stress(MPa)
CA	4.9	1030	1380
CI	4.9	1030	693
CE	4.9	1030	675
CIL	4.9	1030	945
CEL	4.9	1030	936

So, under pressure equal to (68.572MPa) the maximum stress of un-stiffened cylinder exceeds the yield stress so the cylinder is deformed as a plastic deformation. For (CE,CIL,CEL) the maximum von Misses stress for these type is also still elastic, because the maximum stress dose not exceed the yield stress. under hydrostatic pressure equal to (55.65MPa & 48.65MPa) for depth equal to (3.5 & 3)Mile the maximum von Misses stress for un-stiffened and stiffened cylinder is elastic stress because the maximum stress doesn't exceed the limit of yield stress. From the above tables the result of stress analysis of stiffened and un-stiffened cylinder under different pressure the maximum stress of (CA) exceeds the yield stress under depth (4.9&4.3Mile) ,so when adding stiffener the stress is still elastic, but the maximum stress under depths equal to (3& 3.5Mile) the stress for stiffened and un-stiffened is still in elastic.

5.3. Stress analysis for static solution

Figs. 13 to 18 shows the maximum Von Mises stress distribution of (CA, CI, CIL, CE & CEL) under hydrostatic pressure and simply support boundary condition the results of maximum von Mises stress for static solution is shown in Table 8.

Туре	Maximum stress (MPa)	
CA	23.9	
CI	16.9	
CE	17.6	
CIL	21.4	
CEL	20.9	

Table 8. Maximum Von Mises stress of five types of cylinder

From this table the maximum Von Mises stress of un-stiffened cylinder is (23.9MPa) to improve the resistance of this cylinder to the effects of external pressure, the cylinder is usually stiffened by some stiffeners (circumferential, or longitudinal stiffeners) from the table above the stress will decrease when added stiffener this value became 16.9 MPa for CI the percentage of decrease in stress is 29.2%, and the maximum von Mises stress for CE is 17.6MPa so that the percentage of decrease in stress for this type is 22.6%.Symmetry for other cases of cylinders, for CIL the maximum Von Mises stress of this type of cylinder is (21.4) the percentage of decrease in stress is 10.5%, and for CEL is 12.6%. The best type of stiffener which give us the best resistance to the effects of external hydrostatic pressure is cylinder with internal stiffeners.



Fig. 13. Maximum Von Mises stress for CA



Fig. 15. Maximum Von Mises stress for CIL.



Fig. 14. Maximum Von Mises stress for CI.



Fig. 16. Maximum Von Mises stress for CEL.



Fig. 17. Maximum Von Mises stress for CE Fig. 18. Compression between stress and length ratio (L/D).

According to the results shown in Fig. 18, it is observed that there is not a huge difference in stresses between stiffened shell and un-stiffened shell in similar boundary and loading condition. the value of stresses in stiffened shell is a little lower compared to un-stiffened shell.

5.4. Results of stress analysis under effect of buckling

Eigen value buckling analysis predicts the theoretical buckling strength of an ideal elastic structure. It computes the structural Eigen values for the given system loading and constraints. This is known as classical Euler buckling analysis. Buckling loads for several configurations are readily available from tabulated solutions. However, in real-life, structural imperfections and nonlinearities prevent most real world structures from reaching their Eigen value predicted buckling strength. It over-predicts the expected buckling loads. Eigen-value buckling analysis generally yields un conservative results, and should usually not be used for design of actual structures. The procedure of Eigen-value buckling analysis is as follows.

- Building the model.
- Obtaining the static solution.
- Obtaining the Eigen-value buckling solution.
- Expanding the solution.
- Reviewing the results.

, the results of maximum von Mises stress for stiffened and un-stiffened cylinder under hydrostatic pressure and simply support boundary condition is shown in Table 9.

Туре	Maximum stress(GPa)
CA	201
CI	114
CE	118
CIL	188
CEL	164

Table 9. Maximum	Von Mises	stress of five	types of	cylinder
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From this table the Maximum von Mises stress of cylinder under eign value buckling; the results show the stresses will increase comparison with static solution analysis due to buckling and the cylinder will fail because the resistance of this cylinder will decrease.

The Eigen-value solution uses an iterative algorithm that obtains at first the Eigen-values and secondly the displacements that define the corresponding mode shape. The Eigen-value

represents the ratio between the applied load and the buckling load. This can be expressed as follows:

$$\lambda_i = \frac{Buckling \ load}{Applied \ load}$$

That's why it is often said that the Eigen-value is like a safety factor for the structure against buckling. On one hand, an Eigen-value less than 1.0 indicates that a structure has buckled under the applied loads. On the other hand, an Eigen-value greater than 1.0 indicates that the structure will not buckle.

6. CONCLUSIONS

From the buckling theory, it is clear that the circumferential stiffeners for both internal and external type (CI and CE) is better than longitudinal stiffeners (CIL and CEL). The deflection mode for cylinder is increased with increasing the value of critical pressure when comparing the result of cylinder alone (CA) with the result of all type of stiffened cylinder. Also the deflection for stiffened cylinder is function to mode shape and increasing with increasing the mode number and this is logical because when the mode is increased, the strength of cylinder is increased and this leads to lower deflection.

From stress analysis for buckling state shows that under depth equal to (3.5 & 3) Mile the maximum von Mises stress for un-stiffened and stiffened cylinder is elastic stress because the maximum stress doesn't exceeding the limit of yield stress. the results of stress analysis of stiffened and un-stiffened cylinder under different pressure shows the maximum stress of (CA) will be exceed the yield stress under depth (4.9&4.3Mile) ,so that when added stiffener the stress will still elastic, but the maximum stress under depths equal to (3& 3.5Mile) the stress for stiffened and un-stiffened still in elastic. From Stress analysis for static solution shows the best type of stiffener which gives us the best resistance to the effects of external hydrostatic pressure is cylinder with internal stiffeners because it has the higher percentage of decrease in stress about 29.2% comparison with other type.

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