

Some Applications of Fourier Transformation

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الخلاصة:

اجريت في هذا البحث بعض التطبيقات لتحويل فورير في بعد واحد وفي بعدين لبيانات منفصلة، أولاً في بعد واحد، حيث تم دراسة الأشاره من خلال خصائصها وكذلك كيفية ازاله الضوضاء من الاشاره. ثانياً في بعدين، تم إيجاد طريقه للكشف عن الإشكال الأساسية في الصوره من خلال تحديد الجزء المطلوب أولاً ومن ثم مطابقة الطيف لكل شكل موجود في الصورة للكشف عن أمثاله. وفي المرحلة الاخير من البحث تم دراسة نمط حيود من شق دائري وتحسين رؤية هذا النمط من الحيود. جميع الطرق أجريت بواسطة تحويل فورير وقد ظهرت نتائج جيده تم توضيحها من خلال البحث.

Abstract:

In this research some applications to Fourier transform in one dimension and in two dimensions for separated data, first in one dimension, where the signal from the study during their characteristics as well as how to remove noise from the signal. Second, in two dimensions, it was a method to detect basic shapes in the image by selecting the desired first and then match the spectrum of each shape in the image to detect the fold. In the final stage of the research were studied diffraction pattern from a circular slit and improve the visibility of this pattern of diffraction . All methods were conducted by Fourier transform has appeared good results have been clarified through research.

1-Introduction

1-1 One-Dimension Fourier Transformation

The Fourier transform is a mathematical operation that decomposes a signal into its constituent frequencies. Thus the Fourier transform of a is a mathematical representation of the amplitudes of the individual notes that make it up. The original signal depends on time, and therefore is called the time domain representation of the signal, whereas the Fourier transform depends on frequency and is called the frequency domain representation of the signal. The term Fourier transform refers both to the frequency domain representation of the signal and the process that transforms the signal to its frequency domain representation[1].

1-2 Two-Dimension Fourier Transformation

The Fourier transform is a representation of an image as a sum of complex exponentials of varying magnitudes, frequencies, and phases. The Fourier transform plays a critical role in a broad range of image processing applications, including enhancement, analysis, restoration, and compression.

If $f(m,n)$ is a function of two discrete spatial variables m and n , then the *two-dimensional Fourier transform* of $f(m,n)$ is defined by the relationship[2].

$$F(\omega_1, \omega_2) = \frac{1}{N} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m,n) e^{-i\omega_1 m} e^{-i\omega_2 n} \quad \text{-----}(1)$$

The variables ω_1 and ω_2 are frequency variables; their units are radians per sample. $F(\omega_1, \omega_2)$ is often called the *frequency-domain* representation of $f(m,n)$. $F(\omega_1, \omega_2)$ is a complex-valued function that is periodic both in ω_1 and ω_2 , with period 2π . Because of the periodicity, usually only the range $-\pi \leq \omega_1, \omega_2 \leq \pi$ is displayed. Note that $F(0,0)$ is the sum of all the values of $f(m,n)$. For this reason, $F(0,0)$ is often called the *constant component* or *DC component* of the Fourier transform. (DC stands for direct current; it is an electrical engineering term that refers to a constant-voltage power source, as opposed to a power source whose voltage varies sinusoidally.)[3]

The inverse of a transform is an operation that when performed on a transformed image produces the original image. The inverse two-dimensional Fourier transform is given by

$$f(m,n) = \frac{1}{4\pi^2} \sum_{\omega_1=-\pi}^{\pi} \sum_{\omega_2=-\pi}^{\pi} F(\omega_1, \omega_2) e^{-i\omega_1 m} e^{-i\omega_2 n} d\omega_1 d\omega_2 \quad \text{-----}(2)$$

Roughly speaking, equation (2) means that $f(m,n)$ can be represented as a sum of an infinite number of complex exponentials (sinusoids) with different frequencies. The magnitude and phase of the contribution at the frequencies (ω_1, ω_2) are given by $F(\omega_1, \omega_2)$ [4].

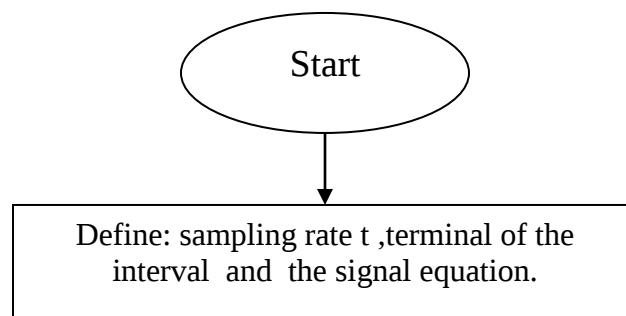
In this research we will deal with some applications of Fourier transform in both 1-D, like signal processing ,also 2-D like template detection and diffraction enhancement.

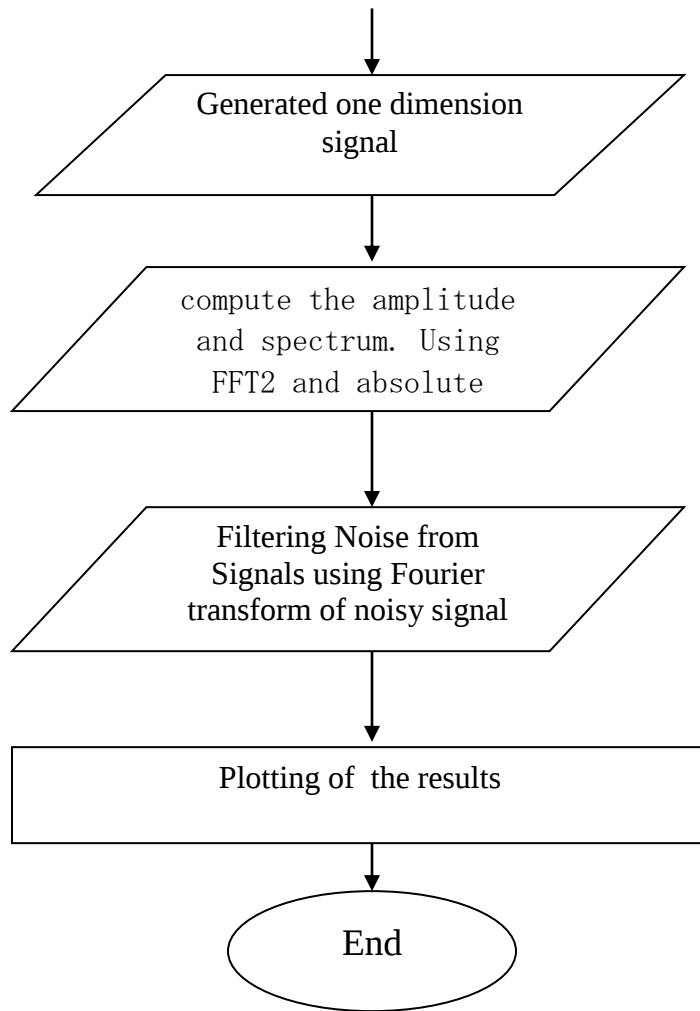
2- Fourier Transformation in 1-D

2-1 Signal Processing

Signal processing is an area of systems engineering, electrical engineering and applied mathematics that deals with operations on or analysis of signals, in either discrete or continuous time[5]. Signals of interest can include sound, images, time-varying measurement values and sensor data, for example biological data such as electrocardiograms, control system signals, telecommunication transmission signals, and many others. Signals are analog or digital electrical representations of time-varying or spatial-varying physical quantities. In the context of signal processing, arbitrary binary data streams and on-off signalling are not considered as signals, but only analog and digital signals that are representations of analog[6,7]. The process indicated in fig(1).

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Figure(1):The Flowchart of filtering noise from signal.

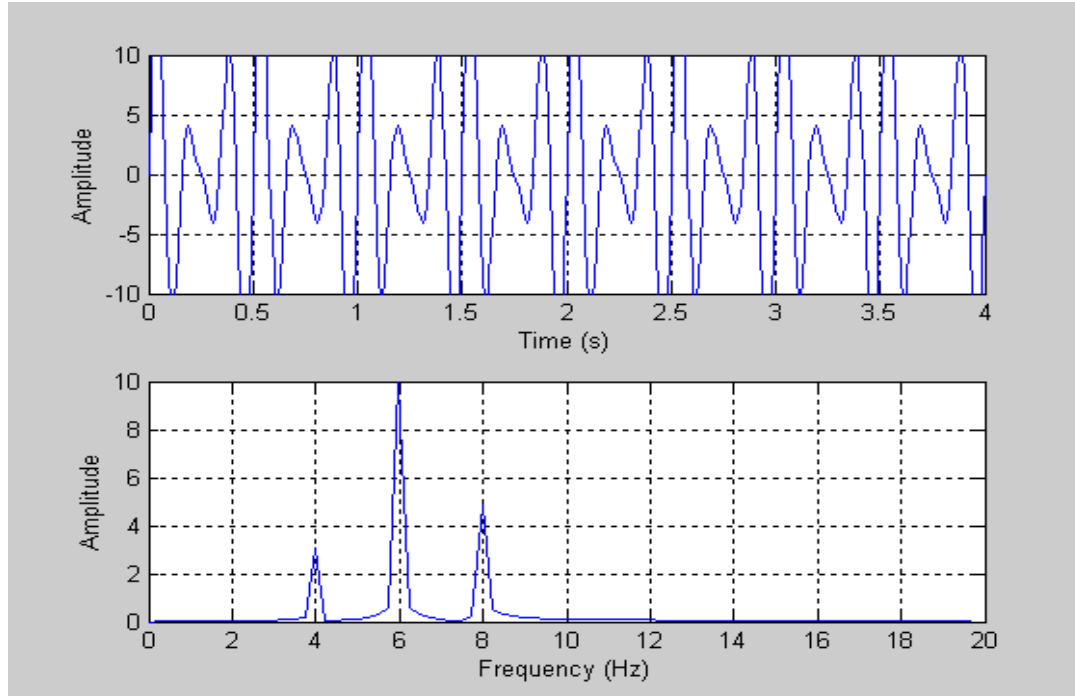
2-2-Results and Discussion

a- Compute the Amplitude Spectrum of Signal

By Fourier Transform we can study what main characterizes of the signal, like the computation of the amplitude spectrum. Every signal can be written as a sum of sinusoids with different amplitudes and frequencies[8].For example:-

$$y = 3 \times \sin(4 \times 2 \times \pi \times t) + 5 \times \sin(2 \times 2 \times \pi \times t) + 10 \times \sin(6 \times 2 \times \pi \times t) \text{ -----(3)}$$

The heights of the peaks are the amplitude of the signal 3 , 5 and 10 of the sine signal respectively located at 4 , 6 and 8 After we defined the t and frequency. A normal plot is that of amplitude versus time as shown in figure(2):



Figure(2) sine signal with its amplitudes

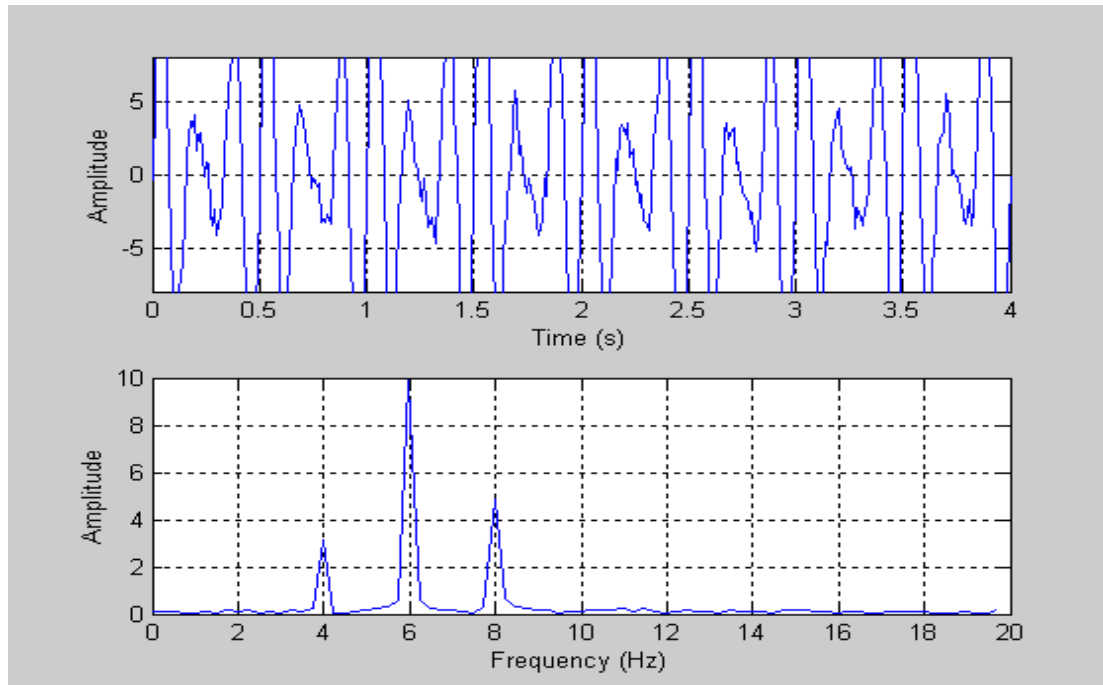
Also with Fourier Transform we can see what the characterizes of the signals in eq.(3) (compute the amplitude spectrum).

b- Adding Noise to Signals

Now we will see how to use forward Fourier transformation and inverse Fourier transformation to filter out the noise from signals. First we add random noise to the signal using the function ***rand*** and compute the amplitude spectrum.

Then, we apply Fourier transform of noisy signal then, and plotted the noisy signal with its amplitude fig(3). From the figure(3) we see the noisy signal in time domain, also figure(3)

indicates another small peaks (wobbles) that distribute into both sides around the main peaks in frequency domain. This is one of the important Fourier transformation features to get the amplitude of the all noise values.



Figure(3): Noisy signal with its amplitude(another small peaks appear due to noise addition is less than that of the original signals).

The wobbles we see around the main three peaks show that the amplitude of the noise is less than that of the original signal. We can visualize the output of the Fourier transforms for the noisy and the origin signals in fig(4)

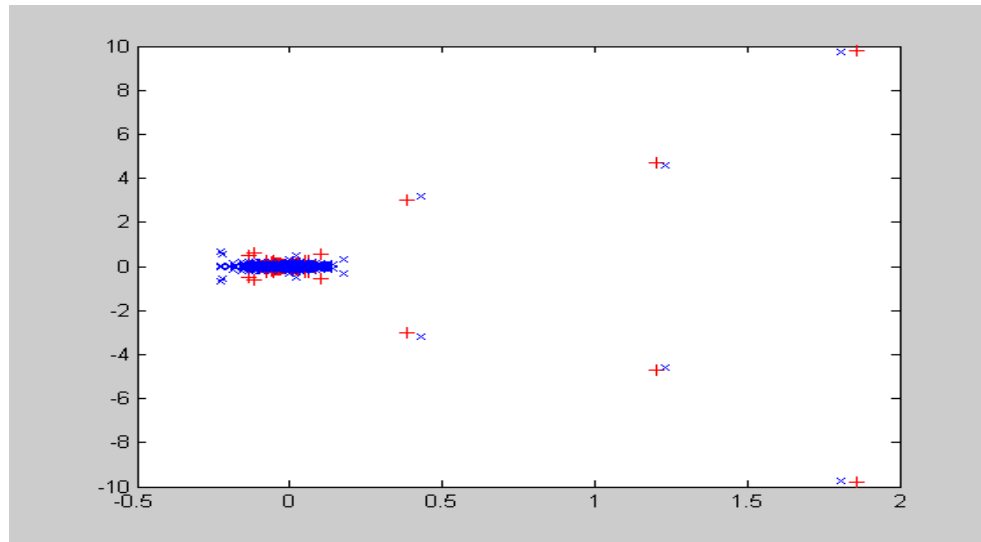
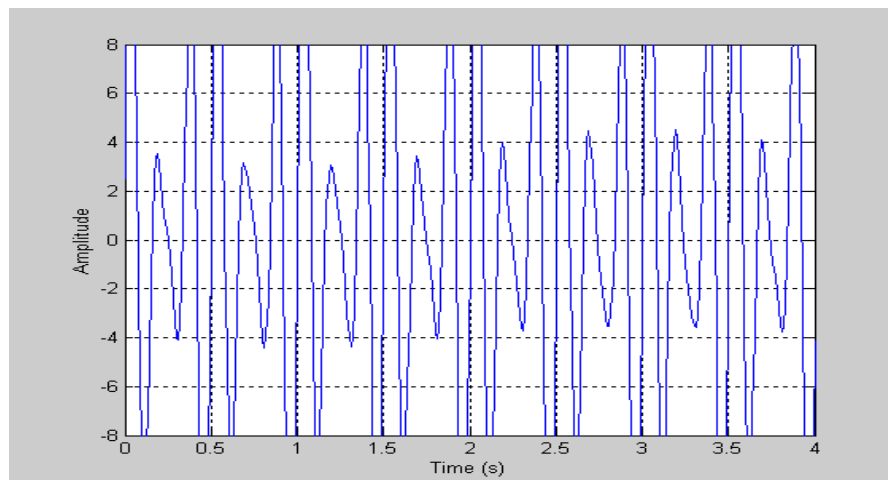


Figure (4) Spectrum of the noisy signal

c- De-Noising Process

we filtered out the noise by using the inverse Fourier Transform. In Matlab multi functions which are works properly for this reason like **Floor**, **Ciel** and **Fix**. The command **Fix** rounds the elements of its argument to the nearest integers towards zero. For this example, we use **Fix** to set all elements in noisy signal less than 100 to zero. Fig(5) illustrate the de-nosed signal using **Fix** function.



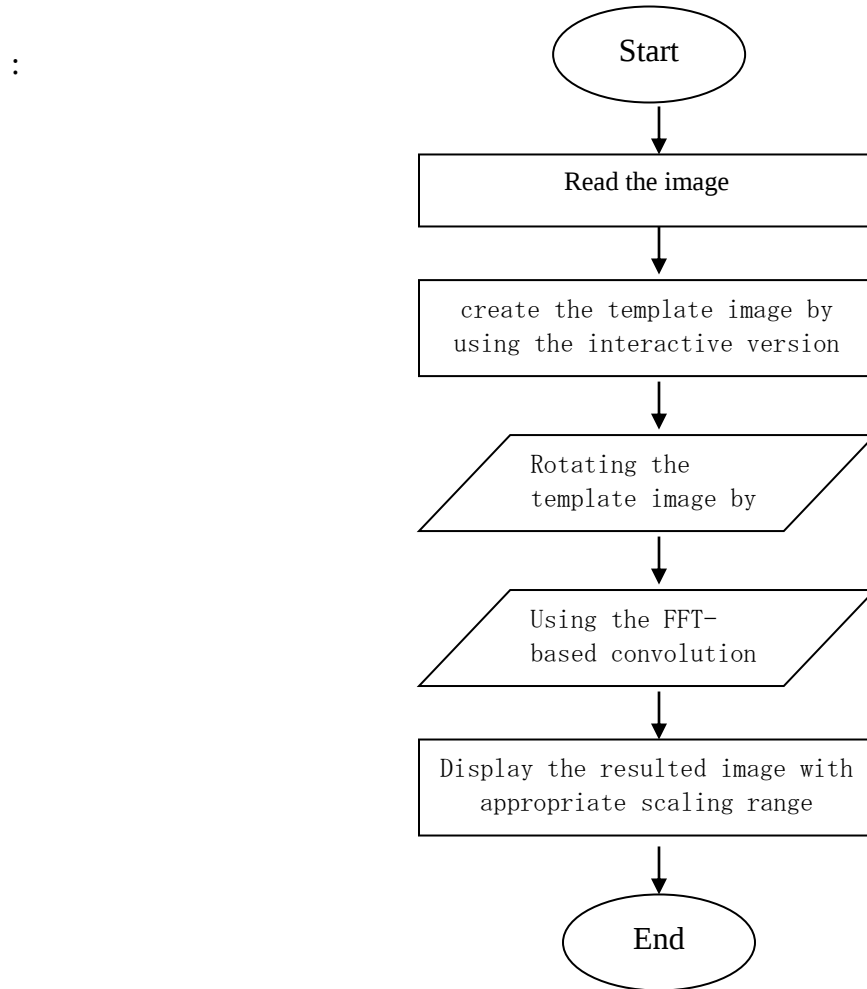
Figure(5) De-noised signal

3- Fourier Transformation in 2-D

3-1 Detection of Image Template

The second application is Template matching. Template matching is a technique in digital image processing for finding small parts of an image which match a template image. It can be used in manufacturing as a part of quality control, a way to navigate a mobile robot, or as a way to detect edges in images[10],[11].

The Fourier transform can also be used to perform correlation, which is closely related to convolution. Correlation can be used to locate features within an image; in this context correlation is often called template or character and some figure finding[12],[13].



Figure(6):The Flowchart of Detection of Image Template.

3-2 Results and Discussion

The flowchart of in fig(6) illustrates how to use correlation to locate occurrences of the basic objects in an image containing multi figures :

Compute the correlation of the template image with the original image by rotating the template image by 180 degree and then using the FFT-based convolution technique described in Fast Convolution.

(Convolution is equivalent to correlation if you rotate the convolution kernel by 180° .) To match the template to the image, use the FFT2 and IFFT2 functions.

```
C = real(IFFT(FFT2(bw) .* FFT2(rot90(a,2),m,n)));
```

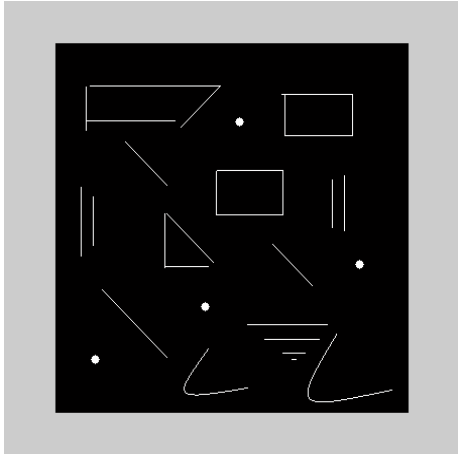
The following image shows the result of the correlation. Bright peaks in the image correspond to occurrences of the letter.

The command ***imshow(C,[])*** Scale image to appropriate display range.

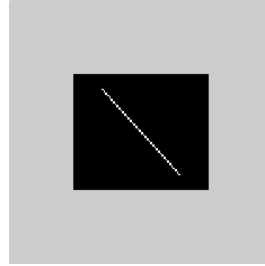
To view the locations of the template in the image, we must find the maximum pixel value and then define a threshold value that is less than this maximum. The locations of these peaks are indicated by the white spots in the thresholded correlation image (To make the locations easier to see in the figure, the thresholded image has been dilated to enlarge the size of the points.)

We use a threshold that's a little less than max.

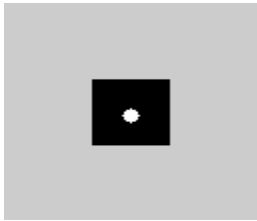
The little amount that subtracted from the threshold is defined by user; Figure(7),(8) and (9), Display pixels over threshold. Correlated, Thresholded Image Showing how many Template occur which its cropped with its Locations.



a-Original Image



b-crop sloped line sample

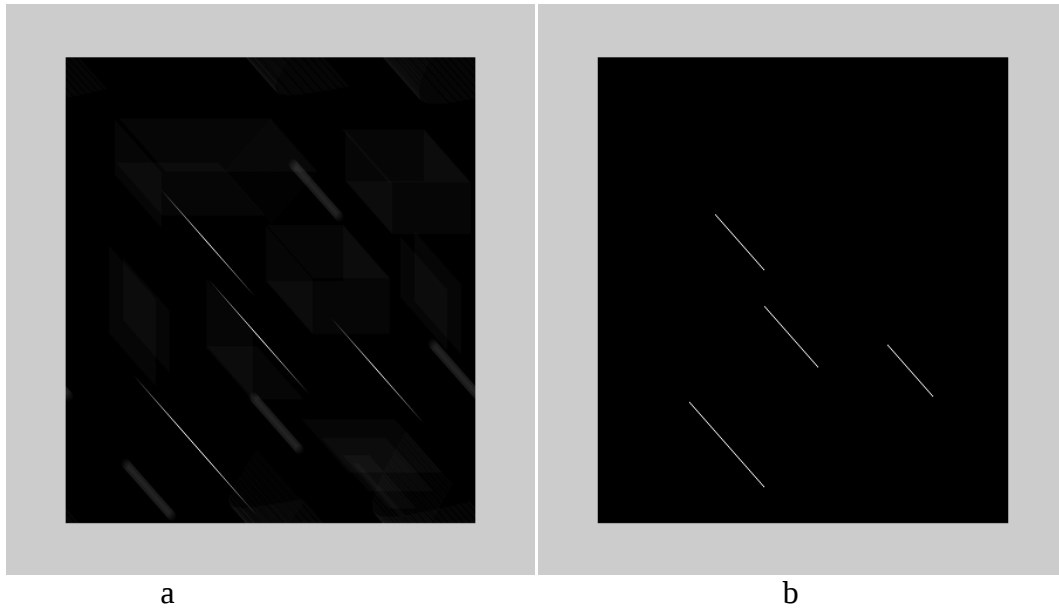


c-crop dot sample



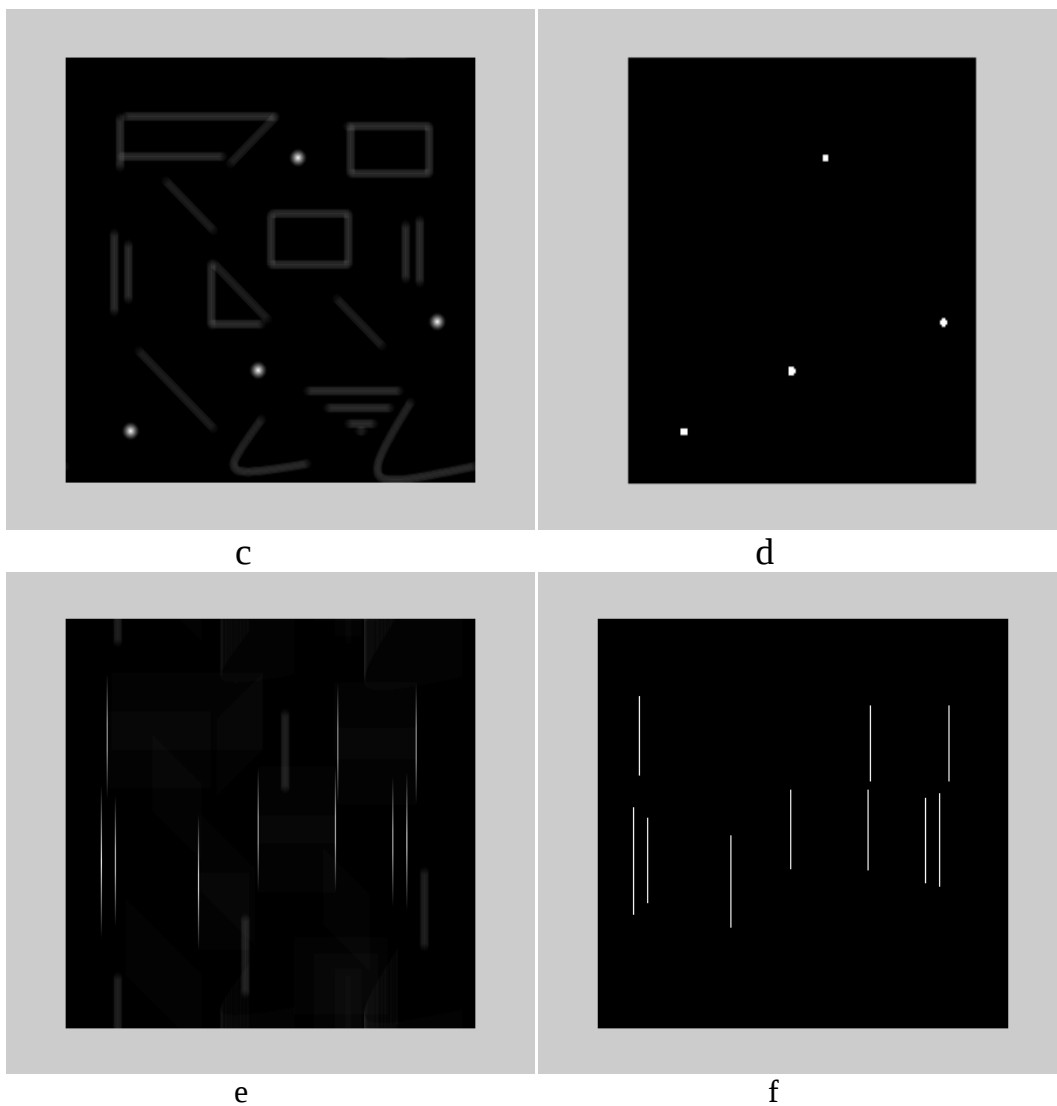
d- crop vertical line sample

Figure(7)a- original image with crop sample(b,c,d).

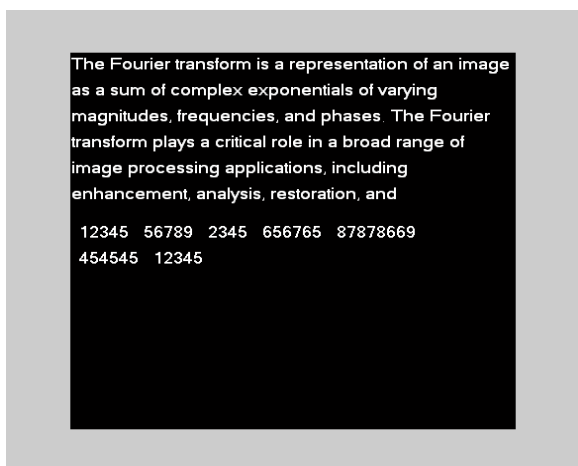


a

b



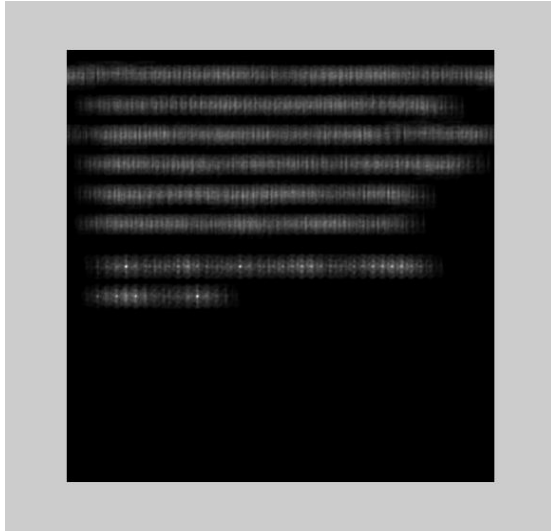
Transformed images(correlated image) (a,c,e) Thresholded images(b,d,f)
 Figure(8) Transformed images with Thresholded images



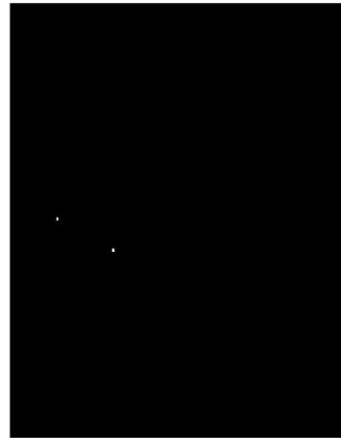
a-Original image



b-crop the sample



c-Transformed image (correlated image)



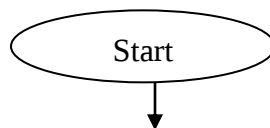
d- Thresholded Image

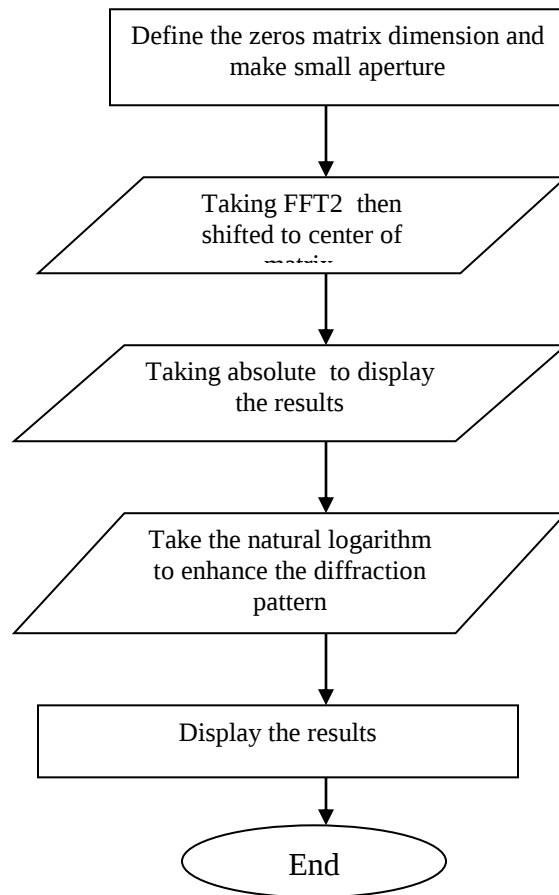
Figure(9)Correlated image with Thresholded Image Showing Template Locations

4- Diffraction Patterns

Diffraction occurs when light deviates from a straight line path and enters a region that would otherwise be shadowed. This deviation from general optics behavior occurs when waves pass through small openings, around obstacles, or past sharp edges. These patterns can only be described by using the wave theory of light which predicts these "smeared" areas of constructive and destructive interference[14].

The idea of optics predicts that the diffraction pattern produced by a plane wave incident on an optical mask with a small aperture is described, at a distance, by the Fourier transform of the mask[15]. The following creates a logical array describing an optical mask M with a circular aperture A of radius R . Fig(10) illustrate the Flowchart of the Diffraction pattern using Fourier Transformation.





Fig(10)flowchart of the Diffraction pattern.

4-1 Results and discussion

Fig (11)represents the circular aperture with radius $R=18$ in matrix with dimension 1000 by1000 and the diffraction pattern using FFT2 command.

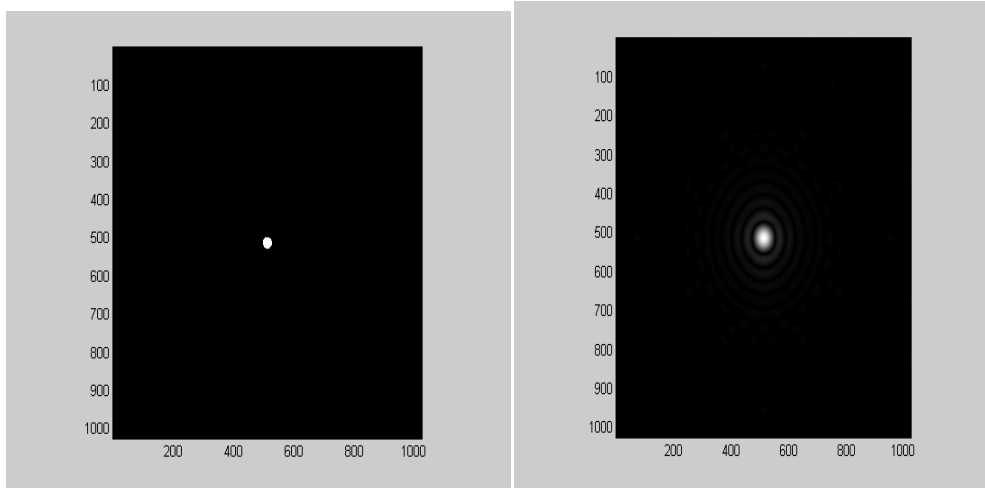


Figure (11) Circular aperture with diffraction pattern using FFT.

To see the intensity distribution in one dimension we can plot pixel value of column or row passes through the center of matrix as fig(12).

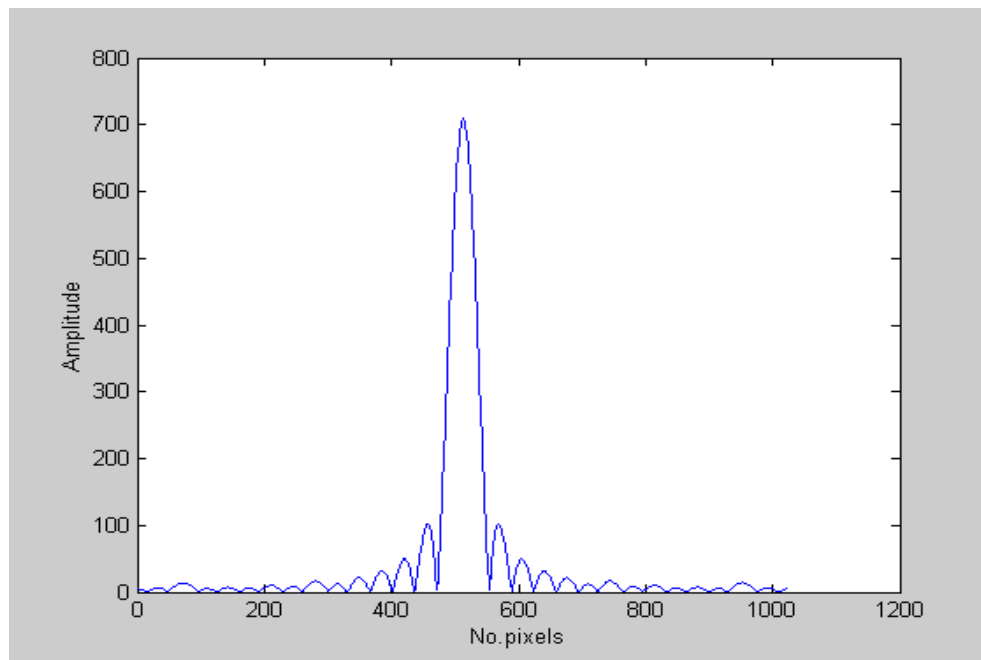
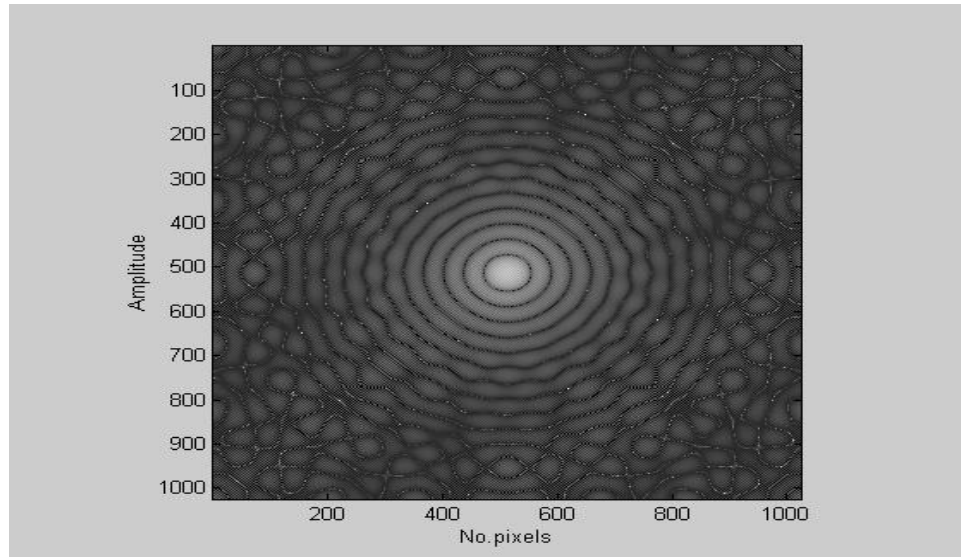


Figure (12) One dimension plot of intensity distribution.

The logarithm helps to bring out details of the DFT in regions where the amplitude is small illustrate in figure (13).



Figure(13)Diffraction enhancement using log method.

Very small amplitudes are affected by numerical round-off. The lack of radial symmetry is an artifact of the rectangular arrangement of data.

The peak at the center of the plot is $F(0,0)$, which is the sum of all the values in $f(m,n)$. Also we can see the effect of the enhancement by the one dimension plotting which indicated as figure (14).

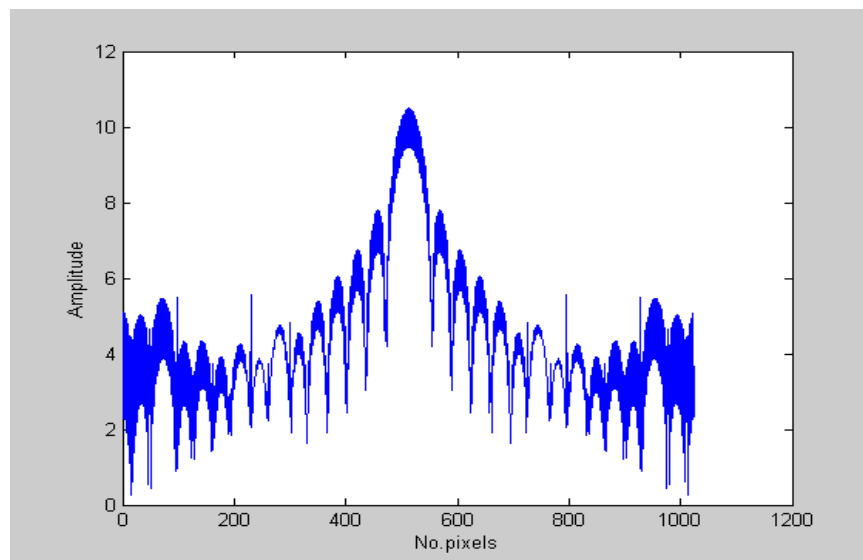


Figure (14): One dimension plotting after enhancement process

5- Conclusion:

There are three principal reasons for using this form of the transform:

- The input and output of the DFT are both discrete, which makes it convenient for computer manipulations.
- There is a fast algorithm for computing the DFT known as the fast Fourier transform (FFT) this done by calculating the zeros values of the transformation data.
- Utilizing Fourier Transforms can provide new ways to do familiar processing such as enhancing brightness and contrast, blurring, sharpening and noise removal. But it can also provide new capabilities that one cannot do in the normal image domain. These include de-convolution (also known as de-blurring) of typical camera distortions such as motion blur and lens defocus and image matching using normalized cross correlation.

6- References

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