

مقاومة الثني في السقوف المستوية ذات الفتحات المعرضة للأحمال المنتظمة التوزيع والأحمال الخطية

الخلاصة

يقدم هذا البحث دراسة تحليلية نظرية لاحتساب قابلية تحمل للبلاطات الخرسانية المسلحة ذات الفتحات المحملة بالأعمدة فقط و التي تكون تحت تأثير الأحمال الخطية على حواف فتحاتها إضافة للأحمال التصميمية الموزعة على سطوحها. تم إجراء تحليل لدن وآخر مرن لهذه السقوف حيث استخدم برنامج حاسبة الكترونية لإيجاد السلوك المرن أثناء التحميل إلى مرحلة ما قبل التشقق. أستند التحليل للدن في هذه الدراسة على نظرية الخط اللدن في البلاطات باعتماد آلية الانهيار التي لوحظت عمليا عند فحص هذا النوع من البلاطات من قبل باحثين آخرين. هذا وقد استخدم برنامج الحاسبة الكترونية الذي يعتمد علي طريقة الشرائح المحددة في التحليل المرن. لقد أجريت مقارنات بين نتائج التحليل النظري المقترح في هذه الدراسة والنتائج العملية التي نشرت من قبل باحثين آخرين . لقد بينت النتائج النظرية إن نسبة التحمل القصوى للسقوف إلى مقاومة الثني تتناسب عكسيا مع نسبة حجم الفتحات إلى فضاء البلاطة. إن المعادلة المقترحة في هذه الدراسة تبين بان مقاومة الانحناء للبلاطات المنتخبة في هذه الدراسة تتناقص بدرجة كبيرة كلما ازداد حجم الفتحة في السقف. لقد أظهرت هذه الدراسة توافقا جيدا بين النتائج النظرية والنتائج العملية المنشورة من قبل باحثين آخرين. لقد بين التحليل المرن بان حمل التشقق و الإزاحة المركزية الناتجة عن طريقة الشرائح المحددة تتوافق جيدا مع النتائج العملية التي نشرت من قبل باحثين آخرين.

1-Introduction

Reinforced concrete flat slabs are the most economical type of construction in multistory buildings. It is necessary in these buildings to pass many pipes and ducts through slabs to accommodate essential services such as electricity, computer network, water supply, sewerage and air conditioning ducts. Therefore, providing openings in slabs will be the best solution for building services.

In general, elastic solutions are available only for restricted conditions, usually uniformly loaded rectangular slabs and slab system. They do not account for the effect of inelastic action except empirically. Furthermore, it is difficult to predict the flexural strength of flat slab with openings by elastic solution. However, yield line theory gives a powerful tools for analyzing flat plate slab with opening under any type of loading. It generally underestimates the actual test results Freguson (1981).

2-Review of literature

Article 13.4 of ACI-Code (2005) permits openings of any size in slab if shown by analysis that the required strength can safely be achieved. Limited sizes of openings in slabs is permitted in ACI-Code with special precautions. On the other hand, location of openings in slabs with respect to columns or concentrated load may affect the critical slab sections for shear as given in article 11.12.5, ACI-Code (2005). However, ACI-Code does not take in consideration the effect of line load around the opening edge resulting from partition wall. Generally, the analysis of slab with opening having line load around its own edge requires rational analysis with special attention for flat slab.

Mansur and Tan (1999) had proposed analysis and design procedure for beams with circular and rectangular openings. The analytical model proposed is able to handle combined bending, shear and torsion in beams with openings, and subsequently design the reinforcements required for this combined action. However, the proposed analysis and design procedure are not applicable to reinforced concrete slabs.

Park and Gamble (2000) conducted a review on analysis of reinforced concrete slabs with openings and reported that an opening in a simply-supported square slab with dimension of 0.2 to 0.3 times of the slab dimension would cause a reduction of 11% in terms of ultimate load per unit area. Larger opening with dimension of 0.5 or more times the slab dimension would not result in reduction of ultimate load per unit area.

El-Salakawy et al. (1999) tested six full-scale reinforced concrete slabs, of which five were slabs with various arrangements of openings in the vicinity of the column. The openings in the prototypes were square with the sides parallel to the sides of the column; and there were two opening sizes, one which is the same size as the column and the other is 60% of the column size. It was reported that the larger and smaller opening sizes led to reduction in ultimate strengths of 30% and 12% respectively.

Another full-scale testing on reinforced concrete slabs with openings was carried out by Teng et al. (2004). In this study, 20 slabs with openings supported on columns of various sizes were tested. This study distinguishes itself from the study carried out by El-Salakawy et al. (1999) by the arrangements of columns and openings. The slabs tested by Teng et al. (2004) had column support in the middle of the slabs whereas slabs tested by El-Salakawy et al. (1999) were having column support in the middle of the longer edges of the slab. It was reported by Teng et al. (2004) that openings reduce punching shear strength of slabs considerably, and the recommended locations for openings in slabs are along the longer side of a column.

Ng et al.(2008), presented theoretical evaluation for the ultimate load-carrying capacity of simply supported slabs with square opening. The approach did not consider the case of flat slab with wall load (linear load) around opening edge which is the usual case in practice. However, the

proposed method has not been verified experimentally.

Due to the difficulty of the problem of opening in flat slab under concentrated loads, Tayel, et. al. (2004), carried out an experimental study on the effect of square and circular opening on the behavior of flat slab supported on four corners. The test specimens have square and circular openings near support and at center of span which have been tested under concentrated load. Twelve square slabs (1.5 m x 1.5 m and 4.0 or 6.0 cm in thickness) models having square opening (200mm, 300mm and 500 dimensions). They were tested under concentrated load. Another two control models without openings were tested under the same conditions for comparison.

The slabs tested by Tayel, et. al. (2004), were supported on four corner and loaded in increments up to failure. Deflections, strains and cracks propagation were recorded after each increment. Also, the first cracking loads and the ultimate failure loads were recorded. Test showed that the opening reduced the strength and rigidity of the flat slab. This reduction is slightly greater for opening near support. It was concluded that the thicker slabs are much better than those with small thickness. The circular openings are most suitable than rectangular ones, and no need to take any precautions regarding the thickness or reinforcement if the opening diameters or side length is less than or equal to one tenth of the slab side length.

3- Objective of the present study

The flexural strength of flat plate slab with opening requires much more extensive research work .Therefore, the main objective of this investigation is to derive theoretical procedure to predict the flexural strength of flat plate slab with opening under both line load and uniformly distributed load. In order to verify of the proposed approach derived here, the theoretical results of this analysis were compared with test results published by Tayel, et al. (2004) and are to be shown in this investigation.

4- Theoretical analysis

4-1 Yield line theory

Generally, yield lines form under concentrated loads, radiating outward from the point of applications as shown in Figures (1) and (2), whereas, they intersect the free edges in flat plate supported on four corners Figure (3) and Figure (4).In case of simply supported slab with opening, the yield line pattern becomes as shown in Figure (5).When line load is applied on simply supported slab the yield line changed into pattern shown in Figure (6). Patterns of Figures (4,5) were obtained experimentally by Elstner and Hognestad (1956).

The analysis is based on the yield lines pattern proposed in this investigation as given in Figure (7) or Figure (8). This pattern is compatible with that found experimentally by Tayel, et. al. (2004)

in test results. This pattern represents the combination of pattern found by Elstner and Hognestad (1956) for flat plate under concentrated load (Figure 4) and well known pattern for simply supported slab with opening (Figure 5).

Figure (8) shows a square flat plate supported on four corners by columns and having a square opening at center. The slab is subjected to uniformly distributed load (W) with linear load around the edges of opening (Q). Using the principle of virtual work for the collapse mechanism shown in figure (8), the relation between the applied loads and the resisting moments of the slab can be obtained:

Let:

a = width of square opening in (m)

L = span length of square slab in (m)

m = resisting moment along yield line of length in (N.m/m)

P = total line load around opening = 4Qa (N)

W = uniformly distributed load in (N/m²)

Q= line load along edge of opening in (N/m)

δ = arbitrary virtual displacement

θ₁ = angle of rotation of yield line parallel to edges in radian

θ₂ = angle of rotation of diagonal yield line

β = Q / W

External work done by loads = Internal work done by resisting moments

External work = P x δ

$P x \delta = P x \delta + 8 x W [(1/2) (L-a) x (1/2) (L-a) x(\delta/3)] + 4[W x a (L-a) x(\delta/2)]$

$\theta_1 = (\delta/2)/(0.5)(L-a)$

$\theta_2 = 2 \delta / N$ For (N) see Figure(8)

$\theta_2 = 2 \delta / [0.5(\sqrt{2}) (L-a)]$

Internal work = m x θ x l

$m x \theta x L = 8 [m x (\delta/2) / 0.5 (L-a)] x 0.5 (L-a) + 4 [m x (2\delta/0.5\sqrt{2})(L-a)x 0.5\sqrt{2}(L-a)]$

$m x \theta x L = 12 x m x \delta$

$P x \delta = m x \theta x L$

$P x \delta + 8 x W [(1/2) (L-a) x (1/2) (L-a) x(\delta/3)] + 4[W x a (L-a) x(\delta/2)] = 12 x m x \delta$

Simplify,

$$P + W (1/3)(L-a)(L+2a) = 12 m \dots\dots\dots(1)$$

Substitute; $P = 4aQ$ & $Q/W = \beta$ in the above equation and simplify ;

$$W = \frac{36}{[12a\beta + (L-a)(L+2a)]} m \dots\dots\dots (2)$$

For practical cases , the design ultimate loads in slabs for most usual cases are calculated to be :

$Q = 9$ kN/m for 0.12m width brick partition

$Q = 18$ kN/m for 0.24 m width brick wall

$W =$ the sum of L.L. and D.L. which ranges between 15 TO 18 kN/m²

Therefore, (β) ranges between 0.5 to 1 for the practical cases

When the slab is subjected to linear load around edge of opening only ($W=0$) and $P= 4Qa$, the failure line load (Q) can be obtained from equation (1) as :

$$Q = \frac{3}{a} m \dots\dots\dots (3)$$

4-2 Effect of opening size on the failure load

To show the effect of opening size on the failure load of slab (flexural capacity of slab), equation (2) is rearrange in the following form;

$$\frac{W}{m} = \frac{36}{[12a\beta + (L-a)(L+2a)]} \dots\dots\dots (4)$$

Equation (4) represents family of curves for various value of (β) . This relationship shows that the ratio of collapse load to yield line moment (W/m) is inversely proportional to the opening size. In other word, the flexural capacity of slab is clearly reduced as the opening size increases.

Equation (4) is plotted for the slabs tested by Tayel, et. al. (2004) considering practical limits of $(\beta = 0.5, \beta = 1)$ as shown in Figure (13). This relation shows the reduction in strength of slab with increasing the size of opening. This is compatible with the test results reported by Tayel, et. al. (2004) and El-Salakawy et. al. (1999). This reduction becomes smaller as the ratio of opening size to the span of slab higher than 0.5. This is again agreed with test results reported by Park & Gamble (2000).However, The derived equation gives reasonable correlation between flexural strength of slab and opening size.

5-Elastic behavior of slab

The flat plate slabs specimens tested by Tayel, et al (2004) are analyzed here by finite element method (F.E.M.) using STAAD/Pro (2007) computer program. Sample of analysis results are

presented in the form of contour lines of bottom stress as shown in Figures (9, 10, 11, 12). These figures show that the maximum tensile stress due to applied line load along edge of opening develops at the corners of the opening. Therefore, the first crack should initiate at the corner. This was clearly agreed with slab specimens tested by Tayel, et al. (2004) as shown in figures (14, 15, 16, 17).

The maximum deflection at mid point of the opening edges were obtained from analysis of slabs (C2A4, C3A4, C5A4) by finite element method (FEM). The deflections corresponding to about first cracking load for both FEM analysis and those observed in test by Tayel, et. al. (2004) were plotted against opening size for slabs C2A4, C3A4, C5A4. as shown in figure (19).It can be seen that the deflection increases as the opening size increases. FEM analysis showed slightly less deflection than that observed by test. This may be due to the procedure of loading through testing in which the loading exerted through rigid box around opening edges whereas point loads on the nodes in FEM analysis. However, figure (19) shows that test results agrees well with FEM results.

6- Comparisons between theoretical Ultimate strength and test results

The proposed equation (1) is used to calculate the theoretical failure loads of slabs tested by Tayel, et. al. (2004). The design distributed load (W) in this case is the self-weight of slab only which is very small compared to the applied linear load (P). However, failure load (P) has been determined for two groups of flat plate slabs with opening having 40mm and 60mm thickness respectively (C2A4,C3A4,C5A4, SOX4, C2A6,C3A6,C5A6,SOX6).

The main variable in this investigation is the opening size. The ratio of opening size to span length for these slabs is ranging from 0 to 0.345. The calculated values by equation (1) and those obtained by test have been given in table (1). The ratio of theoretical to test results is ranging between 0.7 to 0.94 with average value of about 0.82.

Generally, equation (1) slightly underestimated the failure load of slabs tested by Tayel, et. al. (2004). The differences between theoretical values and the test results may be attributed to the flat arch action or membrane action of the test slabs at ultimate stage. However, the proposed equation (1) can predict reasonable failure load of flat plate slabs with opening compared with test results.

Test results and theoretical results have been plotted against the opening size as shown in figure (20). Figure shows that the theoretical values agree well with the test results. Both theoretical failure load and those found from test results show slight decrease as the opening size increased. This behavior is compatible with figure (13). By examining equation (1) carefully, it indicates that; when the slab is subjected to only linear distributed load along opening edges (no distributed load, $W=0$), then, total collapse load P would not affected by opening size ($P = 12m$).Such a case would not practically occur ,since, at least there is self-weight of slab acting besides the linear load around the opening edges.

7- Conclusions

An investigation on the effect of opening on the elastic and flexural strength of the flat plate slab supported on columns was analyzed theoretically. A theoretical equation was derived to evaluate the collapse load of flat plate slab with opening supported on four corners subjected to both uniformly distribute load and uniformly line load along opening edges. The analysis was based upon yield line theory using collapse mechanism which compatible with that observed in test results. The predicted behavior was compared with test results carried by others. Some conclusions can be drawn as;

1- The proposed equation shows that the theoretical collapse load is drastically reduced as the opening size increases. The reduction in flexural strength reaches to about 70% when the opening size increased to about 25% of slab area. A good agreement is found between the theoretical flexural strength proposed here and those observed by test results.

The average ratio of theoretical strength to the test result was about 0.82. On the other hand, finite element results of cracking loads and elastic deflections have shown good agreement with those obtained from test results. The average ratio between them was about 0.8.

2- When the slab subjected to only uniformly distributed line load along the opening edges, elastic center deflection of slab is directly increased as the ratio of opening size to span length increases. The percentage of increase is reached up to 30% when the ratio of opening width to span length increased to about 0.35.

3- The theoretical flexural strength of the slab subjected to only line load distributed uniformly along opening edges is independent on the opening size located in the center of the slab.

4- Compared to test results, the proposed equation in this study can predict safe and reliable failure load of flat plate slab with square opening located at center of slab.

8- References

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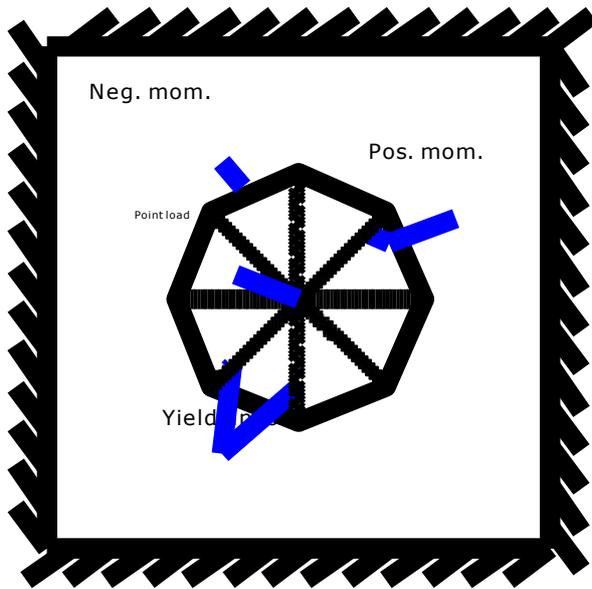


Figure (1): Yield lines pattern in slab under point load (Type 1)

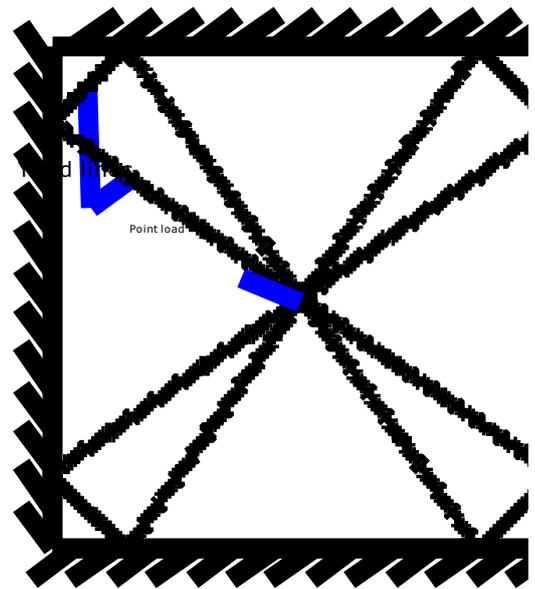


Figure (2): Yield lines pattern in slab under point load (Type 2)

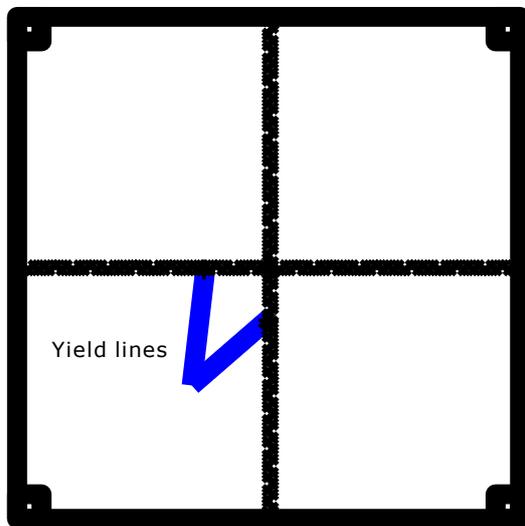


Figure (3): Yield lines pattern in flat slab due to uniformly distributed load

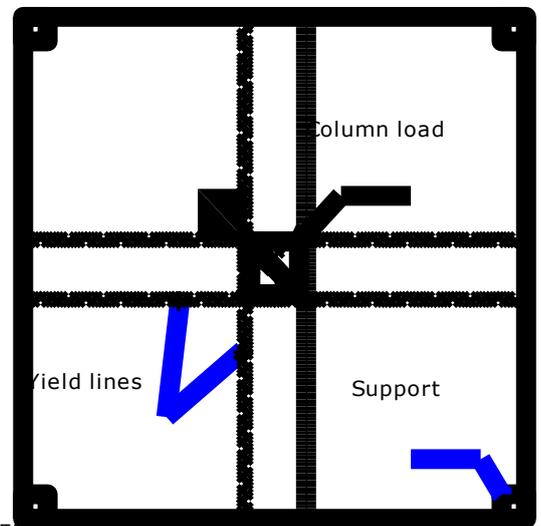


Figure (4): Yield lines pattern in flat slab due to column load

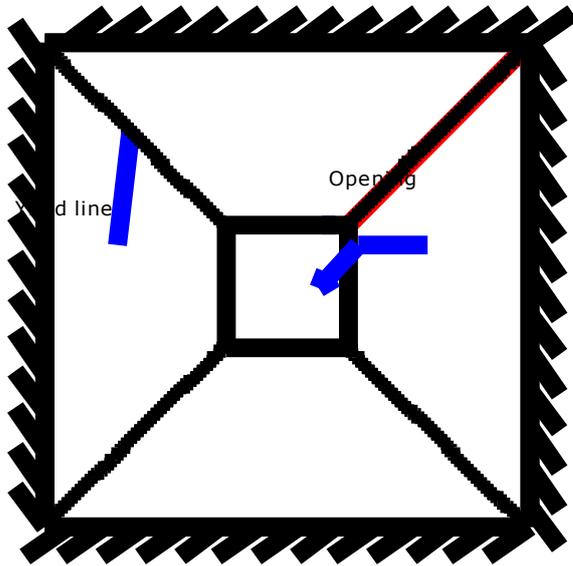


Figure (5):Yield lines pattern in slab with opening under uniformly distributed load

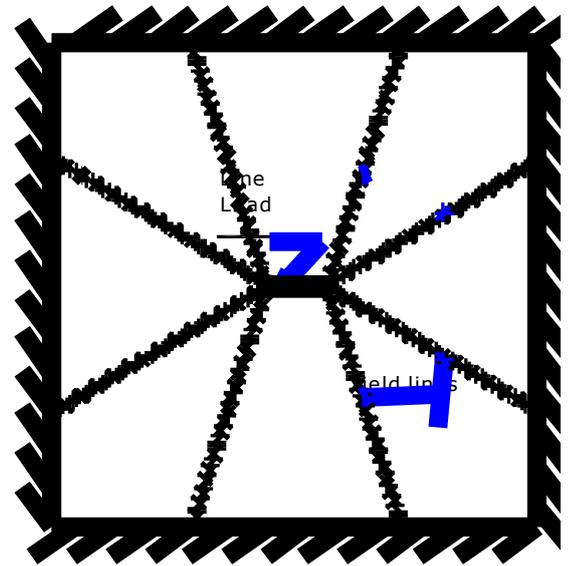


Figure (6):Yield lines pattern in slab under line load

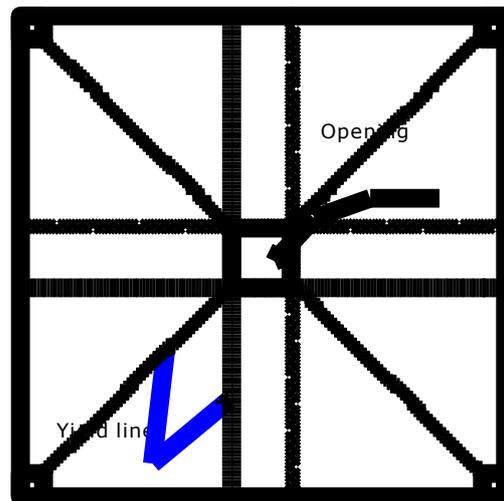


Figure (7):Yield lines pattern in flat slab under line load around opening

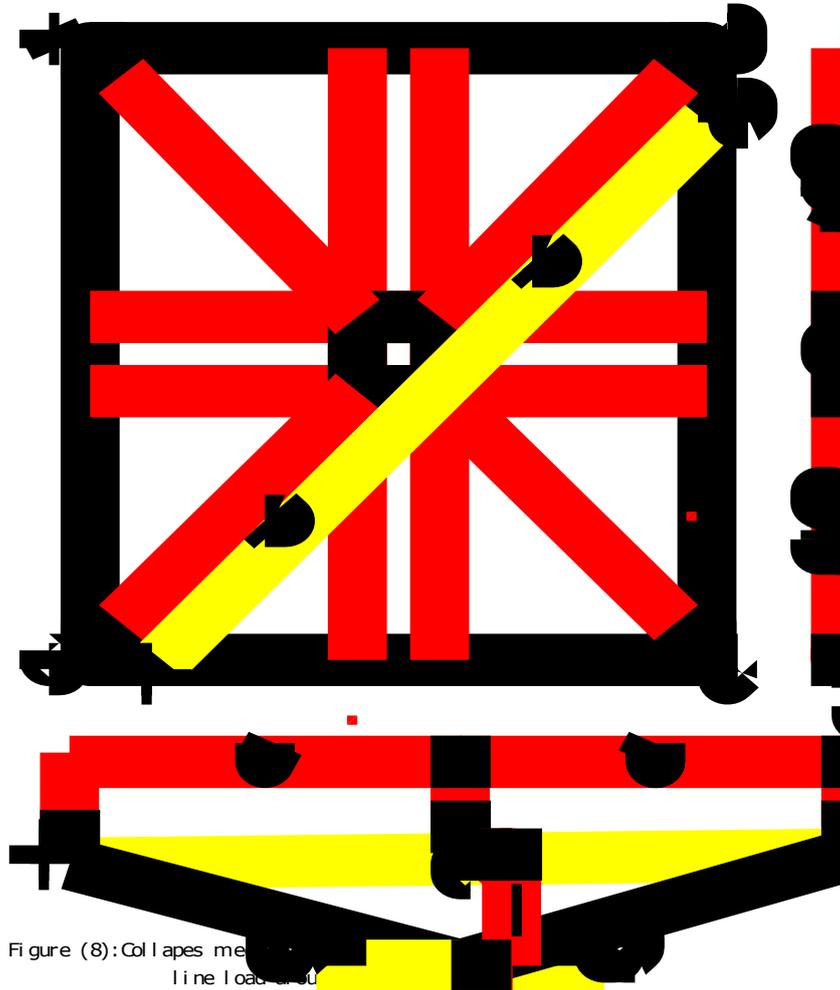


Figure (8): Col laps me
Line Load of 0

Table(1):Comparison between theoretical results of equation proposed in this study and test results of slabs tested by Tayel, et al.(2004).

Slab Mark	Opening size cm x cm	Slab thickness cm	1 st Cracking Load kN	Exp.Failure Load kN	Theo.failure Failure kN	Theo. ——— Exp.	
C2A6	20X20	6	14.5	26.84	19.07	0.71	
C3A6	30X30	6	13.88	24.9	19.05	0.77	
C5A6	50X50	6	13.42	22.9	19.06	0.83	
C2A4	20X20	4	7.23	13.5	11.37	0.84	
C3A4	30X30	4	6.84	12.4	11.36	0.92	
C5A4	50X50	4	6.06	12.1	11.37	0.94	
SOX6	None	6	13.59	27.95	19.18	0.70	
SOX4	None	4	7.55	13.5	11.37	0.84	

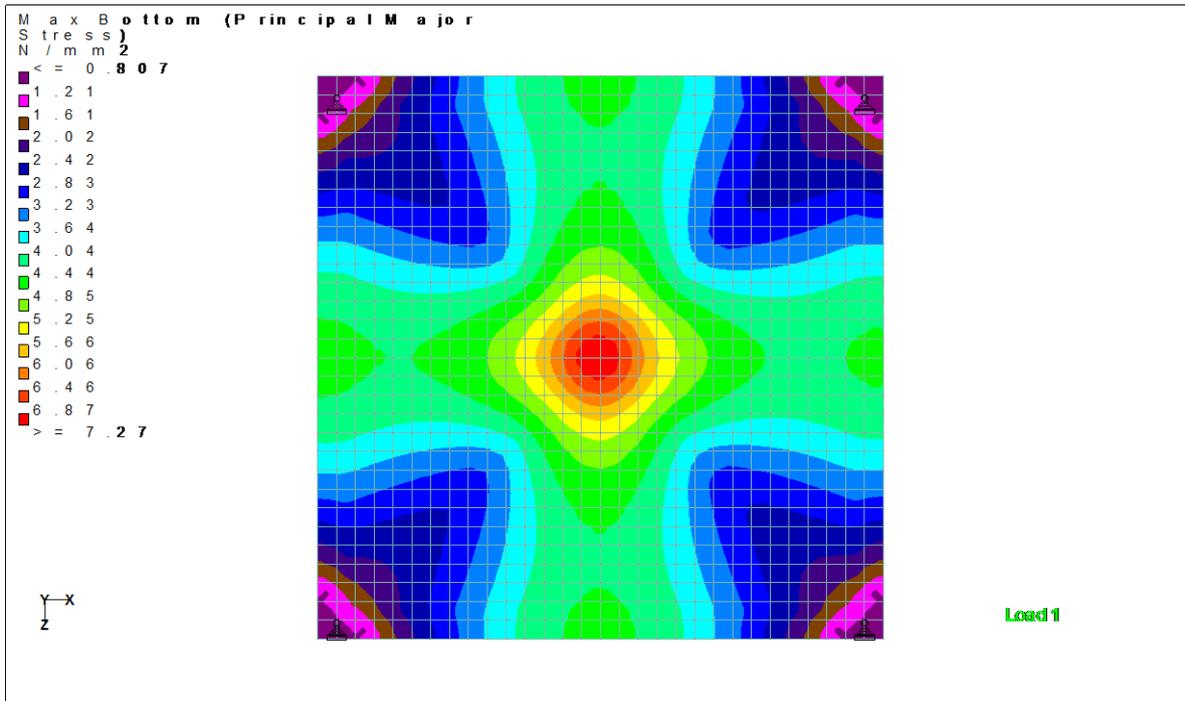


Figure (9) : Bottom stress contour in slab without opening

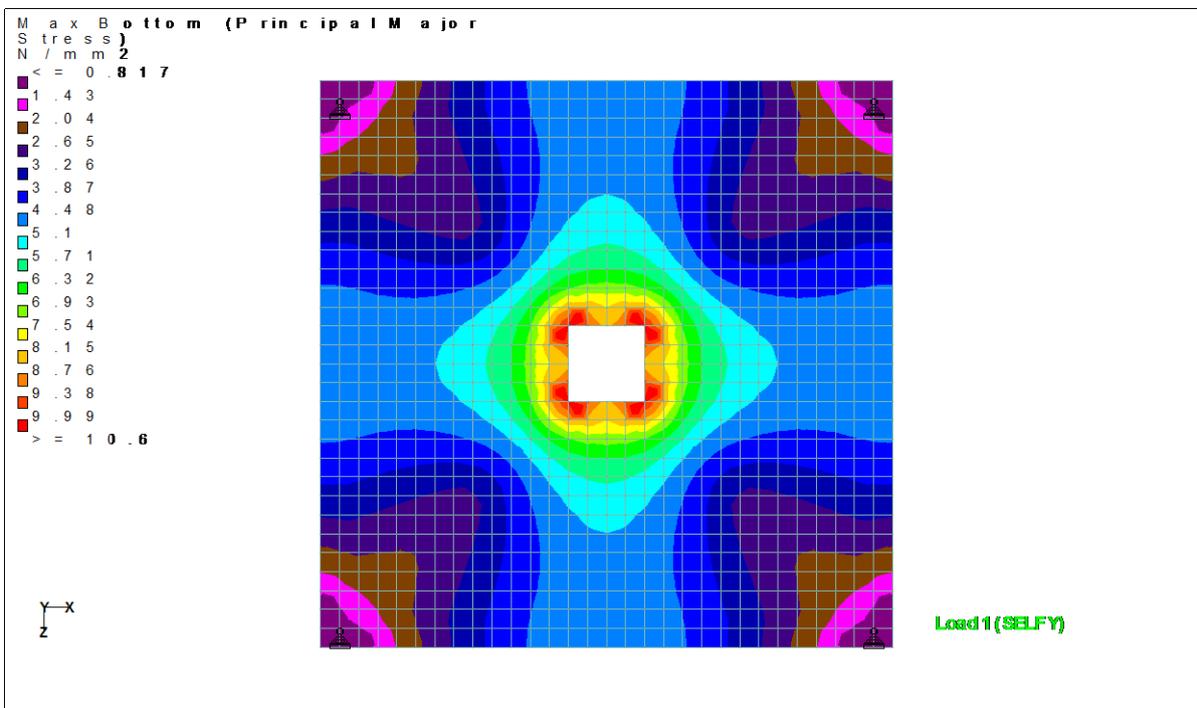


Figure (10): Bottom stress contour in flat plate slab with opening 0.2 x 0.2 m

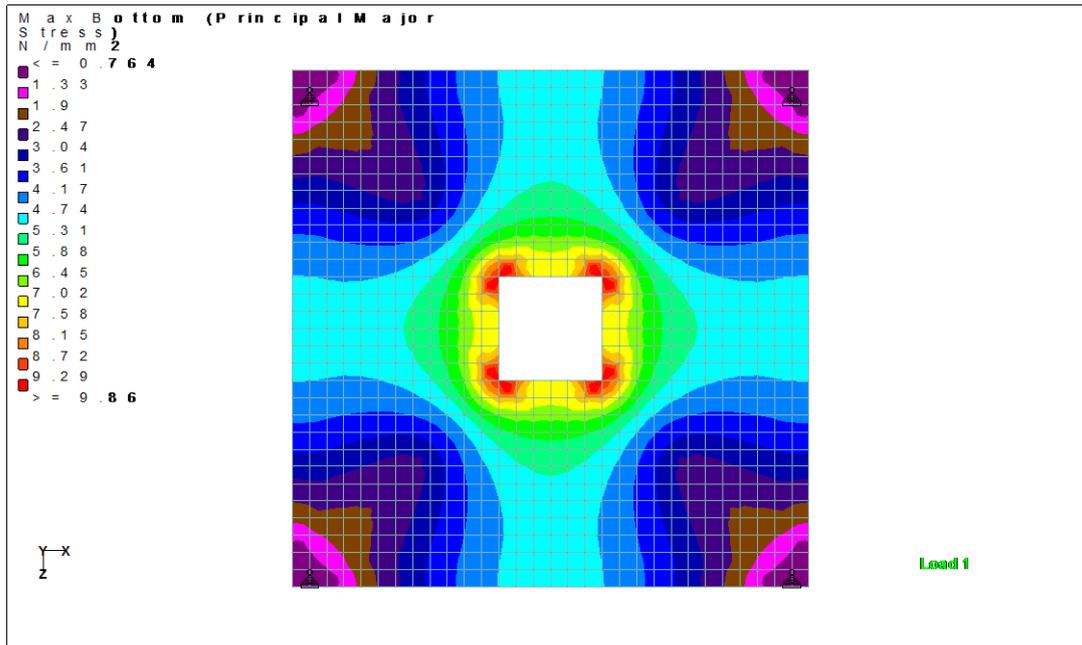


Figure (11): Bottom stress contour in flat plate slab with opening 0.3 x0.3m

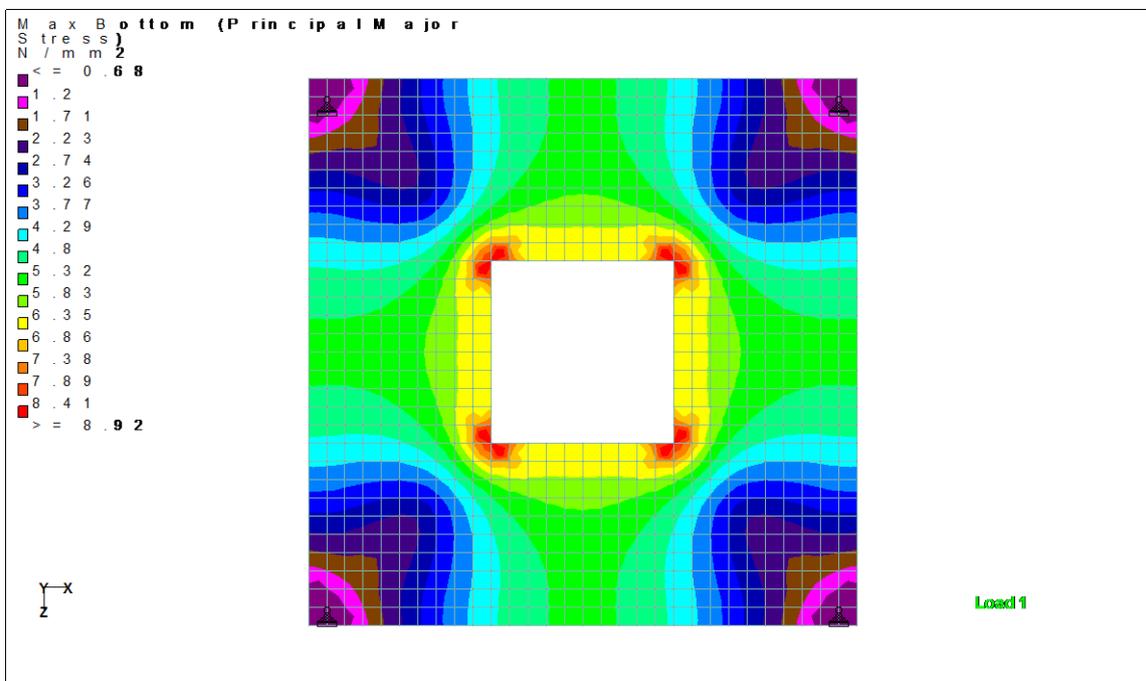
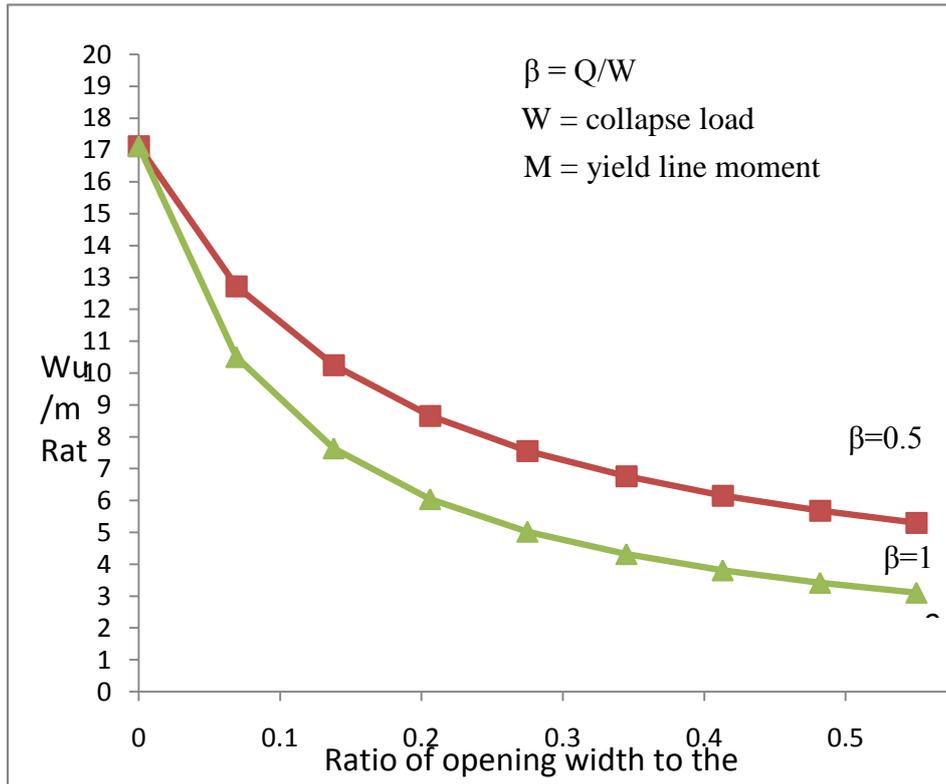


Figure (12) : Bottom stress contour in flat plate slab with opening 0.5 x0.5m



Figure(13): Relation between opening size to span length ratio

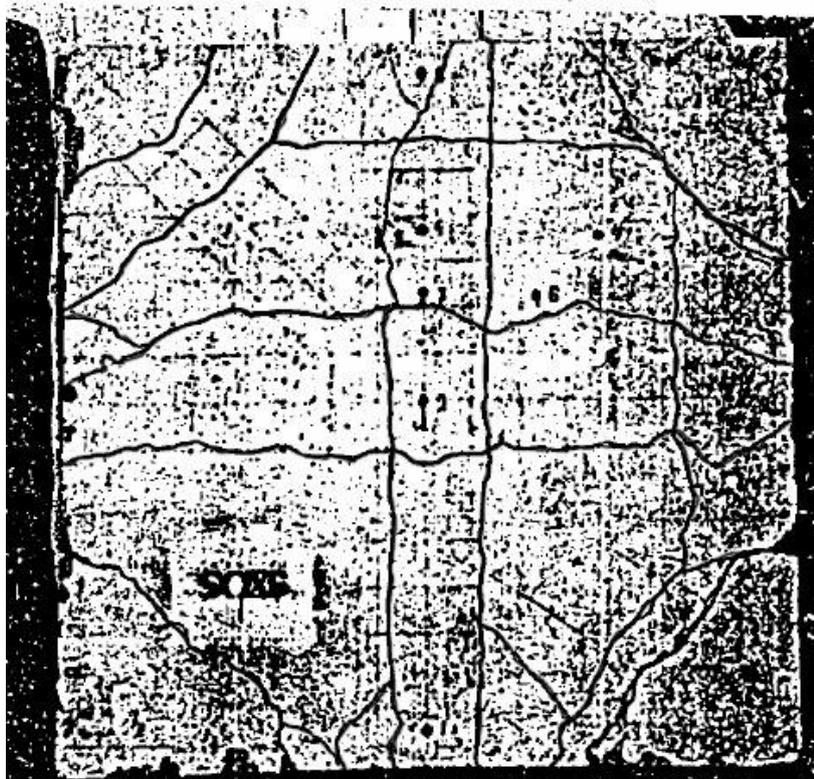


Figure (14) : Cracks pattern of 60mm thick slab (SOX6) without opening

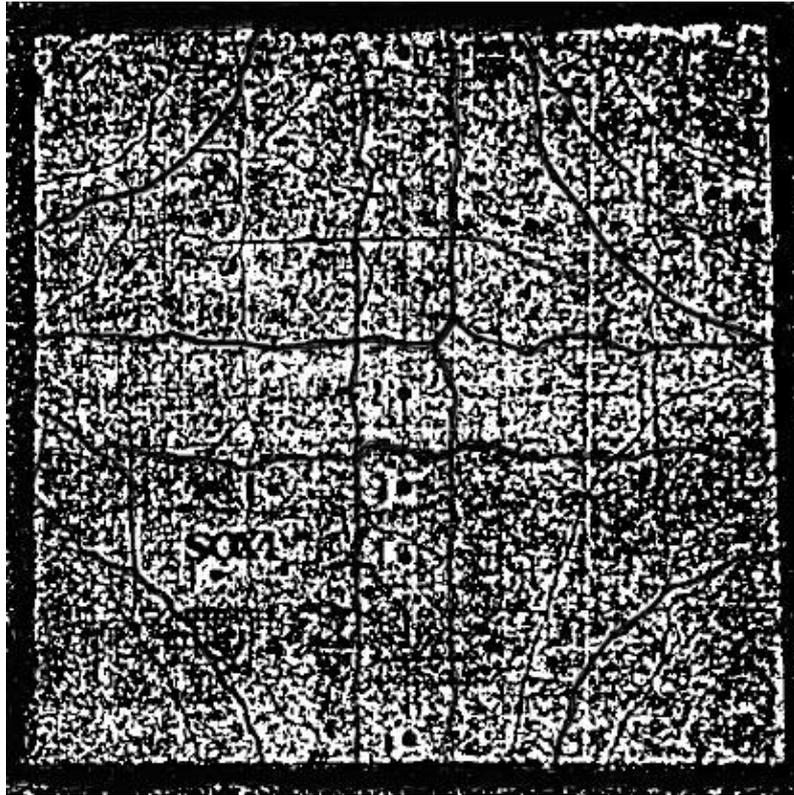


Figure (15): Cracks pattern of 40mm thick slab (SOX4) without opening

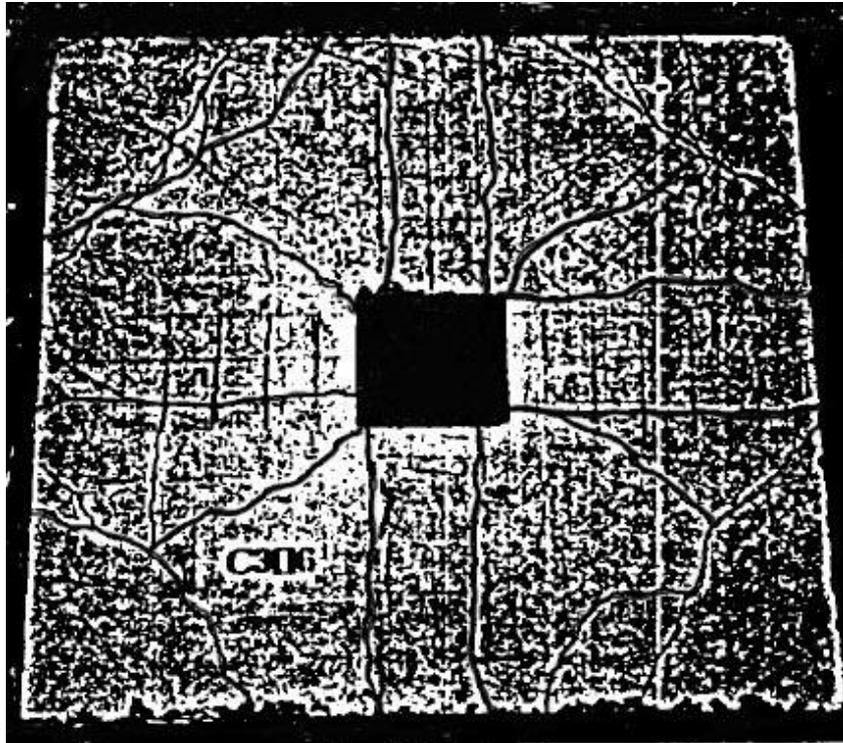


Figure (16) : Cracks pattern of 60mm thick slab (S306) with opening of 300x300 mm

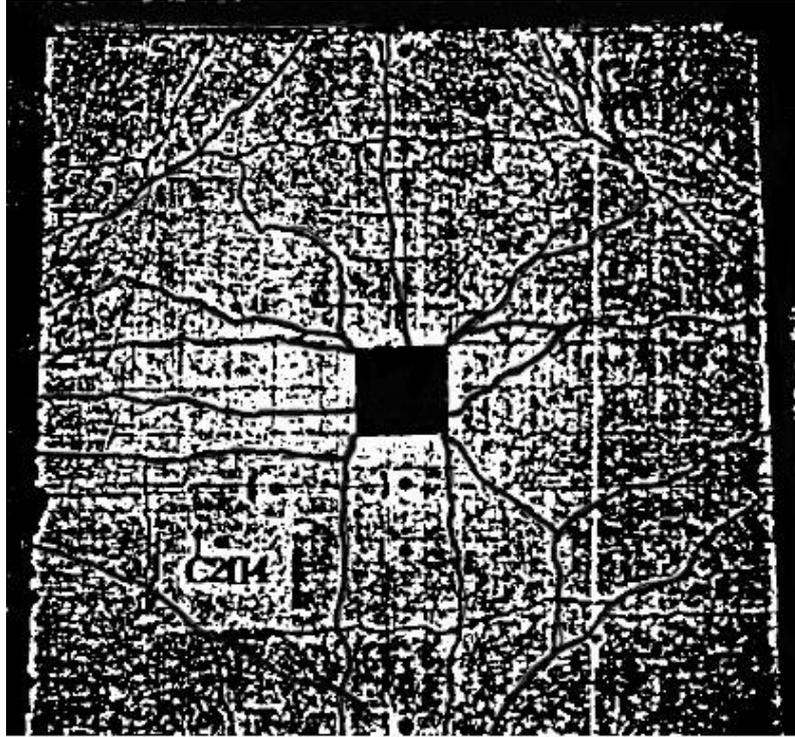


Figure (17) : Cracks pattern of 40mm thick slab (S204) with opening of 200x200 mm

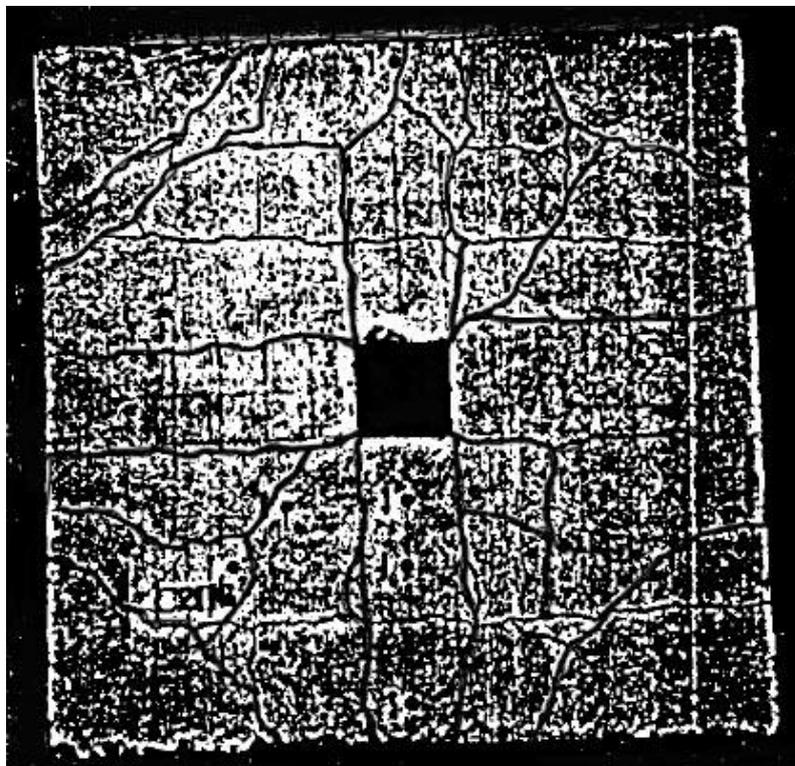


Figure (18) : Cracks pattern of 60mm thick slab (S206) with opening of 200x200 mm

