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Mathematical model for spray drying process

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موديل رياضي لعملية التجفيف الرذاذي

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الخلاصة

تم ايجاد موديل رياضي للمجفف الرذاذي للجريان المتماثل للمادة والوسط المجفف وتطويره وهو نوع من ثنائي الابعاد و ثنائي المزدوج . يتضمن الموديل انتقال الكتلة والزخم والحرارة بين القطرة الواحدة والوسط المجفف . ان النظام المستخدم هو الماء والهواء.

لقد استخدم نظام من المعادلات لحساب التغير في قطر القطرة وسرعتها ودرجة حرارتها ومعدل التبخير ،ودرجة حرارة الهواء ورطوبته وسرعة الهواء في حجرة المجفف الرذاذي.

يتضمن الموديل معادلات تفاضلية غير خطية من الدرجة الاولى كدالة للمسافة العمودية لحجرة المجفف الرذاذي .

لقد تبين بان النتائج التي تم الحصول عليها من الموديل الرياضي متطابقة مع النتائج العملية.

Abstract

Mathematical model for co-current spray dryer is developed . The model is axisymmetric dimensional model and two way coupling .It includes the mass, momentum, and heat transfer for a single drying droplet as well as for drying medium . The system used is water-air.

The model used a system of equations to compute the axial and radial distribution of droplet diameter, its velocity , temperature and rate of evaporation , and air temperature , humidity and velocity in spray drying chamber .

The model includes a system of non linear first order differential equations as a function of the axial distance of the spray drying chamber .

The results obtained from the model are shown to be in agreement with experimental results.

Introduction

Spray drying transforms a liquid feed (slurry) material to a dry powder by spraying the feed into a hot drying medium,. Spraying is done with an atomizer either rotary or nozzle atomizers,where feed is broken into a large number of droplets generally assumed to have a spherical shape.The initial contact between spray droplets and drying air is either co-current ,counter current or mixed flow,and evaporation of water from droplets under controlled air temperature and air flow, resulting in the formation of particles (powder) which are separated from air[1].

Spray drying chamber typically are vertical vessels with a cylindrical cross section and conical bottom.The time and residence time required for complete drying depend on the rate of heat and mass transfer between the droplets and the drying medium.The gas phase temperature and humidity determine the driving force for evaporative heat and mass transfer.

The droplets dry up in a fractions of a second at or near the wet bulb temperature.Thus heat sensitive material can be dried rapidly without damage.

The spray drying process has a variety of applications in food, ceramics, chemical and pharmaceutical industries[2].

Mathematical Model

Models are classified in terms of their geometry and degree of phase coupling. which means the interacting influences of droplet and air phases. Models with one way coupling include only the effect of air on the droplets.Two way coupling include not only the effect of air on droplet but also the effect of the droplet on gas. The geometry is classified as one dimensional (axial only) , quasi-one dimensional (axial variations with properties assumed uniform across each cross section ,or axisymetrical (variation in both radial and axial directions) but not circumferential direction[3].

One Dimensional Models.

1. One way Coupling One dimensional models

It is the simplest models, Marshall and Seltzer [4]presented equations for the life time of a droplet at terminal velocity. Sjenitzer[5] considered the evaporation of droplets at velocities higher than terminal velocity . Marshall[6] presented model for a distribution of droplet sizes and each size treated separately .Dlouhy and Gauvin[7] studied evaporation of spray containing dissolved solids.

2. Two way Coupling,One dimensional models

Dickinson and Marshall[8] developed Marshall model to include thermal coupling between droplet and air .Parti and Palanez[9] presented model for spray containing dissolved solids taking the mean diameter of droplets .Topar[10] make some deviation to Parti and Palanez model by taking variation of tangential , radial and axial velocity with time. Zbicinski,S et al[11] determined the distribution of moisture content and particle diameter and air properties along the dryer height.Antoine Negiz ,and et al[12] develop a model to determine the distribution of moisture content,density,temperature of droplet, along the dryer height.Plaencia, and et al[13] presented a method for estimate outlet product moisture as a function of the outlet air temperaturein the model.Seydel[14] detailed mathematical description of mass and energy processes during solid formation inside the droplet.The nature of the final particle structure is obtained by Seydel et[15] al who employ a population balance in the model to stimulate the solid phase. Mezhericher etal[16] present a model to predict pressure build up and temperature rising within the particle wet core.

Quasi- One dimensional models

1. One way Coupling ,quasi-one dimensional models

   Gluckert[17] developed a model based on an axial velocity of a jet.

2. Two way Coupling , quasi–one dimensional model

Katta and Gauvin[18] created a model based on dividing the chamber into a jet region and an annular free entrainment region and assumed that air velocity is unaffected by the presence of droplets.The energy equations for each region are used to predict air temperature. Miura et al[19] equation used this method for a pneumatic nozzle, equations are integrated using Ranz – Marshall equation and Miura et al equations. Gauvin and Costin[20] made some improvement in the model of Katta and Gauvin.They calculated tangential velocity of air.

Axisymmetric Dimensional models

1. One way Coupling,Axisymmetric Dimensional model

Lapple and shepherd[21] derived equations for droplet velocity components. Cozaloz[22] determined the moisture content of particle where unsteady state diffusion equation was applied.

2. Two way Coupling, Axisymmetric Dimensional models

Baltas and Gauvin[23] proposed model for the free fall zone of a spray dryer by dividing it into a series of annuli, air temperature and humidity is obtained for each annulas at time interval. Crowe et al[24] proposed a model based on regarding the droplet phase as a source of mass,momentum and energy to the gaseous phase . Papadakis and King[25] made some modification to Crowe model, they took grid spacing in the axial direction were closer together near the atomizer. Kincaid and Longley[26] presented model for predicting evaporation and temperature changes in water drops travelling through air used sensible heat transfer and diffusion theory.

The present mathematical Model

The mathematical model is two way Coupling , Axisymmetric dimensional model

1- **Diameter of droplet**

The rate of decrease of droplet mass is[1]

$$\frac{dw}{dt} = - k_g A \rho (x_d - x_b) \quad (1)$$

$$Sh = \frac{k_g d}{D} \quad (2)$$

$$\frac{dw}{dt} = - Sh (\rho D) \pi d (x_d - x_b) \quad (3)$$

according to Ranz-Marshall equation[27]

$$Sh = 2 + 0.6 Re^{0.6} Sc^{0.33} \quad (4)$$

$$\frac{dw}{dx} = \frac{dw}{dt}\ \frac{1}{v_p} \qquad (5)$$

$$\frac{dw}{dx} = \rho_d\ \frac{dV}{dx} \qquad (6)$$

$$V = \frac{\pi}{6}\ d^3 \qquad (7)$$

$$\frac{d(d)}{dx} = \frac{dw}{dx}\ \left(\frac{2}{\pi d^2 \rho_d}\right) \qquad (8)$$

2 – Temperature of droplet

The heat balance for the droplet is

$$Q = h\ A(T_a - T_d) = \lambda\left(-\frac{dw}{dt}\right) + m\ c_d\left(\frac{dT_d}{dt}\right) \qquad (9)$$

$$Nu = \frac{hd}{k} \qquad (10)$$

according to Ranz-Marshall equation[27]

$$Nu = 2 + 0.6\ Re^{0.6}\ Pr^{0.33} \qquad (11)$$

$$\frac{dT_d}{dt} = \frac{hA}{mc_d}(T_a - T_d) - \frac{\lambda}{mc_d}\left(-\frac{dw}{dt}\right) \qquad (12)$$

$$\frac{dT_d}{dx} = \frac{dT_d}{dt}\ \frac{1}{v_p} \qquad (13)$$

3– Humidity of air

Mass balance for air

$$l\ \frac{dx_b}{dx} = -\frac{dm}{dx} \qquad (14)$$

4– Temperature of air

Heat balance for the bulk of air

$$lc_a dT_a = -\ l\ \lambda\ x_b \qquad (15)$$

$$\frac{dT_a}{dx} = -\frac{\lambda}{c_a}\ \frac{dx_b}{dx} \qquad (16)$$

5 – Interfacial humidity of air[9]

$$\text{Log } p_i = 0.622 + \frac{7.5\ t_{wb}}{238 + t_{wb}} \qquad (17)$$

$$x_i = 0.622\ \frac{p_i}{760 - p_i} \qquad (18)$$

6 - Velocity of air

Momentum equation[28]

$$\Delta M_{air} + \Delta M_{droplet} = P_2 A - P_1 A \qquad (19)$$

$$M_i = G\ u_i \qquad (20)$$

$$(M_{i1} - M_{i2})_{air} + (M_{i1} - M_{i2})_{droplet} = P_2 A - P_1 A \qquad (21)$$

7-Terminal falling velocity of droplet

It is obtained from the physical properties of air and droplet[2]

$$\left(\frac{R}{\rho U^2}\right) Re^2 = \frac{2}{3} \frac{d^3 \rho g (\rho_d - \rho)}{\mu^2} \qquad (22)$$

Re is evaluated from the relation of log Re with log $\left(\frac{R}{\rho U^2}\right)$

$$Re = \frac{\rho (V_p - V_a) d}{\mu} \qquad (23)$$

Deaceleration of droplet velocity was applied as Lapple and shepherd[21] model.

The equations were integrated numerically using Euler method for first order differential equations [29].

The model incorporates a finite difference for the gas and droplet phase. A cross section of the dryer is divided into a series of annuli and calculation was performed for each annulus before proceeding the next axial locations.

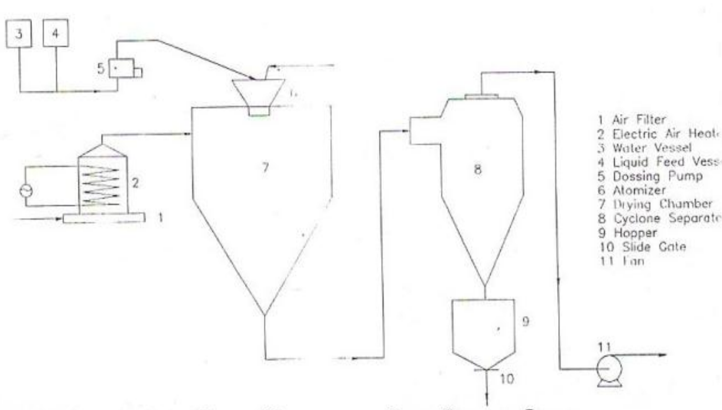
Input parameters are the column dimension , the temperature , humidity and flow rate of air entering the column and the temperature and flow rate of water as shown in table1 and size of distribution of the droplets from the nozzle as shown in table 2.

Calculations for droplets of each initial size continue until those droplets evaporate completely.

The droplet equations are integrated over the time required to traverse the length of the trajectory inside each annuli, the result is multiplied by the flow rate of droplets associated with this trajectory and finally the contributions of all drops trajectories crossing the particular annuli are summed up.

Experimental Measurements

Local measurements of air temperature using thermocouples within a laboratory spray dryer as shown below in flow diagram.



Flow Diagram of a Spray Dryer

Table 1. Input Data for model (Experimental conditions)

column length	2.2 m
column radius	0.28 m
air flow rate	86.4 g/sec
inlet air temperature	474 K
inlet air humidity	0.0081 g water/ g moist air
water flow rate	2.81 g/ sec
water temp	320 K
atomizer	centrifugal pressure nozzle

Table 2. Droplet diameter-mass fraction distribution (from the manufacturer company of the atomizer).

Drop diameter μm	Mass fraction	Drop diameter μm	Mass fraction
5	0.01	44	0.06
12	0.04	48	0.10
17	0.05	53	0.10
21	0.05	58	0.05
24	0.05	62	0.05
28	0.10	68	0.05
33	0.10	76	0.04
37	0.06	90	0.01
40	0.08		

initial velocity for droplets of all sizes is 47.3 m/sec.
starting position for all droplets sizes at axial distance from the nozzle tip is 1 cm and at radial distance in the column is 1 cm.
starting angle with respect to the column axis for all droplet sizes is 32°.

Discussion

Air and droplet velocity

The air enters the column with an axial velocity parallel to the column, as the droplets start their flight down with very large velocities . The air and droplets velocities entering the column are about 0.5 and 47 m/sec respectively. Momentum exchange between droplet and air, results in acceleration of air to a large value in a direction parallel to the column axis. After small distance from the center of column horizontally , air is drawn inside toward the column center and air is continually drawn inside the column which brought from larger radial distances due to momentum transfer . after large distance a recirculation eddy exists in the region by the wall as aback flow by the wall when air reaches to the wall of the column . Air and droplet velocity affect on the evaporation rate of droplet and on the trajectory of the droplet.

Evaporation Rate

The predicted percent of water evaporated from droplet of different sizes as a function of axial distance from the atomizer is shown in fig.1 and fig.2.

For droplets of small diameter , The distance for complete evaporation of droplets increases as the diameter of droplets increases , and this is shown in fig.1 for diameter of droplet up to 33 μ m because of this small size droplets evaporate approximately at the same condition of air temperature and humidity , and it deacelerate rapidly to the terminal velocity which is directly proportional to the square of the droplets diameter.

For larger droplets size, the evaporation rate is different , since it travel radially at a larger distance from the column center than the smaller ones ,and it evaporate at condition of higher air temperature and lower humidity of air and the driving force for mass transfer and heat transfer is higher ,in addition that the velocity of droplet and air are lower and the turbulence is less than in the center of the column so its evaporation happens at axial distance shorter than the smaller ones as shown in fig.2.

For the last remaining larger droplets ,the evaporation rate is high since its positon is at farther distances from other droplets and to be nearer to the column walls , so it evaporates at condition of higher air temperature and lower humidity of air as the same environment of larger ones in addition that its quantity is little,and the therefore evaporation happen at shorter axial distance than other droplets as shown in fig.2.

Temperature of air

Predicted air temperatures versus radial distance at various axial distances are shown in fig. 3. The variation of air temperature with radial distance is the same for various axial distances.There is a sharp fall in air temperature where the smaller droplets commences complete evaporation near the center of the column , and then the temperature of air begins to increase since evaporation of larger droplets take place.

This increase in temperature of air is different according to the axial distance ,as the axial distance increases ,the increase of temperature of air is decreases depending on the evaporation of droplets at this region Eventually,temperature of air is constant for various axial distances because of complete evaporation of all droplets present at this axial distance.and this differs with axial distance which depend on the evaporation of the droplets passing through it.

Humidity of air

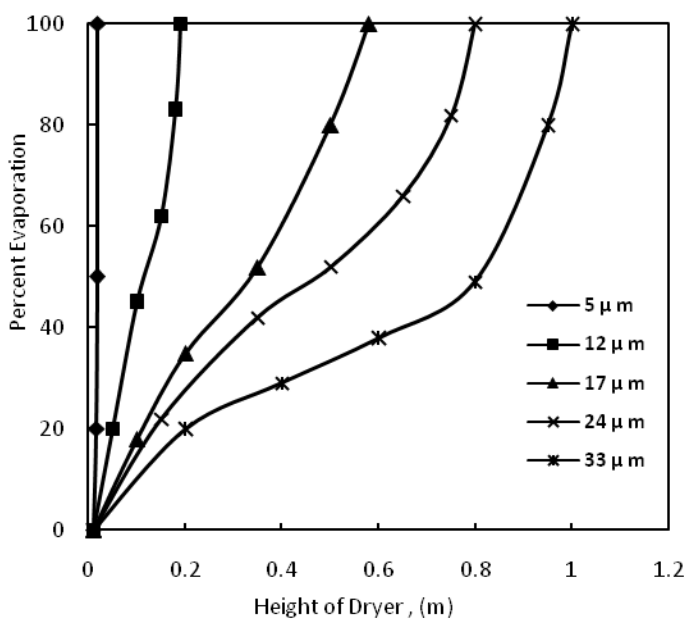
Predicted air humidity which is expressed as water vapor mass fraction versus radial distances at various axial distance is shown in fig.4 . The shape of the curves of air humidity is similar to the curves of air temperature but inverted because the same phenomena occurred.

Comparisions of experimental results with predictions of present model.

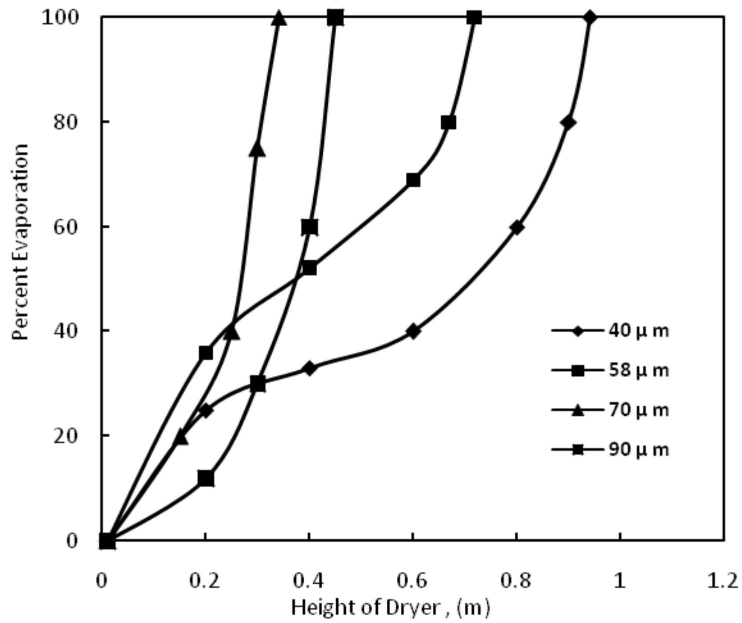
The results of the present model are in a greement with the experimental results as shown in fig.5 and fig.6 . Experimental temperature of air are higher than predicted near the center line of the column and this depend on the exact information about the droplets at the point where they start to fly down the column and this disagreement decrease with the axial distance at which the droplet evaporation are completed .

Experimental temperature of air are lower than predicted far away from the center line of the column is due to the high turbulence and mixing of droplets and air and by the wall and this disagreement differs with axial distance.

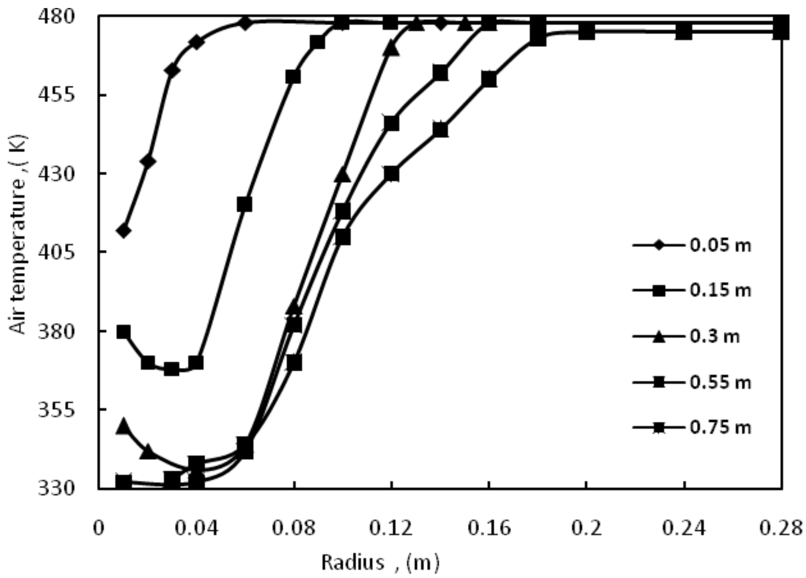
In fig. 7 and fig. 8 predicted air humidity radial profiles area shown with experimental at the same axial distances as for the temperatures.The agreement is the same as for the temperatures.



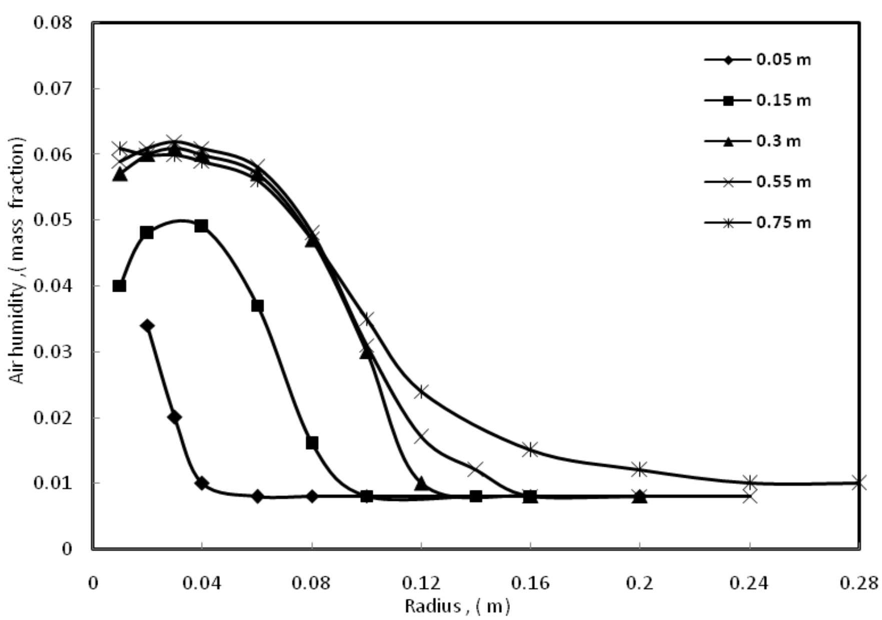
FIG(1) Predicted percent evaporation along the height of dryer from the nozzle for selected initial drop sizes.



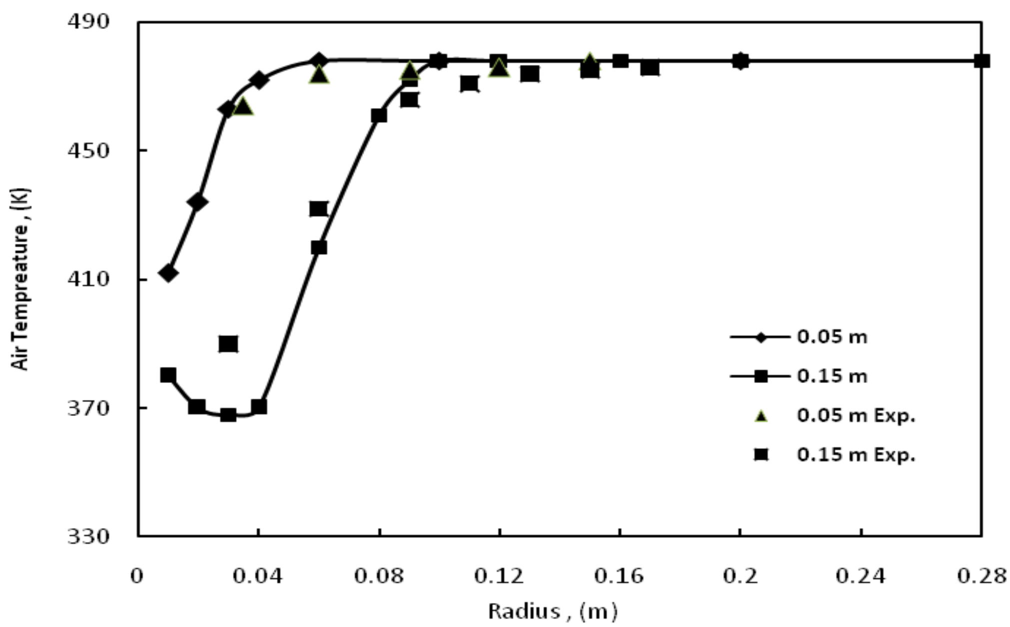
FIG(2) Predicted percent evaporation along the height of dryer from the nozzle for selected initial drop sizes.



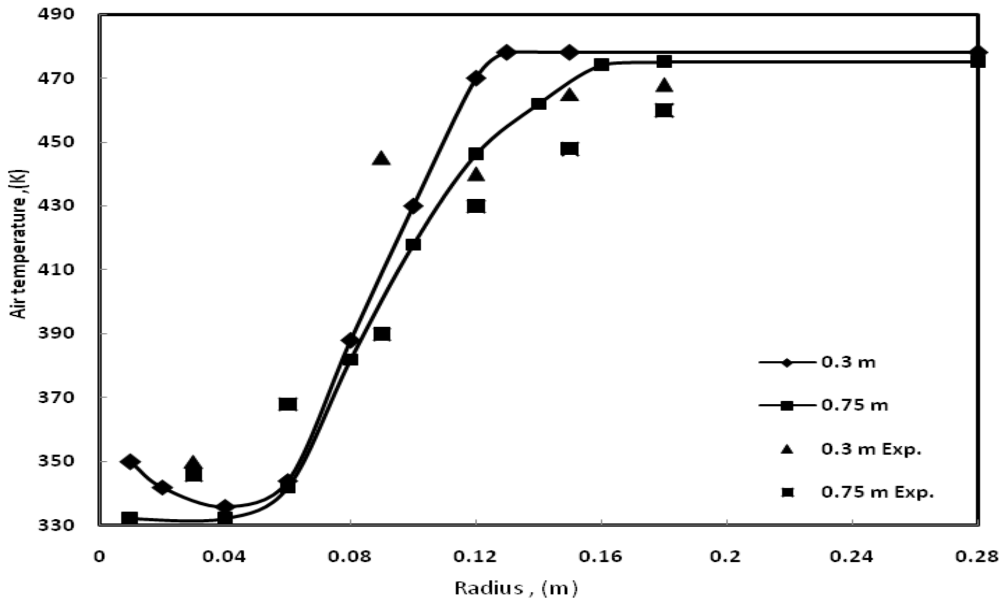
FIG(3) Predicted air temperature along radial distance from centerline of dryer for different axial distances from the atomizer.



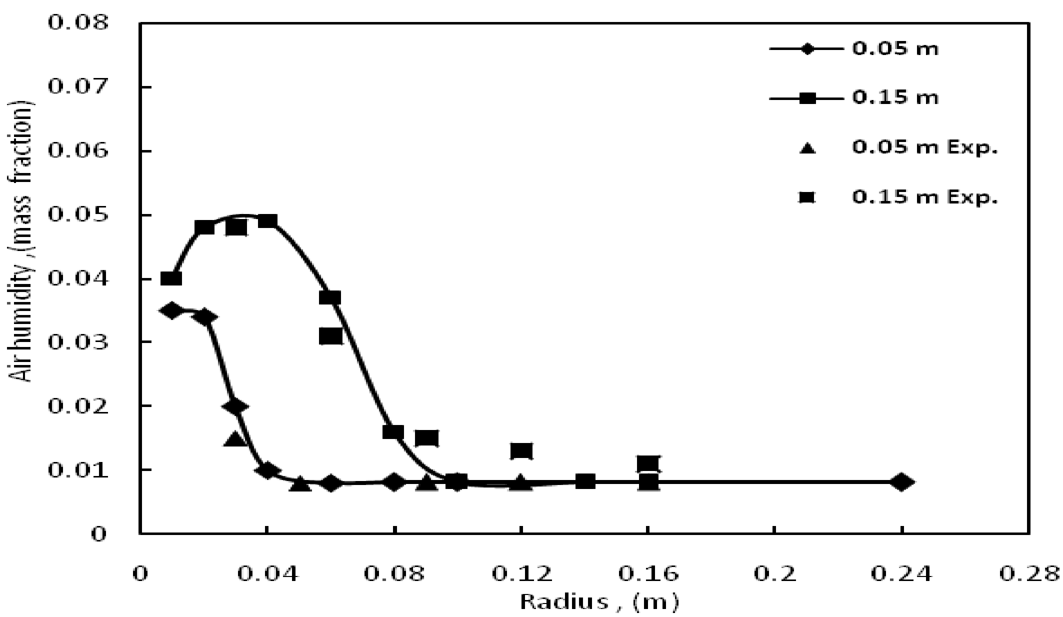
FIG(4) Predicted air humidity (mass fraction water vapor) along radial distance from centerline of dryer for different axial distances from the atomizer.



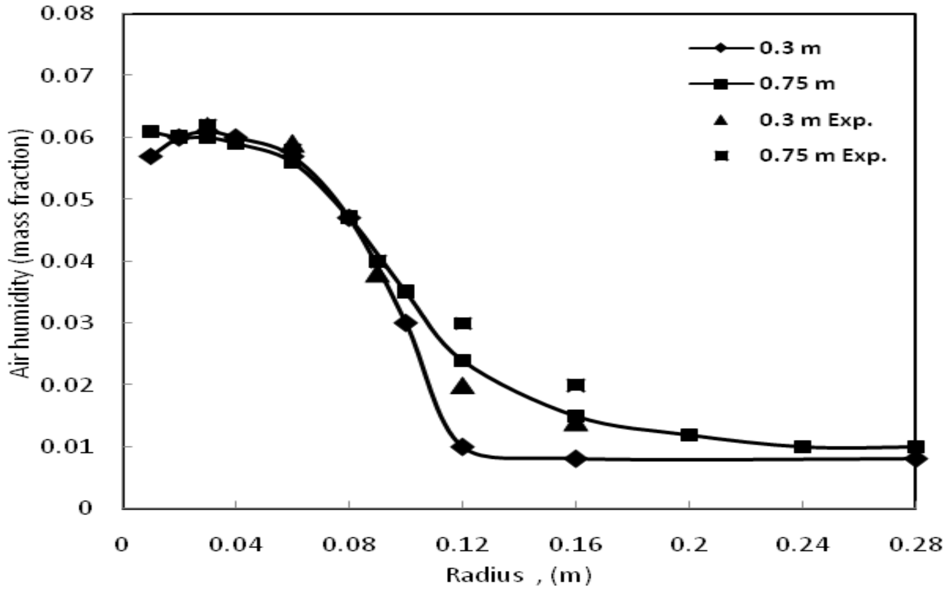
FIG(5) Predicted and experimental air temperature along radial distance from centerline of dryer at different axial distance from the nozzle.



FIG(6) Predicted and Experimental air temperature along radial distance from centerline of dryer at different axial distances from nozzle.



FIG(7) Predicted and Experimental air humidity along radial distance from centerline of dryer at different axial distances from nozzle .



FIG(8) Predicted and Experimental air humidity along radial distance from centerline of dryer at different axial distances from nozzle.

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Nomenclature

A	Area of droplet	m^2
A_c	Area of section	m^2
c_a	Specific heat of air	J/ kg K
c_d	Specific heat of droplet	J/ kg K
D	diffusivity of water vapor in air	m^2/sec
d	Diameter of droplet	m
G	Mass flow rate	g/sec
g	gravitational acceleration	m/sec^2
h	Heat transfer coefficient	J/ m^2K
k	Thermal conductivity of air	J/m K
kg	Mass transfer coefficient	kg/m^2sec
l	Mass flow rate of air	kg/hr
M_i	Momentum in x and y direction	kg.m

m	Mass of droplet	kg
Nu	Nusselt number	
Pr	Prandtl number	
P_1	Pressure of air at inlet	N/m^2
P_2	Pressure of air at outlet	N/m^2
P_i	Partial pressure of air	N/m^2
Q	Heat transfer rate to droplet	J
R	shear stress	N/m^2
Re	Reynold number	
Sc	Schmidt number	
T_a	Temperature of air	K
T_d	Temperature of droplet	K
T_{wb}	Wet bulb temperature of air	K
t	Time	s
U	Resultant velocity	m/s
V	Volume of droplet	m^3
v_a	Velocity of air	m/s
v_p	Velocity of droplet	m/ s
W	Weight of droplet	kg
x	axial distance	m
x_b	mass fraction of vapor in bulk	
x_d	mass fraction of vapor at droplet surface	
ρ	Density of air	kg/ m^3
ρ_d	Density of droplet	kg/ m^3
λ	Latent heat of vaporization	J/kg
μ	Viscosity of air	kg/m s

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