Study some of the nuclear characters of Samarium ^{148,150,152,154}Sm isotopes in the IBM-1 framework

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دراسة بعض الخصائص النوويه لنظائر الساماريوم ^{148,150,152,154}Sm في إطار عمل أنموذج IBM-1

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الخلاصة :

تم تطبيق أنموذج البوزونات المتفاعلة الأول IBM-1 على بعض نظائر عنصر $_{62}$Sm الزوجية- زوجية.
تضمنت الدراسة حساب حزم الطاقة الجماعية ذات الأزدواج الموجب والمتمثلة بحزمة الطاقة الأرضية g-band والحزم المنحلة β,γ-bands للنظائر $^{148}{}_{62}$Sm ، $^{150}{}_{62}$Sm ، $^{152}{}_{62}$Sm و $^{154}{}_{62}$Sm وتمت مقارنتها بقيم الطاقات المتوفرة عمليا .
وضعت متغيرات مؤثر دالة هاملتون بحيث تكون منسجمه مع وصف قريب من المنطقه الإنتقالية U(5)-SU(3) .
لقد تم الحصول على سطوح طاقة الجهد E(β,γ) بدلالة المتغيرين الهندسيين β و γ لتلك النظائر حيث رسمت سطوح طاقة الجهد ممثله بخطوط كنتوريه وتم تحديد أصغر قيمة لعامل التشوه β_{min} المقابل لأصغر قيمة لطاقة الجهد E_{min}، وكذلك قيمة العامل γ .
فضلا عن ذلك أمكن مشاهدة الإنتقال من الشكل الكروي U(5) الى الشكل الدوراني SU(3) في تلك النظائر .
نستنتج من دراسة طيف الطاقة والصورة الهندسـية لسـطوح طاقة الجهد لتلك النظائر في إطار عمل أنموذج BM-1 ، إنه تلك النوى توزعت بين الشكل الكروي والشكل الدوراني المتطاول prolate وشكل مشوه بين هذين التحديدين التناظريين في المنطقه الإنتقاليه U(5)-SU(3) .

Abstract :

The interacting boson model IBM-1 has been applied to some $_{62}$Sm even-even isotopes .
This study has included calculation of the positive-parity collectiv g-band and β,γ –degenerate bands of $^{148}{}_{62}$Sm, $^{150}{}_{62}$Sm, $^{152}{}_{62}$Sm and $^{154}{}_{62}$Sm isotopes and compared with experimental values of energy level .
The used Hamiltonian parameters correspond to description close to the U(5)-SU(3) .
The potential energy surfaces E(β,γ) as function of geometrical parameter β and γ for these isotopes can be obtained where they were represented by contour lines and the minimum value of deformation factor β_{min} was fixed , which was in front of E_{min} , and so the γ factor .

The transition from the spherical to the rotational shape can be observed in these isotopes.

In the framework of IBM-1 , what we conclude from the energy spectra study and the geometrical picture of potential energy surface for these nuclei is that these nuclei are spherical shape (U(5)symmetry), prolate rotational shape (SU(3)symmetry) and deformed shape between these two symmetry limits in the U(5)-SU(3)transitional region .

1- Introduction

The interacting boson model (IBM) in its simplest form , as originally proposed (Arima and Iachello 1974), describes a system of *s (L=0)* and *d (L=2)* bosons which may interact with one another via one or two-body interactions . The neglect of higher order terms does not represent any fundamental constraint , and indeed has been relaxed in some later applications of the model . [1]

A mapping technique was subsequently developed by Otsuka et al. [2] which related matrix elements of boson operators to matrix elements of fermion's operators in a paired – fermions space , this mapping procedure forms the basis of other calculations which attempt to connect the IBM to some under lying fermionic Shell model . [3,4]

The basic idea of the IBM is to assume that low-lying collective state in even-even nuclei can be described by a system of interacting s and d bosons carrying angular momentum 0 and 2 , respectively .

One may wonder after neglecting the difference between protons and neutrons , it is still possible to get any kind of reasonable description of nuclear properties in the IBM-1 framework . [5]

Only proton-proton pairs (proton bosons) and neutron-neutron pairs (neutron bosons) are allowed in this model , while proton-neutron pair are excluded. The reason is that in medium and heavy nuclei the valence protons and the valence neutrons occupy different major shells so that the formation of proton-neutron pairs becomes very improbable . [6]

The underlying SU(6) group structure of the model basis leads to three limiting symmetries U(5) , SU(3) and O(6) , corresponding in the geometrical description to vibrational , rotational , and γ-unstable nuclei . [7]

2- Theoretical considerations and calculations

2.1- Hamiltonian parameters and low-lying Energy spectra

The most commonly used form of the IBM Hamiltonian , and the one in which it is easiest to understand the role of each term in determining the final structure of the nucleus under consideration , is the so-called multiple expansion . In this parameterization the various boson-boson interactions are grouped so that the Hamiltonian takes the following form (Scholten,Iachello,and Arima,1978) : [1]

$$H = \varepsilon \hat{n}_d + a_o P^{\dagger} P + a_1 \hat{L}^2 + a_2 \hat{Q}^2 + a_3 \hat{T}_3^{\,2} + a_4 \hat{T}_4^{\,2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and

$$P = \frac{1}{2} (\tilde{d}^{\,2} - s^2)$$

$$T_l = (d^{\dagger} \tilde{d}) \quad , \quad l = 0,1,2,3,4,$$

$$Q = (d^{\dagger} s + s^{\dagger} \tilde{d}) - \frac{\sqrt{7}}{2} (d^{\dagger} \tilde{d})^{(2)}$$

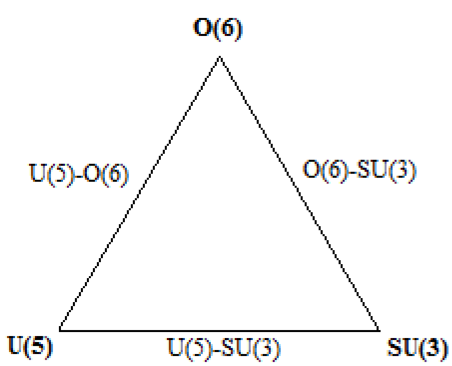
$$= (d^{\dagger} s + s^{\dagger} \tilde{d}) - \frac{\sqrt{7}}{2} T_2 \quad ,$$

$$\hat{n}_d = \sqrt{5}\, T_0 \quad , \quad \hat{L} = \sqrt{10}\, T_1$$

Where : n_d ~ d-bosons number operator, P ~ Pairing operator, L ~ Angular Momentum operator, Q ~ quadrupole Moment operator, T_3 ~ Octupole operator, T_4 ~ Hexadecapole. [5]

The three limiting symmetries U(5) , SU(3) and O(6) are pure and the most of nuclei are included characters of two or three of this limiting symmetries . therefore , there are three transition regions between this limits as shown in figure (1) . [6]

Fig. (1):
Symmetry of the IBM indicating the three limiting symmetries and the transition legs between symmetries



The most of deformed nuclei are by no means good examples of SU(3) .

Many of the properties of broken SU(3) calculation were discussed earlier , this deformed rotor generate energy bands β and γ and this deformed nuclei may be prolate shape ($\gamma = 0^o$) or oblate shape ($\gamma = 60^o$) . [1]

About 100 nuclei in a U(5) – SU(3) transition consisting of about 70 in the A= 150-180 region and about 30 near A= 100 .

Therefore , we expect the isotopes of Sm can be described by U(5) - SU(3) transition region .

The vibrator - rotor (U(5) - SU(3)) transition region near A=150 was treated very early by Scholten , Iachello and Arima (1978) , using the schematic Hamiltonian . [8]

$$H = \varepsilon \hat{n}_d + a_1 \hat{L}^2 + a_2 \hat{Q}^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The parameter ε constrained to decrease linearly with increasing bosons number, this decreases the ratio of ε / a_2 and induces a U(5)-SU(3) phase transition . [9]

In many IBM calculations the parameters of the Hamiltonian are adjust to experiment.[10]

We can obtain the values of a_2 and a_1 parameters by

$$a_2 = -\frac{E_{2^+_\beta} - E_{2^+_g}}{3(2N-1)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$a_1 = \frac{E_{2^+_g}}{6} + \frac{3a_2}{8} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Finding this two parameters gives start point to make our calculations and when chose the other parameters of Hamiltonian , we must based on some beginning information for this nuclei like the ratio R_4 which is equal to $E4_g^+ / E2_g^+$. usually , the region $2 \le R_4 \le 2.4$ refers to the U(5) limit , the region $2.4 \le R_4 \le 3$ refers to the O(6) limit and the region $3 \le R_4 \le 3.33$ refers to the SU(3) limit . [9]

The ratio of R_4 values calculate to the experimental energy level of ground state and its for ^{148,150,152,154}Sm equal to 2.1446 , 2.3155 , 3.0090 and 3.2524 respectively , thus the first and second of these nuclei are example for vibrational nuclei and the third and fourth are example for rotational nuclei .

2.2- The Potential Energy Surface

The expectation value

$$E_N(\beta , \gamma) = \langle N ; \beta \gamma \setminus H \setminus N ; \beta \gamma \rangle \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

defines an energy surface in β and γ space whose minimum bounds the ground-state energy, and N represents bosons number

Study of the ground state for nucleus gives important information about nuclear forces .

The shape of any nucleus can be obtained by the tow deformation factors β and γ .

The value of β is measured radially from the center ,while γ is given by the angle between the radius vector and the horizontal axis ,where the minimum of the energy surface is not at the origin . [11]

In general , the equation of potential energy surface is given as

$$E_N(\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{1+\beta^2} + \frac{N(N-1)}{(1+\beta^2)^2}\,(\alpha_1\beta^4 + \alpha_2\beta^3\cos3\gamma + \alpha_3\beta^2 + \alpha_4) \quad \text{………… (6)}$$

There are general forms represent the potential energy surface in the three limits of symmetry are given by Dieperink et al. [12]

$$E_N(N,\beta,\gamma) = \varepsilon_d N\,\frac{\beta^2}{1+\beta^2} \qquad ;\ \text{for}\ \ U(5)\ \text{……… (7)}$$

$$E_N(N,\beta,\gamma) = kN(N-1)\,\frac{1+\frac{3}{4}\beta^4-\sqrt{2}\beta^3\cos3\gamma}{(1+\beta^2)^2} \qquad ;\ \text{for}\ \ SU(3)\ \text{……… (8)}$$

$$E_N(N,\beta,\gamma) = c_1\,\frac{N\beta^2}{1+\beta^2} + c_2\,N(N-1)\left[\frac{1-\beta^2}{1+\beta^2}\right]^2 \qquad ;\ \text{for}\ \ O(6)\ \text{………… (9)}$$

Where $k \propto a_2$, $c_1, c_2 \propto a_0$

The three equations above give (when N very large) $\beta_{min} = 0, \sqrt{2}, 1$ for U(5) , SU(3) and O(6) respectively and the value of γ in SU(3) limit equals to 0^o for prolate triplet symmetric and 60^o for oblate triplet symmetric. [1,12]

3- Results and Discussions

3.1- Low- lying Energy Spectra

Most of the nuclear models generate the low-lying collective states like in IBM-1, such that the upper level of energy in our study is $I^\pi = 8^+$.

The $^{148}_{62}Sm$, $^{150}_{62}Sm$, $^{152}_{62}Sm$ and $^{154}_{62}Sm$ isotopes have atomic number Z= 62 protons which is closer to closed shell 50 (valance proton bosons $N_\pi = 6$) and have neutron number equal to 86,88,90,92 respectively which are closer to closed shell 82 (valance neutron bosons $N_\nu = 2,3,4,5$ respectively) . therefore , the numbers of valance bosons for these isotopes are $N_{boson} = 8,9,10,11$ respectively .

When study this isotopes under IBM-1 , we obtain the energy spectra for these isotopes as shown in table (1) and we have a good agreement with experimental energy spectra [13] as shown in figures (2,3) .

From the ratio R_4 for calculated (Theoretical) energy state , we have obtained a good agreement values with the experimental results from a point of view the symmetry limits . where it is equal to 2.1517, 2.3805, 3.1610 and 3.2968 for $^{148}_{62}Sm$, $^{150}_{62}Sm$, $^{152}_{62}Sm$, $^{154}_{62}Sm$ respectively . therefore , the first and second isotopes can be described by U(5) limit and the third and fourth isotopes can be described by SU(3) limit .

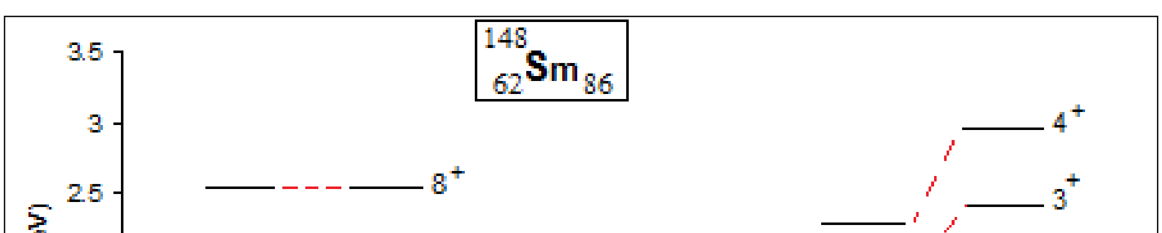
Table (1):

Theoretical energy levels to the g , β and γ bands for $^{148,150,152,154}_{62}Sm$ isotopes, calculated in the framework IBM-1 (in unites MeV), and values of N_{boson} and R_4 .

Nucleus	Band	Energy Levels (MeV)								N_{boson}	R_4
		Angular Momentum (I^{π})									
		0^+	2^+	3^+	4^+	5^+	6^+	7^+	8^+		
$^{148}_{62}Sm$	g	0	0.5722		1.2312		1.9155		2.5390	8	2.1517
	β	1.4502	1.5878								
	γ		1.8044	2.4112	2.9826						
$^{150}_{62}Sm$	g	0	0.3177		0.7563		1.2905		1.7752	9	2.3805
	β	0.7380	1.0903		1.6257						
	γ		1.3726	1.7132	1.8926	2.3385					
$^{152}_{62}Sm$	g	0	0.1285		0.4062		0.7992		1.2156	10	3.1610
	β	0.7013	1.0021		1.3547		1.7588		1.9258		
	γ		1.2052	1.4125	1.5982	1.9125	2.6481				
$^{154}_{62}Sm$	g	0	0.0822		0.2710		0.5502		0.8842	11	3.2968
	β	1.0084	1.1621		1.3852						
	γ		1.3372	1.4425	1.6525						

Fig. (2):

Comparison between experimental(Expt.) [13] and theoretical(ibm-1) energy level (in unites MeV) for $^{148,150}Sm$ isotopes



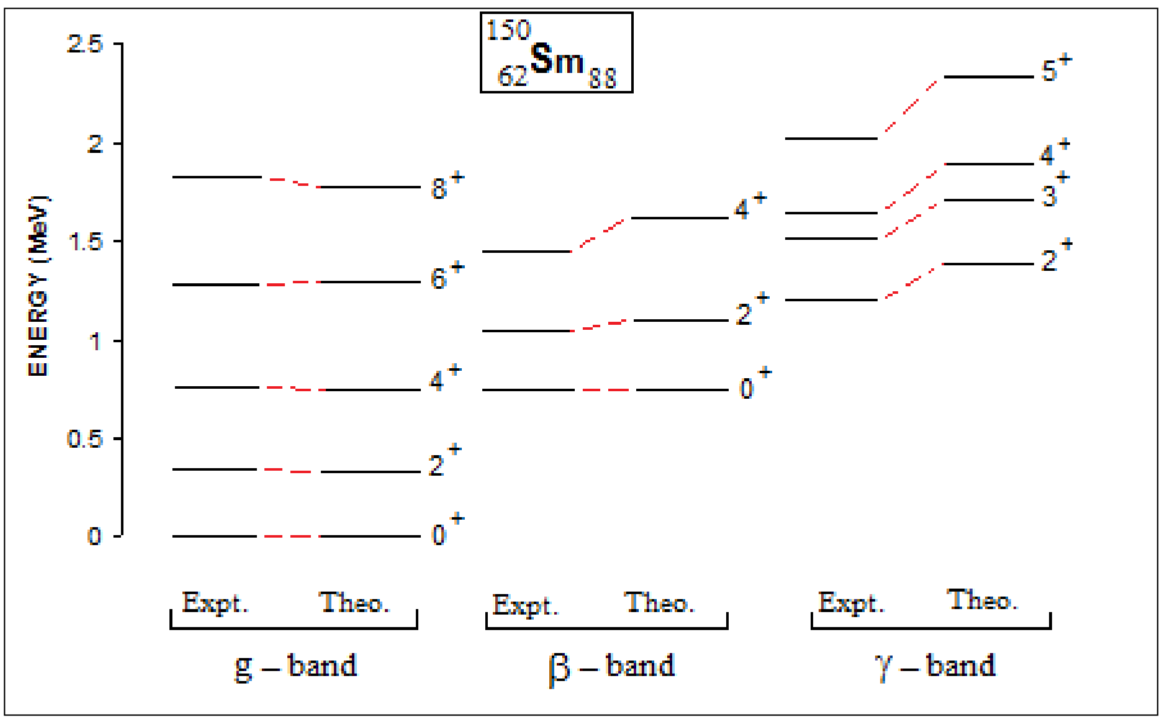
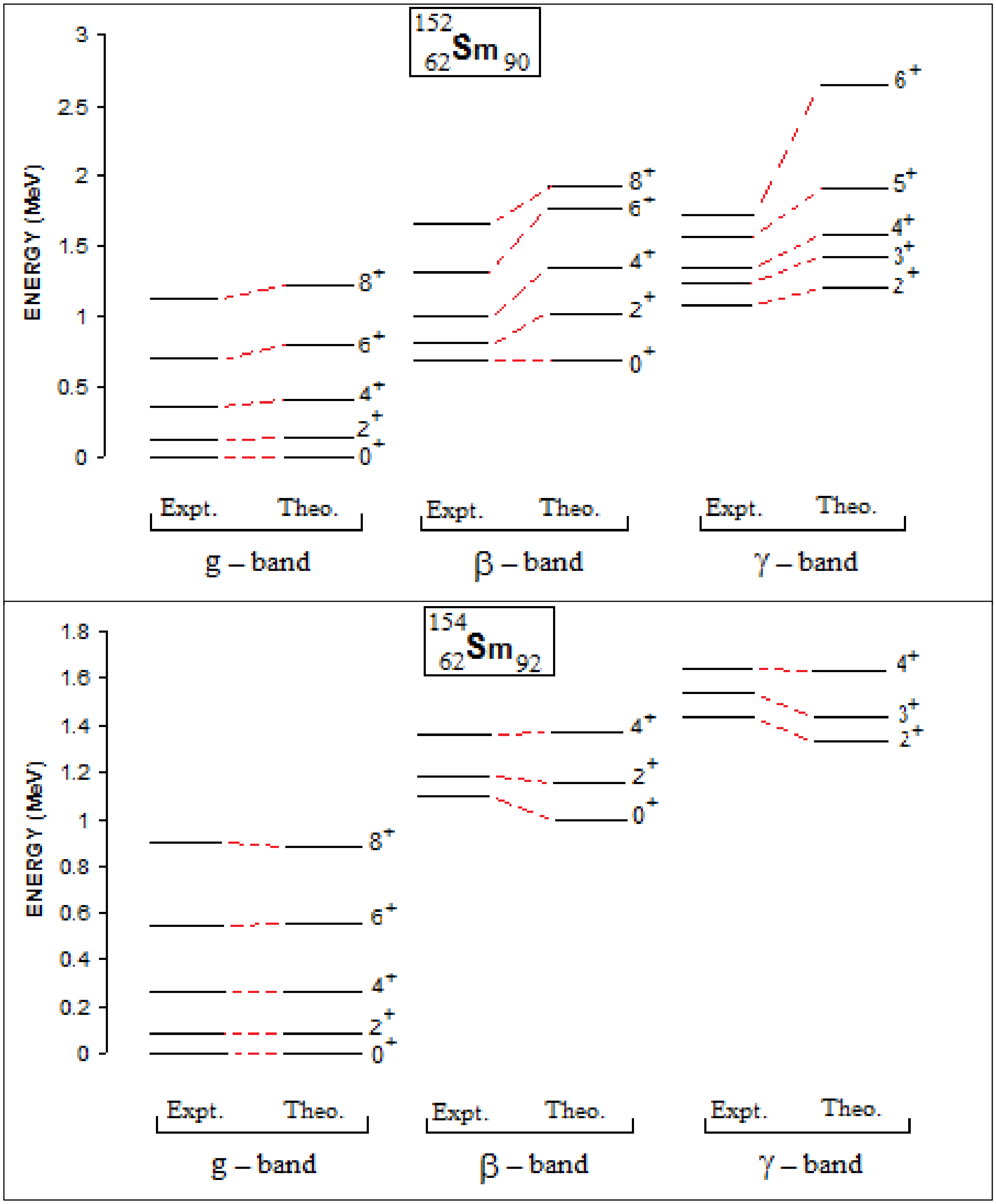


Fig. (3):
Comparison between experimental(Expt.) [13] and theoretical(IBM-1) energy level (in unites MeV) for $^{152,154}Sm$ isotopes



We can plot the relation shape between the R_4 as a function of neutron number for these Sm isotopes to obtain figure (4) and we note from this figure that the nucleus transition from the U(5) limit to the SU(3) limit when the number of neutrons increases.

In addition, when plotting excitation energies at 2^+ and 4^+ as a function of neutrons number to the g, β and γ-bands, we obtain the figure (5) and note that both β and γ-bands begin to increase in energy after N>90.

Fig. (4):
Relation shape between the R_4 as a function of neutron number for the Sm isotopes

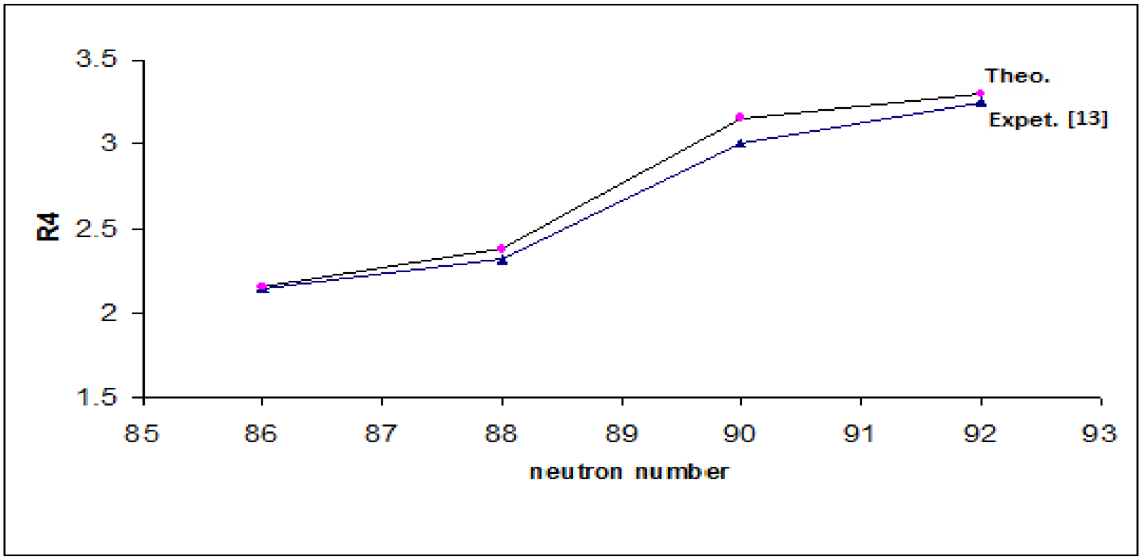
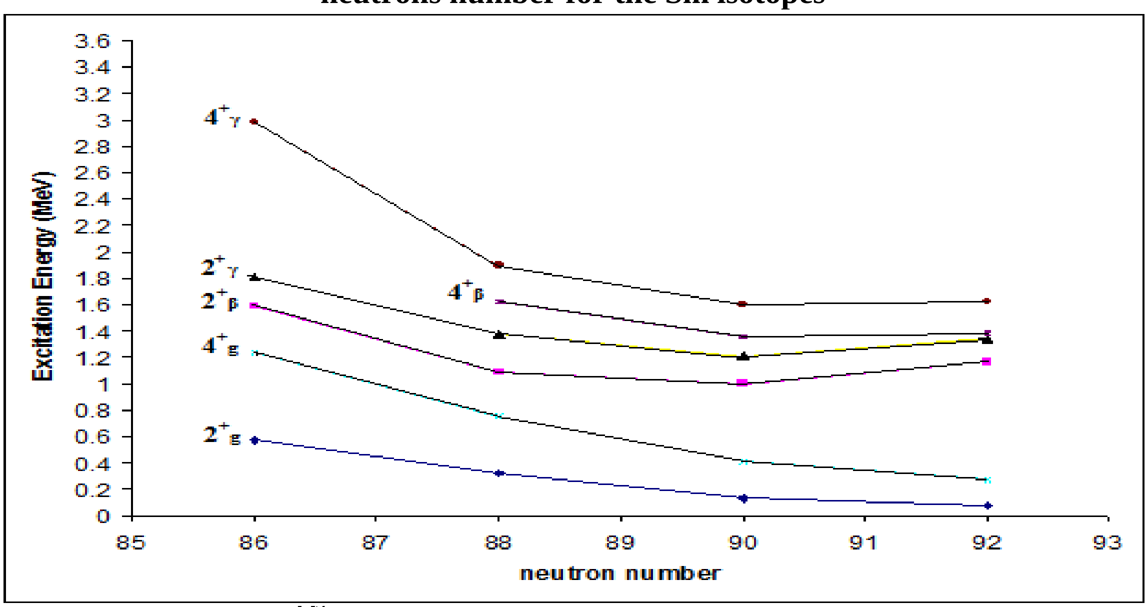


Fig. (5):
Relation shape between the excitation energies (in unites MeV) as a function of neutrons number for the Sm isotopes



From study of $_{62}$Sm isotopes in IBM-1 , we can plot the contour lines to the energy surfaces for these isotopes in the β and γ plane as shown in figure (6) .

The shape of $^{148}_{62}Sm$ and $^{150}_{62}Sm$ isotopes are deformed vibrator nuclei (near to spherical shape or U(5) limit) and the minimum value of β is $\beta_{min}= 0$ for each of them , but the shape of $^{152}_{62}Sm$ isotope is deformed rotor nucleus where $\beta_{min}=$ 0.9 and have prolate shape ($\gamma = 0^o$) and the shape of $^{154}_{62}Sm$ isotope is deformed rotor nucleus (near to SU(3) limit) where $\beta_{min}= 1.15$ and with prolate shape ($\gamma = 0^o$) .

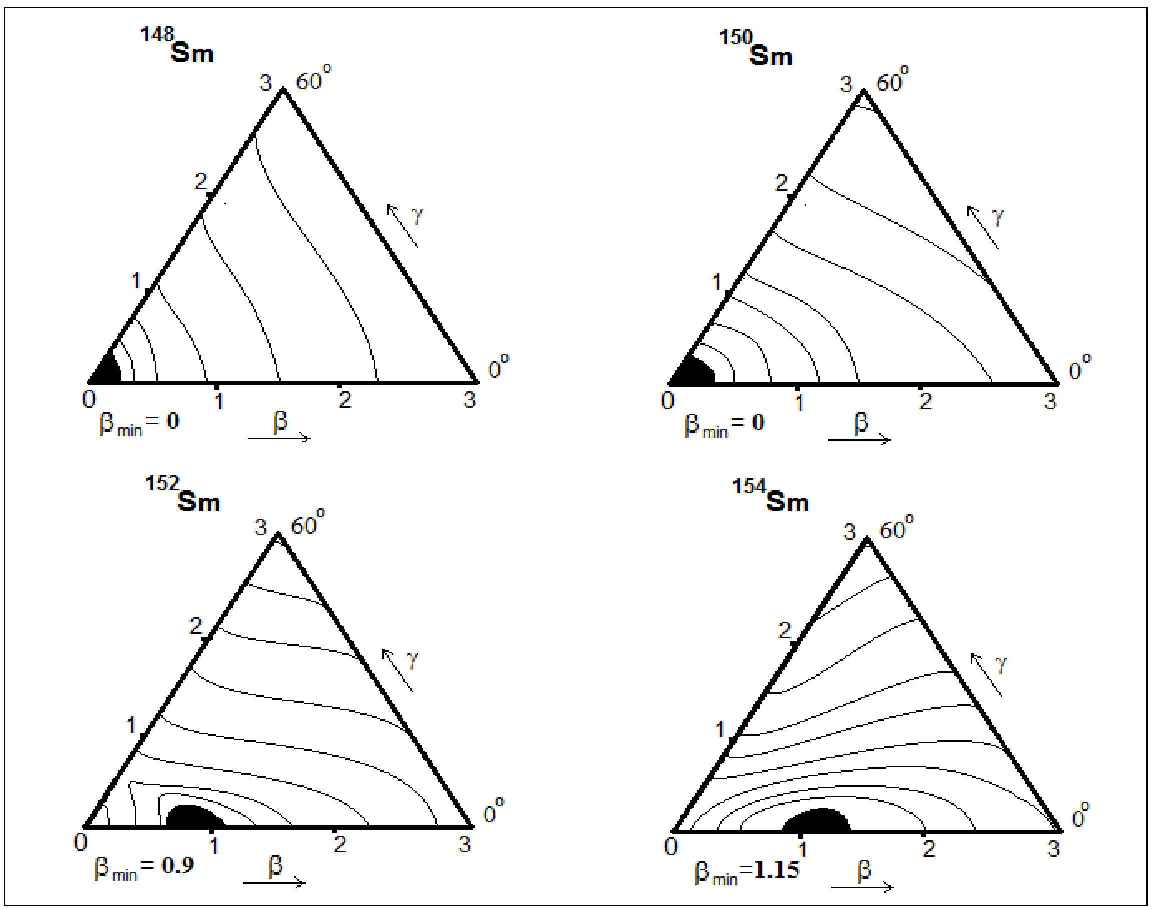
In the fact , the transition from the vibrational shape (U(5) limit) to the rotational shape (SU(3) limit) comes from decrease the ratio ε / a_2 .

These results are in agreement with the description of these nuclei by D. Bonatsos et al. where the low lying spectra of $^{148}_{62}Sm$ look like a spherical vibrator or the U(5) limit of

IBM and for $^{154}_{62}Sm$ which is an example of the axially symmetric deformation or the SU(3) limit of IBM . [10]

Fig. (6):

Contour lines of potential energy surfaces for $_{62}Sm$ isotopes in the β,γ plane and the value of β_{min} for each isotope



In these isotopes one sees the ε parameter decrease linearly with increasing boson number in going from nuclei close to the N = 82 closed shell towards the deformed nuclei with N >90 . This decreases the ratio of ε / a_2 and induces a U(5)- SU(3) phase transition ; therefore, one can say that the $^{148,150,152,154}Sm$ isotopes have different shapes and each of them has vibrational and rotational properties with different ratio.

So the energy levels in g-band decrease when the neutron numbers increase and near the phase transitional point around N = 90 , the Q^2 interaction which depends on N^2 begins to dominate. In accord with this one sees that both the β and γ-bands begin to increase in energy once deformation has set in.

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