ULTIMATE BEARING CAPACITY OF RING FOOTINGS ON SAND

Lamyaa Najah Snodi - Associate Lecturer of Civil Engineering ,
Tikrit University, Tikrit, Iraq.

الخلاصة

تم إيجاد قابلية التحمل القصوى للأسس الحلقية المستندة على تربة رملية لحالات مختلفة ، منها : تغيّر نسبة القطر الداخلي إلى القطر الخارجي للأساس مع تغيير في زاوية الاحتكاك الداخلي للتربة باستخدام برنامج إلْبِلاَ نسخة المحترفين ، الإصدار ٩.٠ الذي يستند بالحل على طريقة العناصر المحددة.

علاقة شبه تجريبية لإيجاد قابلية تحمل التربة للأسس الحلقية التي طورت من قبل باحثين آخرين أيضا" وصفت واستخدمت في المقارنة.

وجد بأن قابلية التحمل القصوى للتربة تحت الأساس الحلقي تقل مع زيادة نسبة القطر الداخلي إلى القطر الخارجي للأساس ($r = d_i/d_e$) حيث ان d_i القطر الداخلي وd_e القطر الخارجي للأساس الحلقي ، على التوالي.

وللقيم الكبيرة من زاوية الاحتكاك الداخلي للتربة ϕ ، كانت قيمة قابلية التحمل القصوى أعلى منها للقيم الأصغر. كما قورنت النتائج المستخلصة من هذا البحث مع نتائج بحوث سابقة.

ABSTRACT

The ultimate bearing capacity of ring footings for different conditions such as internal to external diameter ratio and angle of internal friction for sand is determined using the finite element program ELPLA Professional Edition, Version 9.0.

A semi-empirical relation for determining the bearing capacity of ring footings that developed by other researchers is also described and used in compared. The value of ultimate bearing capacity is found to decrease significantly with an increase in ($r = d_i/d_e$), where d_i and d_e are the internal and external diameter of the ring, respectively.

For larger values of internal friction angle of soil (ϕ), the magnitude of ultimate bearing capacity is seen to be higher than smaller values. The obtained results are compared with those available in the literature.

Keywords: Ring footing, Bearing capacity, Sand, Finite element.

INTRODUCTION

The bearing capacity of soil is a problem in foundation design. Overestimating bearing capacity may lead to structure failure and catastrophic damage to buildings while underestimating it results in an uneconomical design.

Equations for ultimate bearing capacity of circular, square and rectangular footings have been proposed before (1). Ring footings which are often used as foundations of overhead water tanks, chimneys, towers and dome structures seem to receive less attention in this regard, although they may be more effective and economical circular footings.

Solutions are available in the literature to compute the elastic settlements of ring foundations (2, 3). Few experiments have also been performed to determine the bearing capacity of ring foundations (4, 5).

Al-Sanad, *et al.* (6) reported the results of a series of plate loading tests on dense sand using circular and ring plate. No significant difference in the settlement of ring and full plates is found, while the ratio of inside to outside diameter of ring, plate is 0.531.

More recently, Martin (7) investigates numerically the vertical bearing capacity of a rigid, ring shaped shallow foundation on cohesive-frictional soil. He shows that as *r* approaches unity the bearing capacity of the ring approaches that of a strip footing of width $R_{outer} - R_{inner}$.

PROBLEM DEFINITION

For this study rigid ring footings are specified with internal and external diameters as shown in **Table (1)**, and resting on a cohesionless medium with a horizontal ground surface. The footings are loaded with distributed loads (rectangle) shown in **Figure (1)**. The footings are 0.65m thick in order to provide enough rigidity.

Table 1: Dimensions of footings used in this study

Internal to external diameter ratio (*r*)	External diameter (m)	Internal diameter (m)
0.0	15	0.0
0.2	15	3.0
0.3	15	4.5
0.4	=	6.0
0.5	=	7.5
0.6	=	9.0
0.7	=	10.5
0.8	=	12.0
0.9	=	13.5

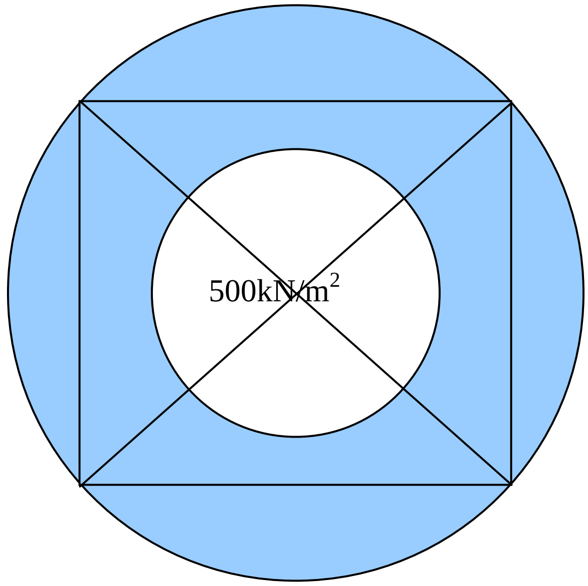

Figure 1: Footing loaded used in this study

MATERIAL PROPERTIES

The properties of the soil and the footing are presented in **Table (2)**:

Table 2: Properties of the soil and footing

Parameter	Value of the soil	Value of the footing
Modulus of Elasticity (kN/m^2)	10000	2.5×10^7
Unit weight (kN/m^3)	18	0.0
Angle of internal friction (degree)	10,20,30,40,50	-
Cohesion	0.0	-
Poisson's ratio	0.25	0.15

ANALYSIS

The finite element program ELPLA is used (8) to analyze the bearing capacity of ring footings. Axisymmetric solid elements are used to model the soil and footing. The Rigid raft method is used to determine the bearing capacity of the ring footing. Analysis is repeated for different internal to external diameter ratio of ring footings and for different values of internal friction angle of sand (ϕ).

RESULTS

For each analysis the ultimate bearing capacity (q_u) is obtained then the variation of ultimate bearing capacity against different ratios of internal to external diameters (*r*) are plotted and is illustrated in **Figure (2)**, with different angle of internal friction of sand. It can be noted that the magnitude of q_u decreases continuously with an increase in the value of *r*. An increase in ϕ causes an increase in the value of q_u.

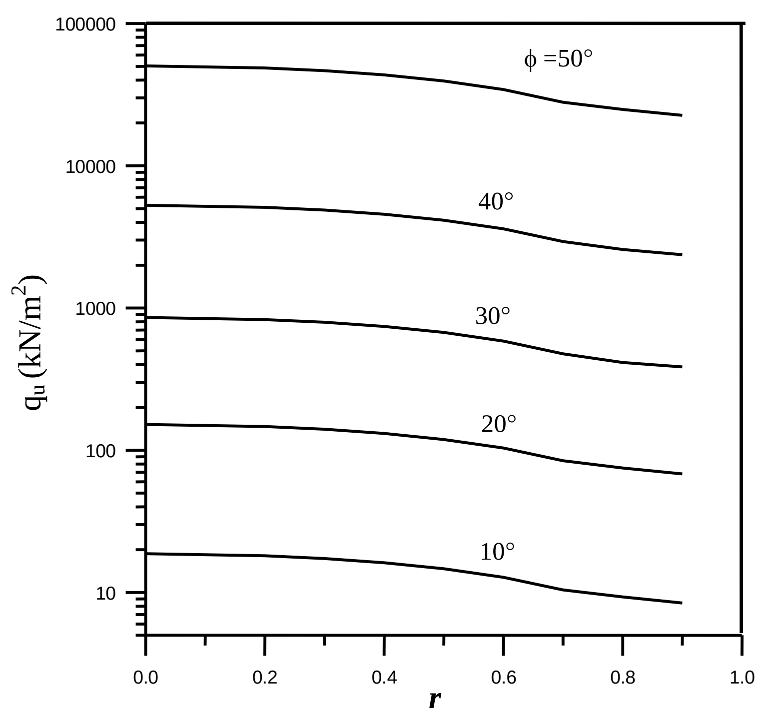

Figure 2: Variation of the ultimate bearing capacity with *r*.

SEMI – EMPIRICAL BEARING CAPACITY RELATION

In order to describe the relation for predicting bearing capacity of ring footing on sand, the failure mechanism observed in laboratory testing by Hataf and Razavi (9), is studied further and an idealized failure mechanism shown in **Figure (3)**, is suggested for the development of the bearing capacity equation. The section shown in this figure consists of soil wedges with axially symmetrical properties.

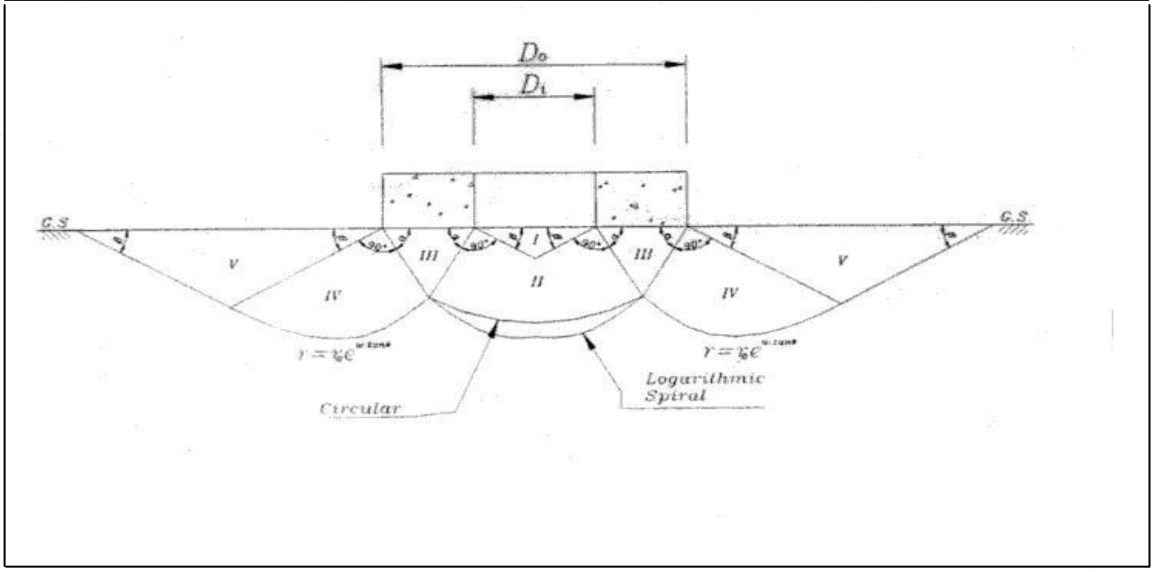

Figure 3: Idealized failure mechanism under ring footing [after Hataf & Razavi, (9)]

Writing the equilibrium equations at limit state for zones I to V and using the Mohr-Coulomb criteria, an equation for the bearing capacity of ring footings on sand is obtained. For simplicity in zone II, in **Figure (3)**, the spirals logarithmic curve is replaced by a circular arch. Using the limit equilibrium method, the following relation is derived:

$$q_u = \frac{1}{2} \gamma D_o N_\gamma \qquad \text{……(1)}$$

Where

$$N_\gamma = \frac{8}{1-r^2}((\lambda-1)c_1(|c_2|c_3(1-r)^3 - c_4) + (\lambda-1)c_5c_6(1-r)(1-r^2) - c_7(1-r)(1-r^2) - c_8(1+c_9(1-r))(1-r)^2)$$

and

$$\alpha = \frac{\pi}{4} + \frac{\phi}{2} \qquad \beta = \frac{\pi}{4} - \frac{\phi}{2}\ , \quad c_1 = \tan\frac{\pi}{4} = 1,$$

$$c_2 = \frac{9\cos\phi\,(\sin\beta - \tan\alpha + \cos\beta\,\tan\alpha)\frac{32}{3}\cos^2\alpha}{18\beta\cos\phi - 64\cos^2\alpha}$$

$$c_3 = \frac{9\beta}{16\cos^2\alpha} - \frac{2}{\cos\phi}\ ,\ c_4 = \frac{1}{24}\tan\beta\ ,\ c_5 = \frac{e^{\pi\tan\phi} - 1}{64\cos^2\alpha\tan\phi}\ ,\ c_6 = 0.3e^{\frac{\pi}{2}\tan\phi}\tan\alpha - 0.2$$

$$c_7 = \frac{1}{32}\tan\alpha\ ,\ c_8 = \frac{1}{32}e^{\pi\tan\phi}\tan\alpha\ ,\ \ c_9 = e^{\frac{\pi}{2}\tan\phi}\tan\alpha$$

In these equations, q_u is the ultimate unit bearing capacity of ring footing on surface of sand layer, D_o is external diameter, *r* is internal to external diameter ratio, ϕ is angle of internal friction of sand and γ is unit weight.

Derivation procedures of the above equations[for more details see (10)]. The idealized failure mechanism shown in **Figure (3)**, is valid for:

$$0 \le r \le \frac{e^{(\pi-\alpha)\tan\phi}}{e^{(\pi-\alpha)\tan\phi} + 2\cos\alpha}$$

When $r \rightarrow 1$ the footing behavior approaches to that of a strip footing (7) and the related bearing capacity equations may be adopted. To obtain the value of λ, the bearing capacity equation for circular footing (i.e. r=0) is adopted. The values of λ for different values of friction angle are shown in **Table (3)**.

Table 3: Variation of λ with friction angle

ϕ°	λ
5	2.54
10	2.68
15	2.92
20	3.20
25	3.50
30	3.87
35	4.26
40	4.70
45	5.38
50	6.45

COMPARISONS

Because of the limited availability of results for a ring footing, the obtained values of the ultimate bearing capacity for this footing with various combinations of ϕ and *r* are compared with (i) the experimental results of Saha (4), (ii) an empirical expression of Saran *et al*. (5). The comparison of all these results for values of ϕ between 30° and 40° is shown in **Figures (4–7)**. The values of ultimate bearing capacity obtained from the numerical analysis are close to the experimental results of Saha. The ultimate bearing capacity values provided by Saran *et al.* (5) in all cases are greater than those from the present analysis and the experimental data of Saha.

On the other hand, equation (1) is used to calculate the ultimate bearing capacity of ring footing with different values of *r*. It is observed that pattern of variation of q_u with *r* is similar to that obtained by Saran and the maximum bearing capacity is at *r* = 0.2.

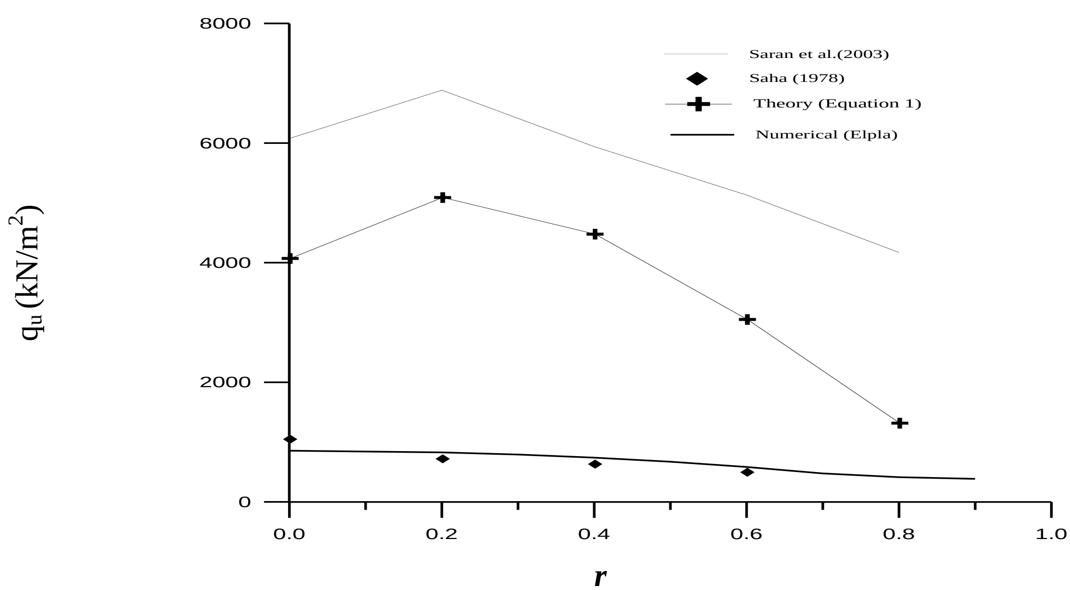

Figure 4: Comparison of q_u values from this study with results after Saha (4) and Saran *et al.* (5) for ϕ = 30°.

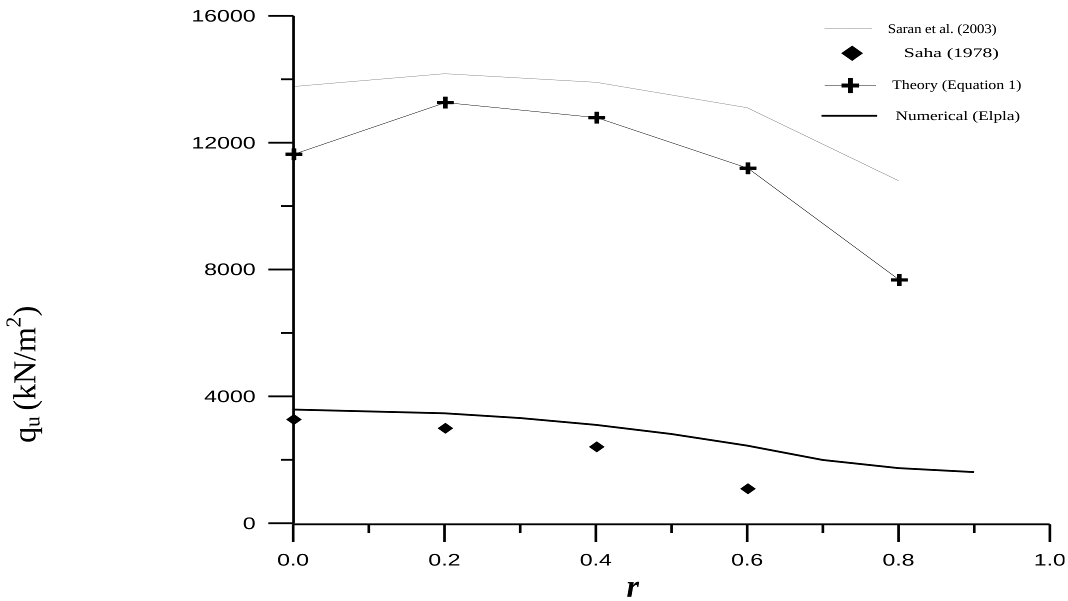

Figure 5: Comparison of q_u values from this study with results after Saha (4) and Saran *et al*. (5) for $\phi = 35°$

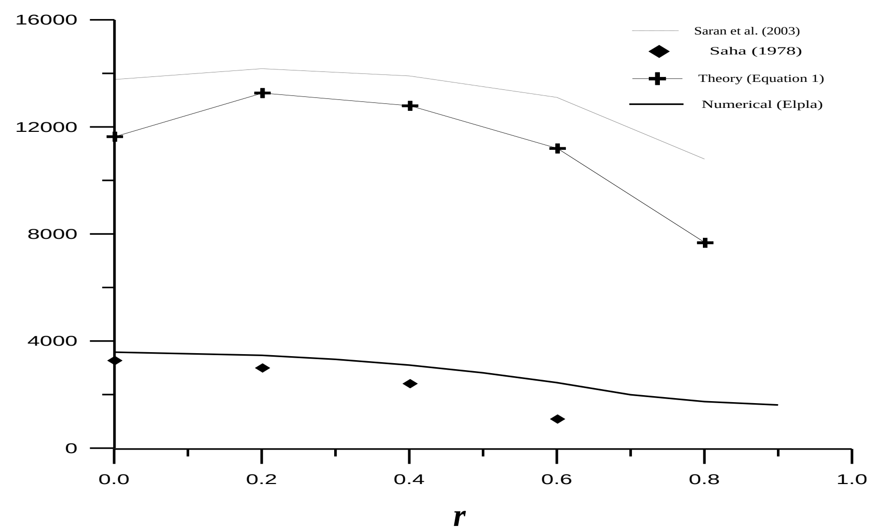

Figure 6: Comparison of q_u values from this study with results after Saha (4) and Saran *et al*. (5) for $\phi = 38°$

$q_u\,(kN/m^2)$

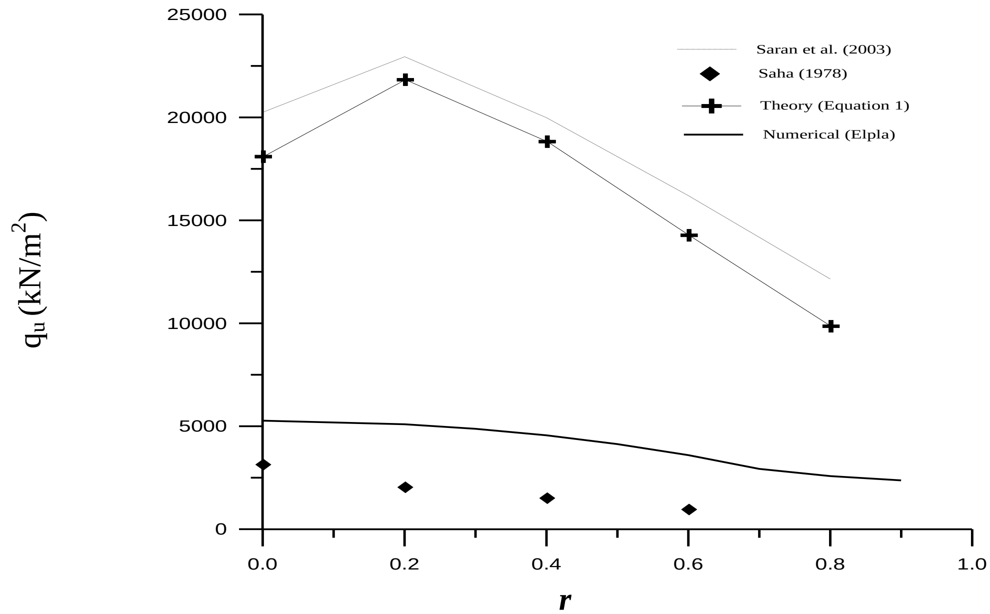

Figure 7: Comparison of q_u values from this study with results after Saha (4) and Saran *et al.* (5) for ϕ = 40°

DISCUSSION AND CONCLUSIONS

The analysis results show that the ultimate bearing capacity of ring footings on sand depends upon the ratio of internal to external diameter. The computed values of ultimate bearing capacity are found to decrease continuously with an increase in *r*. It reaches its maximum value for $0.2 \le r \le$ o.4. This may be due to the interaction between the failure wedges forming beneath the ring footing. The values of ultimate bearing capacity with greater values of ϕ, are higher than those for smaller values of ϕ.

For $0 \le r \le \dfrac{e^{(\pi-\alpha)\tan\phi}}{e^{(\pi-\alpha)\tan\phi} + 2\cos\alpha}$ **, the behavior of the footing is considered**

axially symmetric, and the ring behavior for greater *r* values becomes as a strip footing and results in the ultimate bearing capacity reduction.

The failure of soil under footing starts from the external edges of the ring. This confirmed by results of numerical analysis and is reported as edge effect by others (7, 11).

The computed values of ultimate bearing capacity when ϕ =30°- 35° are identical with the results reported in the literature.

REFERENCES

1. Bowles, J.E. (1982). ***"Foundation analysis and design"***. McGraw-Hill, New York.

2. Egorov, K.E. (1965). "***Calculation of bed for foundation with ring footing"***. Proceeding of the 6th International Conference on Soil Mechanics and Foundation Engineering, 2: p.p.41.

3. Bowles, J.E. (1997). "***Foundation analysis and design***". McGraw-Hill, New York.

4. Saha, M.C. (1978). "***Ultimate bearing capacity of ring footings on sand***". M. Eng. Thesis, University of Roorkee, Roorkee, U.P., India.

5. Saran, S., Bhandari, N.M., and Al-Smadi, M.M. (2003). "***Analysis of eccentrically – obliquely loaded ring footings on sand***". Indian Geotechnical Journal, 33(4): p.p.422-446.

6. Al-Sanad, H.A., Ismael, N.F., and Brenner, R.P. (1993). "***Settlement of circular and ring plates in very dense calcareous sands***". American Society of Civil Engineering, Journal of Geotechnical Engineering, 119: p.p.4.

7. Martin, C.M., (2001). "***Theoretical analysis of the bearing capacity of shallow ring foundations***". Proceeding of the XVth International Conference on Soil Mechanics and Geotechnical Engineering.

8. Geotech software. (2005). ***Elpla User's Guide, Version 9.0***, Professional Edition. Germany/ Egypt.

9. Hataf, N. and Razavi, M.R. (2003). "***Model test and finite element analysis of bearing capacity of ring footings on losse sand***". Iranian Journal of Science and Technology, vol.27.

10. Razavi, M.R (1999). " ***Bearing capacity of ring footings on sand*** ". M. Sc. Thesis, Shiraz University.

11. Jumkis, A.R. (1969). "Theoretical soil mechanics". Van Nostrand Reinhold company, Canada.

Recived …………………………………………..………..………… (28/12/2009)
Accepted ……………………………………..…...……….....….. (8 /4 /2010)