Wasit Journal for Science & Medicine **2010 3 (2): (42 - 47)**

Mixed Approximate(Hurewicz) Cofibration

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اللاتليفات التقريبية– M(المختلطة)واللاتليفات التقريبية هريوز– M (المختلطة)

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المستخلص:-
في هذا البحث درسنا مفهوما جديدا اسمه اللاتليفات التقريبية –M(المختلطة) واللاتليفات التقريبية هريوز- M (المختلطة) التي يرمز لها *(M- approximate cofibration)* و *(M- approximate Hurewicz cofibration)*.
معظم النظريات الصادقة في اللاتليفات تكون صادقة في اللاتليفات المختلطة واللاتليفات هريوز المختلطة اذا أضفنا عليها بعض الشروط . وعليه حصلنا على النتائج الآتية:-

١- جداء اللا تليفين – M(اللا تليفين هريوز – M) يكون اللا تليف – M (اللا تليف هريوز – M)

٢- السحب الخلفي- M الى اللا تليف – M(اللا تليف هريوز – M) يكون اللا تليف – M (اللا تليف هريوز – M)

ABSTRAC:-

In this papers we study a new concept namely (M-approximate cofibration) Mixed Approximate Cofibration and(M-approximate Hurewicz cofibration) Mixed approximate Hurewicz cofibration.
Most of theorem which are valid for cofibration will also be valid for (M-cofibration); the others will be valid if we add extra conditions . Among the results we obtain are:
1-A product of two Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration) is also a Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration)
- The M-pullback of Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration)is also Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration

Key words: Mixed approximate (Hurewicz) Cofibration , M-pullback, Lowering homotopy property.
Mathematics Subject Classification 2000 : 46L85, 55P05.

1- Introduction:

In our papers ,we introduce and study the new concept of M-approximate Cofibration (M- approximate Hurewciz Cofibration).

We also proof some results and study M-pullback approximate Cofibration and T-lifting function.

Let Y be any space $f_1: X_1 \to Y$, $f_2: X_2 \to Y$ are two fiber spaces and $\alpha: X_2 \to X_1$ such that $f_1 o\alpha = f_2$, let $X = \{X_1, X_2\}$, $f = \{f_1, f_2\}$ the $\{X, f, Y, \alpha\}$, has Mixed Lowering homotopy property (M-LHP) w.r.t a space then Z iff given a map $h: Y \to Z$ and a homotopy $g_t: X_1 \to Z$ satisfying $hof_2 = g_0 o\alpha$ then there exist a homotopy $h_t: Y \to Z$ with $h_0 = h$ and $h_t of_1 = g_t$ forall $t \in I$. M-fiber space is called M-cofibration For class $\Re$ if f has (M-LHP) for each $Z \in \Re$.The word map in this work means continuous function, $\Re$ means the classes of topological space and I means [0,1].

2- Preliminaries:

Recalled here same basic concepts and clarify notions used in the sequel

Definition 2-1 [5,6]:

Let f,$g: E \to B$ be mapping and ξ be an open cover of B,we say that f,g are ξ-closed iff given $e \in E$ then there exist $w \in \xi$ such that $f(e), g(e) \in w$

Definition 2-2 [4,5,6]:- A map $p: E \to B$ have to approximate lowering homotopy property (A-LHP) w.r.t X iff given a map $h: B \to X$ and a homotopy $f_t: E \to X$ such that $hop = f_0$ and open cover ξ of X,
then there exist a homotopy $h_t: B \to X$ with $h_0 = h$ and $h_t op, f_t$ are ξ-closed in f_t , for all $t \in I$. Now let $\Re$ be a given class of topological space , a map p is a cofibration w.r.t $\Re$ iff $p: E \to B$ has (LHP) w.r.t each $X \in \Re$

Definition 2-3 [2,3]:-

1- Let X_1, X_2, Y be three topological spaces , let $X = \{X_1, X_2\}$, $f = \{f_1, f_2\}$ where $f_1: X_1 \to Y$, $f_2: X_2 \to Y$ are two fiber space and $\alpha: X_2 \to X_1$ such that $f_1 o\alpha = f_2$ then $\{X, f, Y, \alpha\}$ is a M-fiber space (Mixed fiber space)

$$\begin{tikzcd} X_2 \arrow[rr, "\alpha"] \arrow[dr, "f_2"'] & & X_1 \arrow[dl, "f_1"] \\ & Y & \end{tikzcd}$$

If $X_1 = X_2 = X$, $\alpha = identity$, $f = f_1 = f_2$ then $\{X, f, Y\}$ is the usual fiber space

2- let $\{X, f, Y, \alpha\}$ be a M-fiber space let $y_0 \in Y$ then $F = \{f(y_0)\}$ is the M-fiber over y_0

Definition 2- 4 [2]:- the $\{X, f, Y, \alpha\}$ be a M-fiber structure , X be any space,and $g: Y' \to Y$ be any continuous map into base Y

Let $X'_1 = \{(x_1, y') \in X_1 \times Y' : f_1(x_1) = g(y')\}$ and

$X'_2 = \{(x_2, y') \in X_2 \times Y' : f_2(x_2) = g(y')\}$ then

$\underline{X'} = \{X_1', X_2'\}$ is called a M-pullback of $\underline{f}$ by g and

$\underline{f'} = \{f_1', f_2'\}: \underline{X'} \to Y'$ is called induced M- function of $\underline{f}$ by g, that means $f_1': X_1 \times Y' \to Y'$, $f_2': X_2 \times Y' \to Y'$ are called induced M- function of $\{f_1', f_2'\}$ by g

Define $\alpha': X_2' \to X_1'$ by $\alpha'(x_2, y') = (\alpha(x_2), y')$

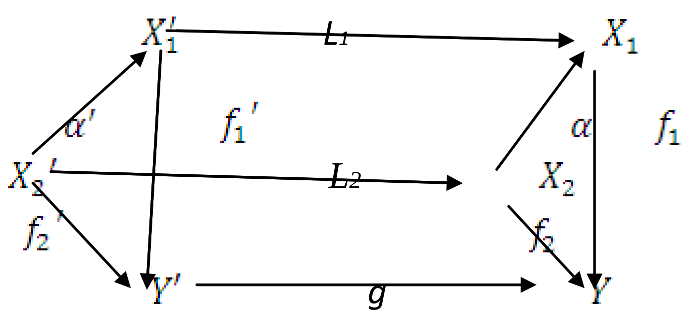
To show α' is continuous

Since $\alpha' = \alpha \times I_{y'}$, α is continuous and $I_{y'}$ is continuous then α' is continuous

To show α' is commutative

$(f'_1 o\alpha')(x_2, y') = f'_1(\alpha'(x_2, y')) = f'_1(\alpha(x_2), y') = y'$, also

$f'_2(x_2, y') = y'$ .therefore $f'_1 o\alpha' = f'_2$



3- M- approximate(Hurewicz) Cofibration

Definition 3-1:- Let Y be any space $f_1: X_1 \to Y$, $f_2: X_2 \to Y$ are two fiber space and $\alpha: X_2 \to X_1$ such that $f_1 o\alpha = f_2$, let $X = \{X_1, X_2\}$, $f = \{f_1, f_2\}$ the $\{X, f, Y, \alpha\}$, has Mixed approximate Lowering homotopy property (M-ALHP) w.r.t a space then Z iff given a map $h: Y \to Z$ and a homotopy $f_t: X_1 \to Z$ such that $hof_2 = g_0 o\alpha$ and open cover ξ of Z, then there exist a homotopy $h_t: Y \to Z$ with $h_0 = h$ and $h_t of_1, f_t$ are ξ-closed in f_t , for all $t \in I$. M-fiber space is called M- approximate cofibration for class $\mathfrak{R}$ if f has (M-LHP) for each $Z \in \mathfrak{R}$,and the $\{X, f, Y, \alpha\}$ be a M-fiber structure over Y ,we say that $\underline{f}$ is M- approximate Hurewicz Cofibration iff f has (M-ALHP) w.r.t all spaces.

Proposition 3-2:-Every approximate(Hurewicz) Cofibration is Mixed approximate (Hurewicz)Cofibration.

Proof:- let $\{X, f, Y, \alpha\}$ be a M-fiber space such that $X_1 = X_2 = X$, $\alpha = identity$, $f = f_1 = f_2$. let $h: Y \to Z$ and a homotopy $g_t: X_1 \to Z$ such that $hof_2 = g_0 o\alpha$ and all open cover ξ of Z then there exist a homotopy $h_t: Y \to Z$ with $h_0 = h$ and $h_t of_1, g_t$ are ξ-closed in g_t for all $t \in I$

Then f has (M-ALHP) w.r.t Z ,or w.r.t all space

Therefore f has M- approximate(Hurewicz) cofibration

Proposition 3-3:- let $\underline{f}: \underline{X} \to Y$ and $\underline{f}': \underline{X}' \to Y'$ be two M- approximate Cofibration then $\underline{f} \times \underline{f}': \underline{X} \times \underline{X}' \to Y \times Y'$ is also M- approximate Cofibration

Proof:- Let Z be any arbitrary space

Let $h^*: Y \times Y' \to Z$ be map where $h: Y \to Z$ and $h': Y' \to Z$ and

Define $g_t{}^*: X_1 \times X'_1 \to Z$ as $h^* o(f_2 \times f_2{}') = g_0{}^* o(\alpha \times \alpha')$ and two open covers ξ ,ξ' of Z such that

$g_t{}': X'_1 \to Z$ and $g_t: X_1 \to Z$ .since $\underline{f}, \underline{f}'$ are M- approximate Hurewicz Cofibration ,then there exist a homotopy $h_t: Y \to Z$ with $h_0 = h$ and $h_t of_1, g_t$ are ξ-closed in g_t ,and a homotopy $h_t{}': Y' \to Z$ with $h_0{}' = h'$ and $h_t' of_1', g_t'$ are ξ'-closed in g_t'

Now for $g_t^{\ *}$ and open cover $\xi \times \xi'$ of $Z \times Z$, then there exist $h_t^{\ *}: Y \times Y' \to Z$ define as $h_0^{\ *} = h^*$ and $h_t^{\ *} o(f_1 \times f_1'), g_t^{\ *}$ are $\xi \times \xi'$-closed in $g_t^{\ *}$,Since Z be any arbitrary

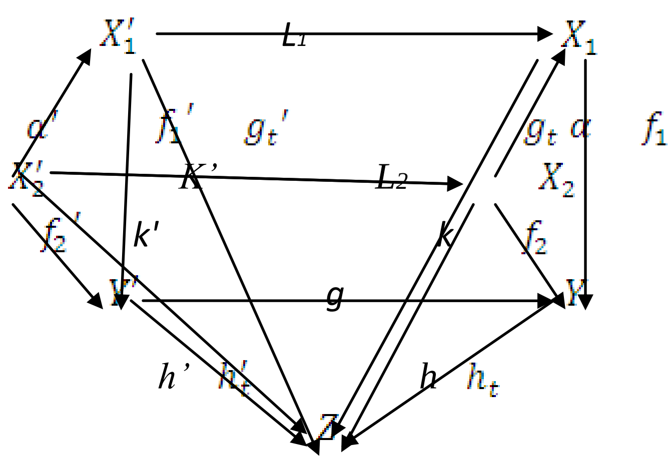
Therefore $\underline{f} \times \underline{f}': \underline{X} \times \underline{X}' \to Y \times Y'$ is M-Hurewicz Cofibration

Proposition 3-4:- The M-pullback of M- approximate (Hurewicz) Cofibration is also M- approximate (Hurewicz) Cofibration

Proof:- Let $h': Y' \to Z$ and $h: Y \to Z$ .Defin a homotopy $g_t: X_1 \to Z$ such that $hof_2 = g_0 o\alpha$ and open cover ξ of Z . since f has M- approximate cofibration then there exist a homotopy $h_t: Y \to Z$ with $h_0 = h$ and $h_t of_1, g_t$ are ξ-closed in g_t

Defin $g_t': X'_1 \to Z$ such that $h'of_2' = g_0'o\alpha'$, $g_t' = g_t oL$ and open cover ξ' of Z, then there exist a homotopy $h_t': Y' \to Z$ with $h_0' = h'$ and $h_t' of_1', g_t'$ are ξ'-closed in g_t'

Therefore $\underline{f}': \underline{X}' \to Y'$ has M- approximate cofibration



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Recived …………………………………………..………………………. (16/2/2010)
Accepted ……………………………………………………...……………... (23/6/2010)