

On 3-semiprime Ideal With Respect To An element Of A near ring

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Abstract

In this paper ,we introduce the notions 3-semiprime ideal with respect to an element x denoted by $(x-3-S-P-I)$ of a near ring and the 3-semiprime ideal near ring with respect to an element x denoted by $(x-3-S-P-I \text{ near ring})$, and we studied the image and inverse image of $x-3-S-P-I$ under epimorphism and the direct product of $x-3-S-P-I$ of near ring and we extension to fuzzify this notions and give some properties and examples about this .

الخلاصة

قدمنا في هذا البحث مفهومي المثالية 3-semiprime ideal بالنسبة لعنصر ما في الحلقة القريبة N والتي يرمز لها بالرمز $(x-3-S-P-I)$, وأيضا حلقة المثاليات القريبة 3-semiprime ideal بالنسبة لعنصر ما في N والتي يرمز لها بالرمز $(x-3-S-P-I \text{ near ring})$. كما ودرشنا الصور المباشرة ومعكوس الصورة للمثالية تحت التشاكل الشامل وأعطينا بعض الخواص التي تتعلق بهذه المثالية. بالإضافة إلى دراسة الحالة الضبابية لهذه المثالية مع بعض الخصائص والأمثلة حول الموضوع .

Introduction

We will refer that all near rings and ideal in this paper are left .In 1905 ,L.E Dickson began the study of a near ring and later in 1930,Wieland has investigated it .Furth material about a near ring can be found [1].In 1965,L.A.Zadeh introduced the concept of fuzzy subset [7] .In 1982 W.Liu introduced the notion of a fuzzy ideal of near ring [13].In 1989 the notion of completely semi prime ideal(C.S.P.I) was introduced by P.DHeena[5].In 1991 N.J .Groenewald introduced the notion 3-semiprime of ideal near ring N [2] and in this year he introduced the notion 3-prime ideal of a near ring . The purpose of this paper is as mention in the abstract .

Key word

Near ring,3-prime ideal, completely prime ideal , union and intersection ideals,3-semi prime ideal.

1.Preliminaries

In this section we give some basic concepts that we need in second section .

Definition (1.1) [1]

A left near ring is a set N together with two binary operations "+" and "." such that

(1) $(N,+)$ is a group (not necessarily abelian)

(2) $(N, .)$ is a semigroup.

(3) $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ For all $n_1, n_2, n_3 \in N$.

Definition (1.2) [1]:

Let N be a near ring. A normal subgroup I of $(N, +)$ is called a left ideal of N if

(1) $I \cdot N \subseteq I$.

(2) $\forall n, n_1 \in N$ and for all $i \in I, (n_1 + i) \cdot n - n_1 \in I$

Definition (1.3) [2]:

An ideal I of near ring N is called a 3-prime ideal for all $a, b \in N, a \cdot N \cdot b \subseteq I$ implies $a \in I \vee b \in I$.

Definition (1.4) [3]

Let $(N_2, +', \cdot')$ and $(N_1, +, \cdot)$ be two near rings

The mapping $f : N_1 \rightarrow N_2$ is called a near ring homomorphism if for all $m, n \in N_1$

$f(m + n) = f(m) +' f(n)$ and $f(m \cdot n) = f(m) \cdot' f(n)$.

Theorem (1.5) [4]

Let $f : (N_1, +, \cdot) \rightarrow (N_2, +', \cdot')$ be homomorphism

(1) If I is ideal of a near ring N_1 , then $f(I)$ is ideal of a near ring N_2 .

(2) If J is ideal of a near ring N_2 , then $f^{-1}(J)$ is ideal of a near ring.

Theorem (1.6)[2]

Let $\{I_j\}$ be a family of ideals of a near ring N , then

(1) $\bigcap_{j \in J} I_j$ is an ideal of N .

(2) if $\{I_j\}$ is a chain, then $\bigcup_{j \in J} I_j$ is an ideal of N .

Definition (1.7) [5]

Let $\{N_j\}_{j \in J}$ be a family of near rings, J is an index set and $\prod_{j \in J} N_j = \{(x_j) : x_j, \text{ for all } j \in J\}$

be the directed product of N_j with the component wise defined operations "+" and "." is called the direct product near ring of the near rings N_j .

Definition (1.8) [5]

A near ring N is called integral domain if has no zero divisions.

Definition (1.9) [6]

The factor near ring N/I is defined as in case of ring.

Definition (1.10) [7]

Let X be a non-empty set. A function $\mu : X \rightarrow [0,1]$ is called a fuzzy subset of X (a fuzzy set in X), where $[0,1]$ is a closed interval of numbers.

Definition (1.11)[7]

Let μ be a fuzzy subset of a non empty set X . If $\mu(y) = 0$, for every $y \in X$ then μ is called empty fuzzy set.

Definition (1.12)[7]

Let μ be a non- empty fuzzy subset of a near ring N , that ($\mu(y) \neq 0$ for some $y \in N$).then μ is said to be fuzzy ideal of N if it satisfies that following conditions :

$$(1) \mu(z - y) \geq \min\{\mu(z), \mu(y)\}$$

$$(2) \mu(z.y) \geq \min\{\mu(z), \mu(y)\}$$

$$(3) \mu(y +z -y) \geq \mu(z)$$

$$(4) \mu(z.y) \geq \mu(y) , \forall y, z \in N.$$

when the subset of N satisfies 1,2 is called fuzzy sub near ring .

Remark (1.13) [8]

If μ is a fuzzy ideal of near ring N then

$$(1) \mu(z + y) = \mu(y + z)$$

$$(2) \mu(0) \geq \mu(z) , \forall y, z \in N.$$

Definition (1.14) [9]

Let $f : (N_1, +, \cdot) \rightarrow (N_2, +', \cdot')$ be a function. For a fuzzy set μ in N_2 , we define

$$(f^{-1}(\mu))(x) = \mu(f(x)) \text{ for every } x \in N_1. \text{ For a fuzzy set } \lambda \text{ in } X, f(\lambda) \text{ is defined}$$

by

$$(f(\lambda))(y) = \begin{cases} \sup \lambda(x) & \text{if } f(x) = y, y \in N_1 \\ 0 & \text{otherwise } y \end{cases}$$

Definition (1.15) [8]

Let μ be fuzzy ideal of a near ring N and f be a function from the near ring N_1 into a near ring N_2 . Then we call μ is f -invariant if and only if for all $y, z \in N, f(z) = f(y)$ implies $\mu(z) = \mu(y)$.

Definition (1.16) [10]

Let μ be a fuzzy ideal of a near ring N then μ^* is a fuzzy subset in N defined by

$$\mu^*(y) = \mu(y) + 1 - \mu(0), \forall y \in N .$$

Definition (1.17) [9]

Let μ be a fuzzy subset of a near ring N and

$t \in [0, 1]$ defined $\mu_t = \{n \in N : \mu(n) \geq t\}$ is called t -cut .

Definition (1.18) [11]

The fuzzy subset n_t of a near ring defined by $\begin{cases} t & y = n \\ 0 & y \neq n \end{cases} \quad \forall y \in N$ is called a fuzzy singleton, where $t \in [0,1]$.

Definition (1.19) [12]

let μ be a fuzzy ideal of N . then the set μ_* is defined by $\mu_* = \{y \in N : \mu(y) = \mu(0)\}$ where 0 is the zero element of N .

Remark [1.20] [13]

let μ is a fuzzy ideal of N if and only if μ_* is an ideal of N .

2. 3-semiprime ideal with respect to an element

This section is devoted to study 3-semiprime ideal with respect to an element of a near ring, and x-3-S-P-I near ring.

Definition (2.1)

An ideal I of near ring N is called 3-semiprime ideal with respect to an element of near ring denoted by (x-3-S-P-I) if for all $a \in N$ $x.(a.N.a) \subseteq I \rightarrow x.a \in I$.

Example(2.2)

Considers the set $N=\{0,a,b,c\}$ be a near ring with addition and multiplication defined by the following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	C
0	0	a	0	a
a	0	a	a	0
b	0	a	b	c
c	0	a	c	b

Let $I=\{0,a\}$ is c-3-semiprime ideal since $c.(a.N.a) \subseteq I \rightarrow c.a \in I$.

Proposition (2.3)

Let $\{I_j\}_{j \in J}$ be a family of x- 3-semiprime ideal of a near ring N for all $j \in J$, $x \in N$ Then $\bigcap_{j \in J} I_j$ is a x- 3-semiprime ideal of N .

Proof

Let $x,a \in N$ $\bigcap_{j \in J} I_j$ is an ideal [1.7] since I_j is x- 3-semiprime ideal of N , $I_j \neq \phi, I_j \subseteq N$, .let $x(a.N.a) \subseteq I$
 $x(a.N.a) \subseteq I, x(a.N.a) \subseteq \bigcap_{j \in J} I_j$ since I_j is x- 3-semiprime ideal $x.a \subseteq I$
 $x.a \subseteq \bigcap_{j \in J} I_j$ implies $\bigcap_{j \in J} I_j$ is an x- 3-semiprime ideal of N .

Proposition (2.4)

Let $\{I_j\}_{j \in J}$ be chain of a x-3-S-P-I of a near ring N , then $\bigcup_{j \in J} I_j$ is x-3-S-P-I of N where $j \in J$.

Proof

Let $\{I_j\}_{j \in J}$ Be chain of x-3-S-P-I of near ring . $\bigcup_{j \in J} I_j$ is an ideal of N [1.7] Now let

$$x.(a.N.a) \subseteq \bigcup_{j \in J} I_j, \quad x.(a.N.a) \subseteq I_j \quad \forall j \in J \text{ since } I_j \text{ is x-3-S-P-I } x.a \subseteq I_j \rightarrow x.a \subseteq \bigcup_{j \in J} I_j \rightarrow \bigcup_{j \in J} I_j \text{ is}$$

an x- 3-semiprime ideal of N.

Definition (2.5)

The near ring N is called 3-semiprime ideal near ring with respect to an element x denoted by (x-3-S-P-I near ring), if every ideal of a near ring N is an x- 3-semiprime ideal of N ,where $x \in N$.

Example (2.6)

considers the near ring $N=\{ 0,a,b,c\}$ with addition and multiplication defined by the following tables

+	0	A	b	c
0	0	A	b	c
A	a	0	c	b
B	b	C	0	a
C	c	B	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	a	b	c
c	0	a	b	c

N is c-3-semi prime ideal near ring since all ideals of N , $I_1\{0\}$ and $I_2=N$ are c-3-semi prime ideal .

Proposition (2.7)

Let N be a near ring with multiplicative identity e' then I is e' - 3-S-P-I of near ring N if and only if I is a 3-semiprime ideal N.

Proof

←

Let $y \in N, e'$ is the identity $e'(y.N.y) \subseteq I$ implies that $y.N.y \subseteq I$ since I is 3-S-P-I of N hence $y \in I$ implies $e'.y \in I$ there for I is e' - 3-S-P-I of N.

→

Let $y \in N$ and I is e' -3-S-P-I of N let $y.N.y \subseteq I, e'(y.N.y) \subseteq I$ since I is e' - 3-S-P-I of N $\rightarrow e'.y. \subseteq I$ implies $y \in I$ since e' the identity of N \rightarrow I is 3-S-P-I of N.

Remark (2.8)

In general not all x-3-S-P-I are 3-prime ideal .

Example (2.9)

considers the near ring $N=\{ 0,1,2,3\}$ be a near ring with addition and multiplication defined by the following tables

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	0	0	0
3	0	1	2	3

Let $I_1=\{0,1\}$ be 2-3-semiprime ideal of N but I is not C.P.I of N, since $2.N.2=0 \in I$ but $2 \notin I$.

Theorem (2.10)

Let $(N_1, +, \cdot)$ and $(N_2, +', \cdot')$ be two near ring , $f : N_1 \rightarrow N_2$ be epimorphism and I be x-3-S-P-I of N_1 . Then $f(I)$ is $f(x)$ -3-S-P-I of N_2 .

Proof

Let I be -3-S-P-I of N_1 $f(I)$ is an ideal of N_2 by using theorem [1.6]

Let $c, y \in N_2$, $\exists a \in N_1$ such that $f(a) = y, f(x) = c$, hence

$f(x) \cdot' (f(a) \cdot N_2 \cdot f(a)) \subseteq f(I)$ since f be an epimorphism $f(x \cdot (a \cdot N_1 \cdot a)) \subseteq f(I)$ since I is x-3-S-P-I of N_1

$x \cdot (a \cdot N_1 \cdot a) \subseteq I \rightarrow x \cdot a \in I \rightarrow f(x \cdot a) \in f(I) \rightarrow f(x) \cdot' f(a) \in f(I)$

$f(x)$ is $f(x)$ -3-S-P-I of N_2 .

Theorem (2.11)

Let $(N_1, +, \cdot)$ and $(N_2, +', \cdot')$ be two near ring , $f : N_1 \rightarrow N_2$ be epimorphism and J be a $f(x)$ -3-semiprime ideal of N_2 . Then $f^{-1}(J)$ is a x-3-S-P-I of N_1 , where $y = f(x)$, $\ker(f) \subseteq f^{-1}(I)$.

Proof

Let $x, a \in N_1$, such that $f^{-1}(J)$ is an ideal by using theorem [1.6] ,

$x \cdot (a \cdot N_1 \cdot a) \subseteq f^{-1}(J) \rightarrow f(x \cdot (a \cdot N_1 \cdot a)) \subseteq J \rightarrow f(x) \cdot' (f(a) \cdot' N_2 \cdot f(a)) \subseteq J$

since J is $f(x)$ -3-S-P-I of N_2

$f(x) \cdot' f(a) \in J \rightarrow f(x \cdot a) \in J \rightarrow x \cdot a \in f^{-1}(J)$

$\rightarrow f^{-1}(J)$ is a x-3-S-P-I of N_1 .

Proposition (2.12)

If N is non zero near ring then I is 0-3-semiprime ideal of N .

Theorem (2.13)

Let $\{N_j\}_{j \in J}$ be a family of a near rings, $x_j \in N_j$ and I_j be x_j -3-S-P-I of N_j for all $j \in J$.

Then $\prod_{j \in J} I_j$ is (x_j) -3-S-P-I of the direct product near ring $\prod_{j \in J} N_j$.

Proof

Let $(a_j), (x_j) \in \prod_{j \in J} N_j$, and $\prod_{j \in J} I_j$ is an ideal of $\prod_{j \in J} N_j$ by using definition (1.9) such that

$(x_j) \cdot (a_j) \cdot \prod_{j \in J} N_j \cdot (a_j) \subseteq \prod_{j \in J} I_j$ for all $j \in J$

$x_j \cdot a_j \cdot N_j \cdot a_j \subseteq I_j$ since I_j is x_j -3-S-P-I of N_j for all $j \in J$

$\rightarrow x_j \cdot a_j \in I_j \quad \forall j \in J \rightarrow (x_j \cdot a_j) \in \prod_{j \in J} I_j \rightarrow (x_j) \cdot (a_j) \in \prod_{j \in J} I_j$

$\rightarrow \prod_{i \in J} I_j$ is (x_j) -3-S-P-I of the direct product near ring $\prod_{i \in J} N_j$.

Theorem (2.14)

Let I be an ideal of the x-3-semiprime ideal near ring N . Then the factor near ring N/I is x+I-3-semiprime ideal near ring .

proof

The natural homomorphism $\text{nat}_I : N \rightarrow N/I$ which is defined by $\text{nat}_I(x) = x+I$, for all $x \in N$, is an epimorphism. Now let J be an ideal of the factor near ring N/I . Then by theorem (1.6) we have $\text{nat}_I^{-1}(J)$ is an ideal of the near ring N . $\Rightarrow \text{nat}_I^{-1}(J)$ is a x -3-S-P-I of N [since N is x -3-S-P-I near ring]. By theorem (2-18) we have $\text{nat}(\text{nat}_I^{-1}(J)) = J$ is $\text{nat}_I(x)$ -3-semiprime ideal of $N/I \Rightarrow J$ is $x+I$ -3-semiprime ideal of factor near ring. Then N/I is $x+I$ -3-semiprime ideal near ring.

3. 3-semiprime fuzzy ideal with respect to an element

This section is devoted to study 3-semiprime fuzzy ideal with respect to an element of a near ring.

Definition (3.1)

A fuzzy ideal μ of a near ring N is called 3-semiprime fuzzy ideal with respect to an element of a near ring N denoted by **x-3-P-F-I** for all $a \in N$ $\mu(x.a) \geq \inf_{n \in N} \mu(x.a.n.a)$.

Example (3.2)

Consider the near ring $N = \{0, 1, b, 2\}$ with addition and multiplication defined by the following tables.

+	0	1	b	2
0	0	1	b	2
1	1	0	2	b
b	b	2	0	1
2	2	b	1	0

.	0	1	b	2
0	0	0	0	0
1	0	1	0	1
b	0	b	0	b
2	0	2	0	2

The fuzzy ideal μ of N denoted by $\mu(y) = \begin{cases} 1 & \text{if } y \in \{0, b\} \\ 0 & \text{if } y \in \{2, 1\} \end{cases}$ is an **b-3-S-P-F-I**.

Theorem (3.3)

Let μ be a fuzzy ideal of a near ring N , and μ is a **x-3-S-P-F-I** of N then μ_t is **x-3-S-P-I** of N for all $t \in [0, \mu(0)]$.

Proof

Let $x, a, n \in N$, $x.(a.N.a) \subseteq \mu_t \rightarrow x.(a.n.a) \in \mu_t$ by using definition (1.11), $\mu(x.(a.n.a)) \geq t \Rightarrow \inf_{n \in N} \mu(x.(a.n.a)) \geq t$ since μ is x -3-S-P-F-I of $N \Rightarrow \mu(x.a) \geq$

$$\inf_{n \in N} \mu(x.(a.n.a)) \geq t$$

$\Rightarrow \mu(x.a) \geq t, x.a \in \mu_t \Rightarrow \mu_t$ is **x-3-S-P-I** of N .

Remark (3.4)

let f be an aepimomorphisem from the near ring N_1 onto the near ring N_2 and let μ be **x-3-S-P-F-I** of N_1 ,then $f(\mu_t)$ is a **f(x)-3-S-P-I** of N_2 .

proof

by proposition (3.3) we have μ_t is **x-3-S-P-I** of N_1 .By theorem (2.10) we get $f(\mu_t)$ is a **f(x) -3-S-P-I** of N_2 .

Theorem (3.5)

Let μ be fuzzy subset of a near ring N , if μ is **x-3-S-P-I** of N , then μ_* is **x-3-S-P-I** of N .

Proof

Let $a, c, x \in N$ such that μ_* is an ideal of N $x.a.N.a \subseteq \mu_*$
 $\rightarrow x.(a.n.a) \in \mu_* \Rightarrow \mu(x.(a.n.a)) = \mu(0)$ by using definition μ_* since μ is **x-3-S-P-F-I** of N
 $\{\mu(x.a) \geq \inf_{n \in N} \mu(x.(a.n.a)) = \mu(0) , \mu(0) \geq \mu(y), \forall y \in N, \text{ since } \mu \text{ fuzzy ideal, or } \mu(x.b) = \mu(0)\}$
 $\rightarrow x.b \in \mu_* \Rightarrow \mu_*$ is **x - 3 - S - P - I** of N .

Theorem (3.6)

Let f be an a epimorphism from the near ring N_1 onto the near ring N_2 .Then μ is **f(x)-3-S-P-F-I** of N_2 if $f^{-1}(\mu)$ is **x-3-S-P-F-I** of N_1 ,for all $x \in N$.

Proof

Let , $x, n, a \in N, f(x), f(n), f(a) \in N_2$,since μ is **(x)-3-S-P-F-I** of N_2 then
 $\mu(f(x).f(a)) \geq \inf_{n \in N} \mu(f(x).(f(a).f(n).f(a)))$
 $\mu(f(x.a)) \geq \inf_{n \in N} \mu(f(x.(a.n.a)))$
 $f^{-1}\mu(x.a) \geq \inf_{n \in N} f^{-1}\mu(x.(a.n.a))$
 $\Rightarrow f^{-1}(\mu)$ is **x-3-S-P-F-I** of N_1

Corollary (3.7)

Let f be an a epimorphism from the near ring N_1 onto the near ring N_2 .Then the mapping $\mu \rightarrow f(\mu)$ defines a onto correspondence between the set of all f -invariant **x-3-S-P-F-I** of N_1 the set of all **f(x)-3-S-P-F-I** of N_2 .

Proof

Directly from theorem (3.6) .

Corollary (3.8)

Let f be an a epimorphism from the near ring N_1 onto the near ring N_2 .Then μ^* Is **f(x)-3-S-P-F-I** of N_2 if and only if $f^{-1}(\mu)$ is an **x-3-S-P-F-I** of N_1 .

Proof

By from theorem (3.6)

proposition (3.9)

let μ be a fuzzy subset of a near ring N , then μ is **x-3-S-P-F-I** of N if and only if μ^* is **x-3-S-P-F-I** of N .

proof

→ let $x, a, n \in N$ since μ is x-3-S-P-F-I of N

$$\mu(x.a) \geq \inf_{n \in N} \mu(x.(a.n.a)), \forall n \in N. \text{ since } \mu(x.a) + 1 - \mu(0) \geq \inf_{n \in N} \mu(x.(a.n.a))1 - \mu(0).$$

$$\mu^*(x.a) \geq \inf_{n \in N} \mu^*(x.(a.n.a)) \Rightarrow \mu^* \text{ is x-3-S-P-F-I of } N.$$

← let μ^* is x-3-P-F-I of N .

$$\mu^*(x.y) \geq \inf_{n \in N} \mu^*(x.(a.n.a))$$

by using definition of μ^*

$$\mu(x.a) + 1 - \mu(0) \geq \inf_{n \in N} \mu(x.(a.n.a))1 - \mu(0).$$

$$\mu(x.a) \geq \inf_{n \in N} \mu(x.(a.n.a)), \forall n \in N.$$

⇒ μ is **x-3-S-P-F-I** of N .

Theorem (3.10)

A fuzzy ideal μ of a near ring N is an x-3-S-P-F-I of N if and only if $x_t.(a_t.n_t.a_t) \in \mu$ implies $x_t.a_t \in \mu$ for all fuzzy singleton $x_t, a_t, n_t \in N$.

Proof

→ Let $x_t, a_t, n_t \in N, t \in [0, \mu(0)]$, $x_t.(a_t.n_t.a_t) \in \mu \rightarrow$

$$(x.(a.n.a))_t \in \mu \rightarrow x.(a.n.a) \in \mu_t$$

$$\mu(x.(a.n.a)) \geq t \text{ since } \mu \text{ is x-3-S-P-F-I of } N, \mu(x.a) \geq \inf_{n \in N} \mu(x.(a.n.a)) \geq t$$

$$\mu(x.a) \geq t \rightarrow (x.a)_t \in \mu \rightarrow x_t.a_t \in \mu$$

← let $x_t, a_t, n_t \in N$. suppose that

$$x_t.(a_t.n_t.a_t) \in \mu \Rightarrow x_t.a_t \in \mu \rightarrow (x.(a.n.a))_t \in \mu$$

$$\Rightarrow (x.(a.n.a))_t \geq t = \inf_{n \in N} \mu(x.(a.n.a))$$

$$x_t.a_t \in \mu \rightarrow x_t.(a_t.n_t.a_t) \in \mu, \mu(x.a) \geq t$$

$$\mu(x.a) \geq \inf_{n \in N} \mu(x.(a.n.a)) \Rightarrow \mu \text{ is } \mathbf{x-3-S-P-F-I} \text{ of } N.$$

Theorem (3.11)

Let $\{\mu_i\}_j \in J$ be a family of is x-3-S-P-F-I of N, then $\bigcap_{j \in J} \mu_j$ is **x-3-S-P-F-I** of N.

Proof

Let $x, a, n \in N$ and $\{\mu_i\}_j \in J$ be a family of x-3-S-P-F-I of N where $\bigcap_{j \in J} \mu_j(x.a) = \inf_{\substack{n \in N \\ j \in J}} \mu_j(x.a)$

$$\mu_j(x.a) \geq \inf_{\substack{n \in N \\ j \in J}} \mu_j(x.(a.n.a))$$

$$\inf_{j \in J} \mu_j(x.a) \geq \inf_{j \in J} (\inf_{n \in N} \mu_j(x.(a.n.a)))$$

$$\inf_{j \in J} \mu_j(x.a) \geq \inf_{n \in N} \{ \inf_{j \in J} \mu_j(x.(a.n.a)) \}$$

$$\Rightarrow \bigcap_{j \in J} \mu_j(x.a) \geq \inf_{n \in N} (\bigcap_{j \in J} \mu_j(x.(a.n.a)))$$

$$\Rightarrow \bigcap_{j \in J} \mu_j \text{ is } \mathbf{x-3-S-P-F-I} \text{ of } N.$$

Remark (3.12)

Let $\{\mu_i\}_{j \in J}$ be chain of a x-3-S-P-F-I of N ,then $\bigcup_{j \in J} \mu_j$ is **x-3-S-P-F-I** of N.

Proof

By using Remark (1.14).

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