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Prediction the gold prices using Grey residual correction Model and Grey Verhulst residual correction model

التنبؤ بأسعار الذهب باستخدام النموذج الرمادي ذو البواقي المصححة والنموذج فرهولست الرمادي ذو البواقي المصححة

Prof. DR. Mohammed H. AL- Sharoot
أ.د.محمد حبيب الشاروط
Faculty of Administration and Economics-Qadisiyah, Iraq.–University of Al
قسم الإحصاء ، كلية الإدارة والاقتصاد، جامعة القادسية ، العراق
mohammed.alsharoot@qu.edu.iq

Nadia Ali Ayed
نادية علي عايد
Faculty of Administration and Economics-Qadisiyah, Iraq.–University of Al قسم
الإحصاء ، كلية الإدارة والاقتصاد، جامعة القادسية ، العراق
nidea.iead@uobasrah.edu.iq

Abstract:

Most researchers face uncertain conditions such as incomplete data and confusing information when they deal with the task of prediction. Assuming that the clear data for a system are displayed in a white color and the unknown data are shown in black color , but in reality we notice that the real data for most existing natural systems are neither purely white color nor black color but rather a mixture of the both so it is called a grey systems. This data's primary specification is its incompleteness. One could argue that one of the primary responsibilities of applying grey system theory is to predict grey models, which have the advantage of being able to be created using an infinite amount of incomplete data. This makes them a useful tool for predicting systems with intricate, unstable, and irregular structures, as opposed to artificial intelligence techniques and Box-Jenkins models, which necessitate a significant investment of time and energy in order to estimate parameters and establish models for various stages. Therefore, we can conclude that the grey models for forecasting are more practical and user-friendly. Despite having fairly simple computations, this model's accuracy is one of its most crucial aspects.The models were also improved through correcting by the Fourier residual errors method to increase the accuracy , and comparison between the applied models using some criterion of accuracy to choose the best one . the results shows the FDGM is the appropriate model for predicting the daily gold prices.

Keywords: Grey Model, Verhulst - grey model, Discrete grey Model, Fourier residual errors correction.

المستخلص:

يواجه معظم الباحثين ظروفًا غير مؤكدة وبيانات غير كاملة ومعلومات مربكة عندما يتعاملون مع مهمة التنبؤ وبافتراض أن البيانات الواضحة لنظام ما يتم عرضها باللون الأبيض والبيانات غير المعروفة تظهر باللون الأسود، إذن فبيانات معظم أنظمة الطبيعة الموجودة ليست بيضاء أو سوداء بحتة بل هي خليط من الاثنين معًا إنه رمادي. تسمى هذه الأنظمة بالأنظمة الرمادية. المواصفات الرئيسية لهذه البيانات هي عدم اكتمالها. إن التنبؤ بالنموذج الرمادي باعتباره الجوهر الرئيسي لنظرية النظام الرمادي يتمتع بامتياز إنشاء نموذج باستخدام بيانات غير محدودة وغير كافية وهو أداة مناسبة للتنبؤ بالأنظمة ذات البنية المعقدة وغير الموثوقة وغير المنتظمة مقارنة بنماذج بوكس جنكينز وتقنيات الذكاء الاصطناعي التي تتطلب الكثيرمن الوقت والجهد من أجل تحديد المعلمات و إنشاء نماذج لمراحل مختلفة، وتكون النماذج الرمادية للتنبؤ أكثر قابلية للتطبيق وأسهل في الاستخدام ومن أهم مميزات هذا النموذج أنه دقيق على الرغم من أن الحسابات بسيطة للغاية . تم تطبيق نموذج فيرهولستالرمادي لمراقبه تطور أسعار الذهب والتنبؤ بالأسعار مستقبلا فضلا ان تحسين النموذج المستخدم عن طريق تصحيح البواقي بفوريبه لزيادة دقه النموذج .

الكلمات المفتاحية: النموذج الرمادي ,نموذج فيرهولست الرمادي ,النموذج الرمادي المنفصل , طريقة تصحيح فوريبه للاخطاء

1. Introduction

Compared to other commodities, gold is the ideal instrument for investing and hedging, and it's also one of the best measures of market performance. Therefore, one of the main goals of many economists is to forecast the price of gold, not just for financial gain, but also for creating financial and investment plans for both public and private enterprises (Dunis & Nathani, 2007) [1] Gold and silver have been used to exchange goods for other things since ancient times. In the past, gold coins were produced and used extensively as standard currency, as shown in the Roman Empire. There are two ways to define gold standards: either define a currency based on the weight of gold that would have an equivalent value, or use gold and silver as coins, this is known as backed currency. The majority of nations in the world now use fiat money, also known as unbacked money. Fiat money is not correlated with the value of other commodities like gold or silver, nor does it hold value over time, because of this, it is less effective as a medium of exchange than precious. The United States was the main hub for global gold trading; however, following the collapse of the Bretton Woods standard system, which linked the US dollar's value to that of gold, the hub of global gold trading shifted back to London, which has long been one of the oldest. According to Jastram and Leyland (2009) [2], the price of gold is still set in US dollars as well as a few other significant currencies . The London Bullion Market Association (LBMA) was founded in 1989 to set the daily gold price because London is regarded as the global hub for gold trade. An ounce, which is equivalent to 31.10347 grams of gold, is used to estimate the global price of gold in US dollars. These days, gold is also valued in major currencies such as the Euro, British Pound, and others (2010) [3].The numerically based gold price forecasting systems available today can't always account for local variations in the price of gold. Predicting the future solely using a subset of the time series' most recent data is known as local prediction. These kinds of forecasts involve creating a curve using the most recent data and then basing predictions on the curve. First put forth by J. Deng in 1982, the grey system theory eliminates the flaws in traditional approaches and just needs a little amount of data. The primary component of grey system theory is the grey model, which has the benefit of allowing for the creation of a model based on incomplete or unknownly distributed data (Deng, 1989) [4]. The Grey Model (GM) or GM(1,1) was initially proposed by Ju-Long in 1982 and further developed in 1989 [4-5]. It is a modeling approach that is specifically designed to handle discrete data, uncertainty data distribution, and situations where there is a limited number of available data points. The GM(1,1) has gained significant popularity and has been widely applied in various fields.

In 2008, Xie and Liu [6] introduced an improvement to the GM(1,1) called the Discrete Grey Model (DGM)has been widely applied in various fields .

In 2008, Xie and Liu [6] introduced an improvement to the GM(1,1) called the Discrete Grey Model (DGM)has been widely applied in various fields .. The DGM is based on the GM(1,1) but incorporates certain enhancements to enhance its effectiveness. Unfortunately, without specific details about Xie and Liu's work, it is challenging to provide further information about the exact improvements made in the DGM. .Overall, the GM(1,1) and its variants, such as the DGM, provide useful tools for modeling and forecasting in situations where discrete data, uncertainty, and limited data availability are present. These models have found applications in various domains, including economics, finance, engineering, and environmental science. ,

The grey system theory has gained rapid development and attracted the attention of many researchers. Numerous systems, including commercial, financial, scientific and technological, agricultural, industrial, transportation, mechanical, meteorological, ecological, hydrological, geological, medicinal, and military ones, have effectively and widely used it. Hsu and Chen (2003) introduced the enhanced grey prediction model [7]. The accuracy of the grey model is one of its most significant characteristics, despite its extremely basic computations(Yang and Huang ,2004)[8]. In the end, it can be said that the grey theory performs better than statistics, probability, and mysterious mathematics because it deals with semi-complex problems and uncertainties. employed the gray model to forecast OPEC's raw oil price. (VanMard & Faghidian, 2014)[9]. In this paper. we divided the time series of the gold prices into two parts, the first part included 80% of the data it was used for estimating the above six grey models, and the second part included 20% of the observations it was used to predict prices of gold in the future . The paper content the following **Section 1** describes the concept of grey system theory .**Section 2** discusses grey models. GM(1,1), the discrete grey model, Verhulst's grey model, and the modification made by using the residuals error Fourier transform **Section 3** using all the grey models in **section 2** to predict the daily time series data for gold prices measured per ounce in US dollars during the time frame of 14/7/2023 to 14/3/2024.The sample size was 245 observations **, Section 4** conclusions and **Section 5** References.

The programming operations, were all done using the R2020a , Eviews 12 and Minitab 21**.**

The aim of the paper

Using daily time series data for gold prices measured in US dollars per ounce for the period from 14/7/2023 to 14/3/2024, this paper aims to predict the daily gold prices using six different grey models: GM(1,1), Verhulst's grey model, Discrete grey model, and three correcting models by employing Fourier residual errors. The best model will be determined by comparing the applied models using some accuracy criteria.

***2.** The Mathematical* Models

a. Grey Model GM

Grey system theory is an interdisciplinary scientific field that was first introduced by Deng (1982) in the early 1980s. Since then, the theory has acquired a lot of attention because of its ability to manage systems with partially understood features. Grey models are superior than classical statistical models because they estimate the behaviors of unknown systems with a minimal amount of data. *Numerous researchers have expressed interest in the gray system theory as it has progressed quickly* . It has been effectively employed by numerous systems in a variety of applications.In grey systems theory, the notation GM(n, m), where n is the order of the difference equation and m is the number of variables, designates a grey model. Although many different types of grey models can be named, the GM(1,1) type, or "Grey Model First Order One Variable," is the most often used grey model in the literature. (Kayacan & Okyay, (2010)) [10].

The steps to build the gray model **GM(1,1)** (Xie, Nai, & Si,2008) [11]

- The existing data with the n sample units are represented as :

$$x^{(0)} = \left(x^{(0)}(1), \dots, x^{(0)}(n)\right) \qquad \dots(1)$$

where $x^{(0)}$ is the series of data .

- Compute the cumulative sum of the current data$x^{(0)}$, in the form of a cumulative sum series

$$x^{(1)}(K) = \sum_{i=1}^{k} x^{(0)}(i) \qquad , k = 1,2, \dots, n \qquad \dots(2)$$

$$x^{(1)} = (x^{(1)}(1), \dots, x^{(1)}(n)) \qquad \dots(3)$$

• Calculate $z^{(1)}$, the means from (1) as:

$$z^{(1)} = \left(z^{(1)}, z^{(2)}, \dots, z^{(n)}\right) \qquad \dots(4)$$

Where:

$$z^{(1)}(K) = 0.5x^{(1)}(K) + 0.5x^{(1)}(k+1), k = 2,3, \dots, \mathrm{r} \quad \dots(5)$$

$$x^{(0)}(k) + az^{(1)}(k) = b \qquad \dots(6)$$

The ordinary differential equation of first order of $X^{(1)}$ as

$$\frac{\mathrm{dX}^{(1)}}{\mathrm{dt}} + \mathrm{aX}^{(1)} = \mathrm{b} \qquad \dots(7)$$

The terms "developing coefficient" and "grey input," respectively, refer to a and b.
They can be calculated as follows:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \qquad \dots(8)$$

Where:

$$= \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \dots & \dots \\ -z^{(1)}(n) & 1 \end{bmatrix} \qquad \dots(9)$$

and:

$$Y = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T \qquad \dots(10)$$

$$x^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \qquad \dots(11)$$

$$x^{(0)}(k+1) = (1 - e^{a})(x^{(0)}(1) - \frac{b}{a})e^{-ak}, k = 1,2, \dots. n. \quad \dots(12)$$

b. *Discrete Grey Model (DGM)* (Wei& Xie. (2020)),[12]
discussed Wang et al by improved. the grey model's forecast accuracy for stochastic oscillation sequences, which helps to overcome the direct's flaw and more accurately portray the data series' growth pattern.

The grey prediction model can successfully derive the system's development trend from sparse samples and is used to address uncertainty issues arising from inadequate data and knowledge. While several expanded iterations of gray prediction models have been created over time, the fixed model structure of the majority of these models restricts their applicability to complicated and nonlinear time series. In order to simulate the growth trend of system sequences with nonlinear, complicated, and fluctuating data characteristics, it is important to design a more versatile and flexible version based on the capacity to extract sparse data information. In light of this, a new information priority principle-based discrete gray prediction model with an adaptive structure is put forth. This model can more easily adjust to the nonlinear and fluctuating features of the system sequence.

The DGM model is defined as follows:

$$\mathrm{x}^{(1)}(\mathrm{k}+1) = \beta_1 \mathrm{X}^{(1)}(\mathrm{K}) + \beta_2 \qquad \dots(13)$$

Eq.(13) It is called the Discrete Grey Mode

Where:

$\beta^{\wedge} = (\beta_1\ \beta_2)^T$is the dcrics of paramctcrs Where $Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}n \end{bmatrix}$ and $B = \begin{bmatrix} X^{(1)}(1) & 1 \\ X^{(1)}(2) & 1 \\ \vdots & \vdots \\ X^{(1)}(n-1) & 1 \end{bmatrix}$

The parameters are estimated using the least squares approach.

$$x^{\wedge(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 \quad \ldots(14)$$

- Let $x^{(0)}(1) = x^{(1)}(1)$ Then the following setting is applied while using the function

or

$$x^{\wedge(1)}(k+1) = \beta_1^k\, x^{(1)}(1) + \frac{1-\beta_1^k}{1-\beta_1} * \beta_2; k = 1,2, \ldots .. n-1 \qquad \ldots(15)$$

$$x^{\wedge(0)}(k+1) = x^{\wedge(1)}(k+1) - x^{\wedge(1)}(k) \qquad \ldots(16)$$

$$, k = 1,2, \ldots n$$

c. The Grey Verhulst mode

Within the grey system, the Verhulst grey model is a unique type of model that explains saturatio processes such as the "S" curve.Li, Z., Liu, Q., Yang, F., Hui, S., & Dong, J. (2009) [13]. This model just needs a small number of data, and as data increases, it is continuously optimized. The majority of processes with saturated states, often known as sigmoid processes, that begin slowly, pick up speed, and then cease growing or grow slowly can be explained by the gray Verhulst model (Ju-Long, D., 1982) [14].

The VGM procedure is define as follows

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b(x^{(1)})^2 \qquad \ldots(17)$$

$$x^{(0)}(k) + az^{(1)}(k) = b\left(z^1(k)\right)^2$$
$$x^{(0)}(k) = -az^{(1)}(k) + b\left(z^1(k)\right)^2 \qquad \ldots(18)$$

Where:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \qquad \ldots(19)$$

Where:

$$= \begin{bmatrix} -z^{(1)}(2) & \left(z^{(1)}(2)\right)^2 \\ -z^{(1)}(3) & \left(z^{(1)}(3)\right)^2 \\ \cdots & \cdots \\ -z^{(1)}(n) & \left(z^{(1)}(n)\right)^2 \end{bmatrix} \qquad \ldots .(20)$$

$$Y = (x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n))^T \qquad \ldots(21)$$

$$x^{(1)}(k+1)=\frac{ax^{(0)}(1)}{bx^{(0)}(1)+\left(a+bx^{(0)}(1)\right)e^{ak}} \qquad \dots(22)$$

$x^{(1)}$ is obtainable from Eq. (22). $x^{(0)}$ be the series that was expected and fitted.

$$x^{(0)}(k)=\frac{ax^{(0)}(1)\left(a-bx^{(0)}(1)\right)}{bx^{(0)}(1)+\left(a-bx^{(0)}(1)\right)e^{a(k-1)}}\times\frac{(1-e^{a})e^{a(k-2)}}{bx^{(0)}(1)+\left(a-bx^{(0)}(1)\right)e^{a(k-2)}} \qquad \dots(23)$$

$$,k=1,2,\dots n$$

d Fourier correction model

One common corrective technique for time series forecasting is residual analysis. Fourier series is one of the residual correction techniques. The Fourier correction method's acceptable performance has led to the approval of increasing the modeling performance of grey models' precision.

The FGM procedure can be define as follows:

$$\varepsilon^{(0)}(k)=x^{(0)}(k)-x^{(0)}(k) \qquad \dots(24)$$

$$\varepsilon^{(0)}(k)\cong\frac{1}{2}a_0+\sum_{i=1}^{z}\left[a_i\cos\left(\frac{2\pi i}{T}K\right)+b_i\sin\left(\frac{2\pi i}{T}K\right)\right] \qquad \dots(25)$$

$$,K=2,3,\dots n$$

$$\mathrm{T}=\mathrm{n}-1\ \text{ and } \mathrm{Z}=\frac{(\mathrm{n}-1)}{2}-1$$

Z will be chosen as an integer number, and T will be an integer number.

Eq. (25) can be rewritten as Eq. (26) where:

$$\varepsilon^{(0)}\cong \mathrm{PC} \quad \dots(26)$$

$$P=\begin{bmatrix} 1/2 & \cos\left(2\frac{2\pi}{T}\right) & \sin\left(2\frac{2\pi}{T}\right) & \cos\left(2\frac{2\pi 2}{T}\right) & \sin\left(2\frac{2\pi 2}{T}\right) & \dots & \cos\left(2\frac{2\pi Z}{T}\right) & \sin\left(2\frac{2\pi Z}{T}\right)\\ 1/2 & \cos\left(3\frac{2\pi}{T}\right) & \sin\left(3\frac{2\pi}{T}\right) & \cos\left(3\frac{2\pi 2}{T}\right) & \sin\left(3\frac{2\pi 2}{T}\right) & \dots & \cos\left(3\frac{2\pi Z}{T}\right) & \sin\left(3\frac{2\pi Z}{T}\right)\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ 1/2 & \cos\left(n\frac{2\pi}{T}\right) & \sin\left(n\frac{2\pi}{T}\right) & \cos\left(n\frac{2\pi 2}{T}\right) & \sin\left(n\frac{2\pi 2}{T}\right) & \dots & \cos\left(n\frac{2\pi Z}{T}\right) & \sin\left(n\frac{2\pi Z}{T}\right)\end{bmatrix} \dots(27)$$

$$\mathrm{C}=[a_0\ a_1\ \ b_1\ a_2\ b_2\ \dots a_n b_n]^{\mathrm{T}} \qquad \dots(28)$$

$$C\cong(P^TP)^{-1}P^T\varepsilon^{(0)} \qquad \dots(29)$$

Eq. (29) provides the correction for the Fourier series.

$$x_r^{(0)}(k)=x^{(0)}(k)-\varepsilon^{(0)}(k) \qquad \dots(30)$$

e. Model accuracy evaluation

Error is defined as the difference between the expected and actual values. This study compares the models it has investigated using the accuracy criteria defined as follows:

• ***The root mean square error (RMSE)*** is the average sum squared of the error's root.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{k=1}^{n} x(k) - x^{(0)}(k)^2}{n-1}} \qquad \ldots(31)$$

•***The mean absolute percentage error (MAPE)*** is the mean of the relative absolute value

$$\mathrm{MAPE} = \left(\frac{1}{n-1}\sum_{k=1}^{n}\left|\frac{x(k) - x^{(0)}(k)}{x^{(0)}(k)}\right|\right) \times 100\% \qquad \ldots(32)$$

•***Correct Direct Prediction (CDP)* criterion** gives the proportion of correctly predicted direct changes during the entire forecast period. This criterion differs from the previous two criteria because the ideal model is the one that gives a greater value to the criterion because both the MSE and MAPE criteria indicate a lower value, and this criterion is calculated as follows : (Geng, N., Zhang, Y., Sun, Y., Jiang, Y., & Chen, D. 2015) .[15]

$$\mathrm{CDP} = \frac{100\%}{n}\sum_{t=1}^{n} d_t \qquad \ldots(33)$$

Where:

$$d_t = \begin{cases} 1 & \text{when } (y_t - y_{t-1})(\hat{y}_t - \hat{y}_{t-1}) \geq 0 \\ 0 & \text{when } (y_t - y_{t-1})(\hat{y}_t - \hat{y}_{t-1}) < 0 \end{cases}$$

•***The Thiel Inequality Coefficient (TIC)* criterion** : This criterion was named by Henri Thiel. It is a relative criterion whose value is limited to zero and one, when its value closer to zero, then the more accurate the model .

The criterion TIC can be shown in Eqs.(34), as follows:

$$\mathrm{TIC} = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t - y_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t)^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t)^2}} \qquad \ldots(34)$$

3. The application

We will use the previously discussed grey models for modeling and prediction in this part. From 14/7/2023 to 14/3/2024, we will examine the daily time series data for gold prices expressed in US dollars per ounce.With 196 observations used for training and 49 for testing, the sample size consisted of 245 observations that represented gold prices for the aforementioned period. The daily gold prices during the aforementioned period are displayed in figure (1).

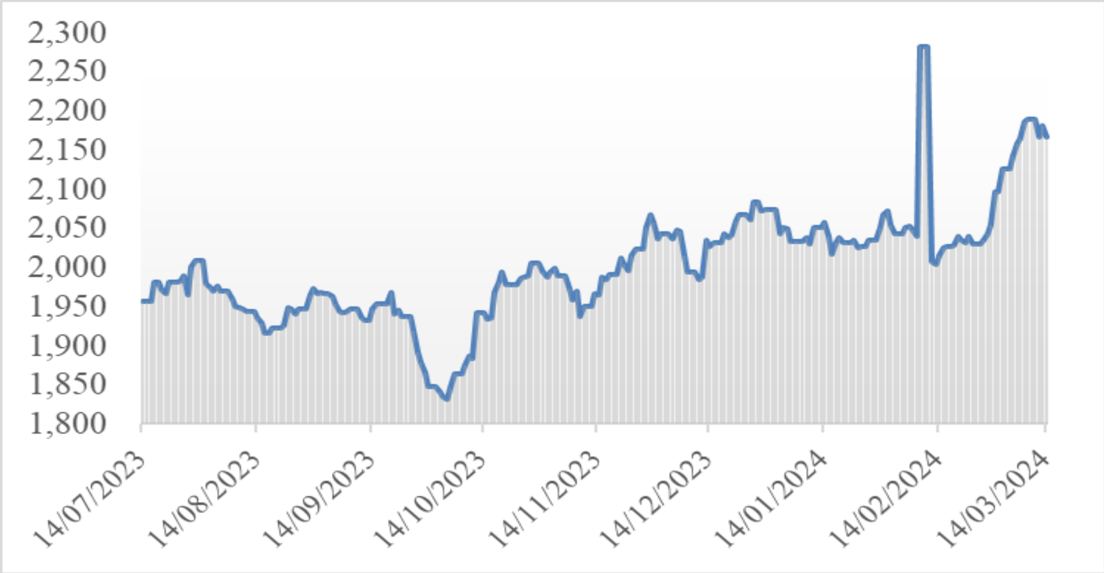

Figure 1: The price of gold each day for the period 14/7/2023 to 14/3/2024.

We notice from Figure (1) that The price of gold each day series is oscillating, and this is supported by the descriptive statistics related to it as shown in table 1:

Table 1: Descriptive statistics

Mean	Minimum	Maximum	Standard Deviation	Jarque-Bera Test	ADF	
					Level	**First Diff**
2001.527	1831.800	2281.400	73.62623	0.0000	0.1706	0.0000

ADF: Augmented Dickey–Fuller

we notice from Table (1) that the highest value of the data is (2281.400) and that the lowest value of the data is (1831.800). This indicates a relatively large ifference between the two values, and that the Jarque-Bera Test's p-value is equal to zero, less than the level of significance. Therefore, we reject the null hypothesis that imposes that the data is distributed Normal distribution, and we accept the lternative hypothesis that indicates that the data is not follow to a normal distribution, and the p-value (ADF) indicates the information is not stationary in mean and may be stationary at the first difference.

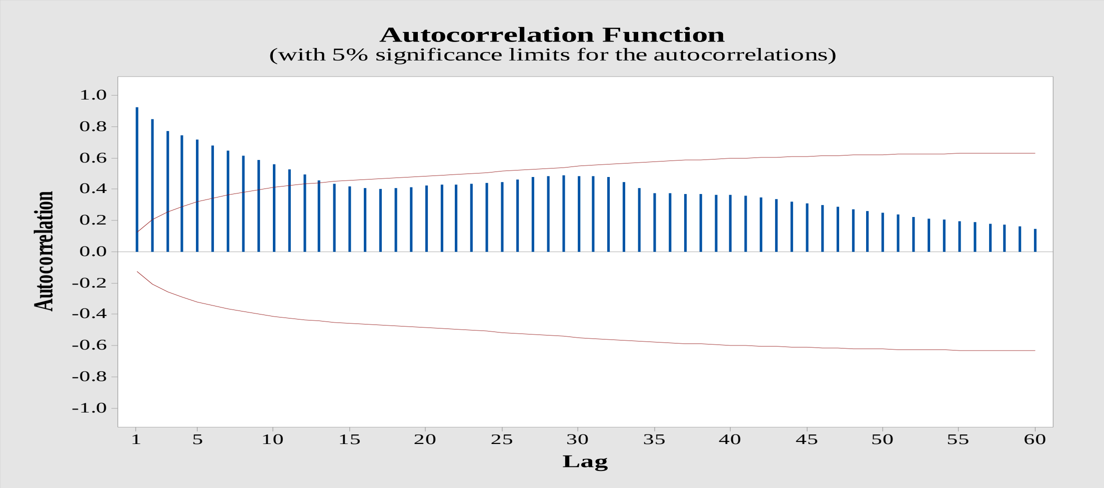

Figure 2: Autocorrelation coefficients gold price

To compare the six grey models (GM, FGM, VGM, FVGM, DGM, FDGM), respectively, four comparison criteria were relied upon, represented by he criteria for mean absolute percentage error (MAPE) and root mean square error (RMSE) , the correct direct prediction criterion (CDP), and the Thiele inequality coefficient (TIC) criterion which are shown in table (2) using equations.(31)-(34).

The model parameters were estimated as follows:

- Estimate the parameters of the gray model(GM) and replace them with equation (12) and in the following form to estimate the model:

GM: a=-0.0003811141, b=1909.324

$$x^{(0)}(k+1) = (1-e^{-0.0003811141})(x^{(0)}(1) - \frac{1909.324}{-0.0003811141})e^{+0.0003811141k} \quad , k = 1,2,\ldots n$$

- Estimate the parameters of the discrete gray model(DGM) and replace them with equation (15) and in the following form to estimate the model:

DGM: $\beta_1 = 1.000381, \beta_2 = 1909.703$

$$x^{\wedge(1)}(k+1) = 1.000381^k\, x^{(1)}(1) + \frac{1-1.000381^k}{1-1.000381} * 1909.703 \quad , k = 1,2, \ldots\ldots n-1$$

$x^{\wedge(0)}$ be the fitted and predicted the series.

$$x^{\wedge(0)}(k+1) = x^{\wedge(1)}(k+1) - x^{\wedge(1)}(k)$$

- Estimate the parameters of the Verhulst gray model(VGM) and replace them with equation (23) and in the following:

VGM: a=-0.02221609, b=-0.000009011

$$x^{(0)}(k) = \frac{-0.02221609x^{(0)}(1)\left(-0.02221609-bx^{(0)}(1)\right)}{-0.000009011x^{(0)}(1)+\left(-0.02221609+0.000009011x^{(0)}(1)\right)e-0.02221609^{(k-1)}} \times \frac{(1-e^{-0.02221609})e-0.02221609^{(k-2)}}{-0.000009011x^{(0)}(1)+(+0.000009011x^{(0)}(1))e-0.02221609^{(k-2)}} \quad , k = 1,2,\ldots n$$

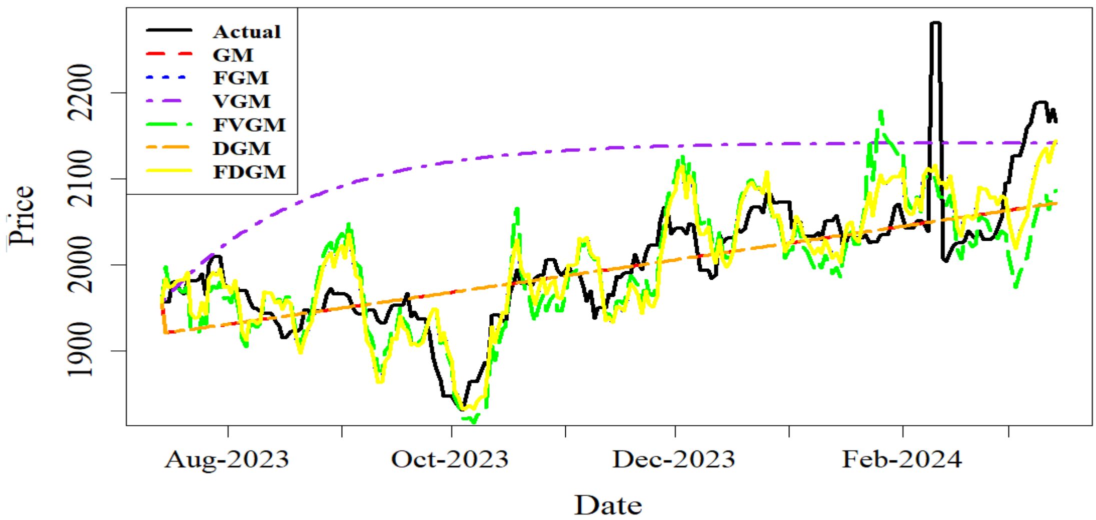

Figure 3: The behavior of the smoothness estimated with the grey models (GM, FGM, VGM, FVGM, DGM, F DGM), respectively, compared to the original series of gold price.

Table 2. Comparison criteria for estimated grey models to predict the daily gold prices.

Model	RMSE	MAPE	MAE	TIC
GM	78.28700	0.02433	52.57298	0.01888
FGM	70.71153	0.02758	58.38579	0.01694
VGM	92.49526	0.04041	83.67752	0.02186
FVGM	92.15844	0.03457	73.59843	0.02219
DGM	78.29505	0.02433	52.57341	0.01889
FDGM	70.71079	0.023757	52.37000	0.01693

Table 2: shows that the discrete grey model modified by Fourier was the best model for estimating the price of gold according to the mentioned comparison criteria.Gold prices were predection according to (FDGM) within the series from 14/7/2023 to 14/3/2024 and then forecasting gold prices from15/3/2024 to15/4/2024 As is clear in Figure 4. and Appendix (1):

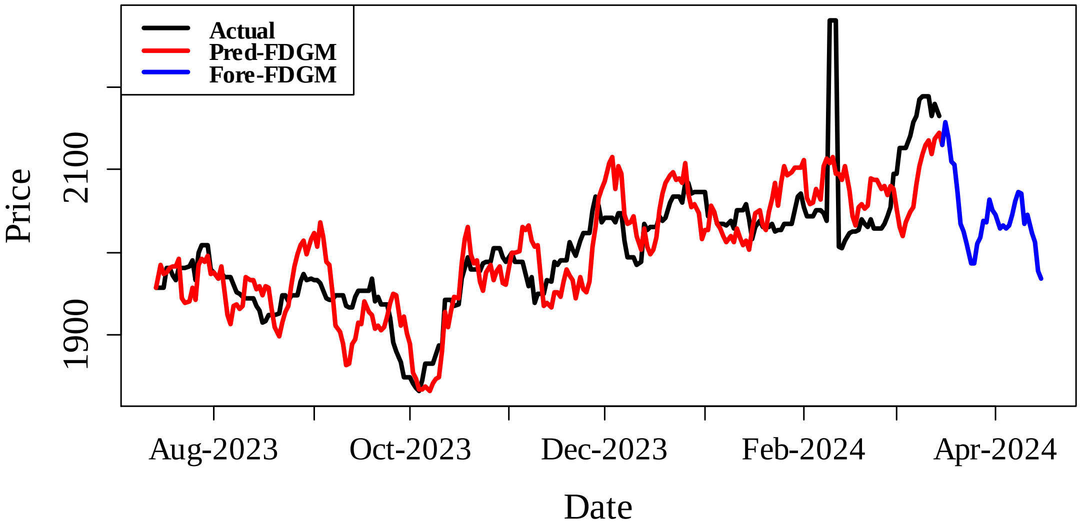

Figure 4: It shows the behavior of the original time series with all observations with predictive values.

Appendix (1) shows the real values, estimated values, and residuals (F DGM).

Table 5: The of Forecasting gold price(F DGM) from15/3/2024 to15/4/2024 In US dollars

4. Conclusions

Date	Gold Price	Date	Gold Price
15/3/2024	2129.434	31/3/2024	2050.099
16/3/2024	2157.887	1/4/2024	2045.451
17/3/2024	2140.227	2/4/2024	2029.295
18/3/2024	2109.698	3/4/2024	2032.638
19/3/2024	2105.775	4/4/2024	2028.173
20/3/2024	2073.201	5/4/2024	2032.081
21/3/2024	2033.571	6/4/2024	2044.611
22/3/2024	2025.973	7/4/2024	2061.823
23/3/2024	2010.023	8/4/2024	2072.145
24/3/2024	1985.764	9/4/2024	2070.910
25/3/2024	1987.023	10/4/2024	2034.155
26/3/2024	2010.133	11/4/2024	2044.467
27/3/2024	2017.364	12/4/2024	2024.044
28/3/2024	2037.514	13/4/2024	2012.334
29/3/2024	2035.795	14/4/2024	1977.477
30/3/2024	2063.520	15/4/2024	1968.815

Through the studied time series of gold prices, we notice

1. The daily gold price time series is a non-stationary.
2. The daily gold prices time series does not follow a normal distribution.
3. that the daily gold prices series is oscillating1 from 4/7/2023 to 14/3/2024.

4. The best model was used for estimating and predicting the daily gold prices according to comparison criteria is the Fourier-corrected discrete grey model(FDGM).
5. Gold prices were predection according to (FDGM) within the series from 14/7/2023 to 14/3/2024 and then forecasting gold prices from15/3/2024 to15/4/2024.

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5. Recommendations

Based on the conclusions reached, the following is recommended:

1. using the grey models in forecasting the ***gold prices.***
2. Use other methods to correct the residuals of grey models.

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Appendix (1) shows the real values, estimated values, and residuals (F DGM) for the period 14/7/2023 to 14/3/2024.

Date	Actual	Estimated	Residuals	Date	Actual	Estimated	Residuals
14/07/2023	1956.40	1956.40	0.00	30/11/2023	2036.30	2077.30	-41.00
15/07/2023	1956.40	1984.55	-28.15	01/12/2023	2042.20	2085.71	-43.51
16/07/2023	1956.40	1973.96	-17.56	02/12/2023	2042.20	2107.63	-65.43
17/07/2023	1980.80	1976.00	4.80	03/12/2023	2042.20	2114.92	-72.72
18/07/2023	1980.80	1981.66	-0.86	04/12/2023	2036.30	2076.97	-40.67
19/07/2023	1970.90	1982.52	-11.62	05/12/2023	2047.90	2103.93	-56.03
20/07/2023	1966.60	1981.90	-15.30	06/12/2023	2046.40	2094.68	-48.28
21/07/2023	1981.60	1991.76	-10.16	07/12/2023	2014.50	2046.13	-31.63
22/07/2023	1981.60	1944.65	36.95	08/12/2023	1993.70	2034.63	-40.93
23/07/2023	1981.60	1938.22	43.38	09/12/2023	1993.70	2036.27	-42.57
24/07/2023	1983.10	1939.36	43.74	10/12/2023	1993.70	2044.38	-50.68
25/07/2023	1989.70	1956.83	32.87	11/12/2023	1983.90	2019.11	-35.21
26/07/2023	1965.30	1942.91	22.39	12/12/2023	1988.10	2002.34	-14.24
27/07/2023	1999.90	1984.05	15.85	13/12/2023	2035.20	2029.21	5.99
28/07/2023	2009.20	1992.01	17.19	14/12/2023	2026.00	2006.38	19.62
29/07/2023	2009.20	1988.10	21.10	15/12/2023	2030.90	1997.75	33.15
30/07/2023	2009.20	1995.14	14.06	16/12/2023	2030.90	2002.62	28.28
31/07/2023	1978.80	1973.80	5.00	17/12/2023	2030.90	2017.75	13.15
01/08/2023	1975.00	1975.22	-0.22	18/12/2023	2042.60	2050.75	-8.15
02/08/2023	1968.80	1967.54	1.26	19/12/2023	2038.10	2071.30	-33.20
03/08/2023	1976.10	1982.58	-6.48	20/12/2023	2041.80	2084.34	-42.54
04/08/2023	1970.00	1953.47	16.53	21/12/2023	2059.60	2092.61	-33.01
05/08/2023	1970.00	1922.81	47.19	22/12/2023	2066.65	2096.55	-29.90
06/08/2023	1970.00	1912.07	57.93	23/12/2023	2066.65	2087.42	-20.77
07/08/2023	1959.90	1934.07	25.83	24/12/2023	2066.65	2089.27	-22.62
08/08/2023	1950.60	1936.32	14.28	25/12/2023	2060.40	2083.31	-22.91
09/08/2023	1948.90	1931.57	17.33	26/12/2023	2083.40	2108.38	-24.98
10/08/2023	1946.60	1934.63	11.97	27/12/2023	2083.50	2071.60	11.90
11/08/2023	1944.00	1969.05	-25.05	28/12/2023	2071.80	2054.41	17.39
12/08/2023	1944.00	1966.84	-22.84	29/12/2023	2072.90	2058.68	14.22
13/08/2023	1944.00	1966.70	-22.70	30/12/2023	2072.90	2047.01	25.89
14/08/2023	1935.20	1955.89	-20.69	31/12/2023	2072.90	2015.03	57.87
15/08/2023	1928.30	1958.28	-29.98	01/01/2024	2073.40	2026.16	47.24
16/08/2023	1915.20	1948.13	-32.93	02/01/2024	2042.80	2026.32	16.48
17/08/2023	1916.50	1959.12	-42.62	03/01/2024	2050.00	2056.43	-6.43
18/08/2023	1923.00	1956.11	-33.11	04/01/2024	2049.80	2048.59	1.21
19/08/2023	1923.00	1931.24	-8.24	05/01/2024	2033.50	2033.67	-0.17
20/08/2023	1923.00	1909.75	13.25	06/01/2024	2033.50	2028.54	4.96
21/08/2023	1926.00	1897.81	28.19	07/01/2024	2033.50	2019.39	14.11
22/08/2023	1948.10	1914.85	33.25	08/01/2024	2033.00	2011.94	21.06
23/08/2023	1947.10	1927.58	19.52	09/01/2024	2037.50	2020.28	17.22
24/08/2023	1939.90	1933.81	6.09	10/01/2024	2028.90	2013.08	15.82
25/08/2023	1946.80	1961.15	-14.35	11/01/2024	2051.35	2028.93	22.42
26/08/2023	1946.80	1981.90	-35.10	12/01/2024	2051.35	2018.51	32.84
27/08/2023	1946.80	1997.59	-50.79	13/01/2024	2051.35	2007.59	43.76
28/08/2023	1965.10	2007.56	-42.46	14/01/2024	2057.85	2014.50	43.35
29/08/2023	1973.00	2014.04	-41.04	15/01/2024	2039.70	2002.72	36.98
30/08/2023	1965.90	1997.41	-31.51	16/01/2024	2015.90	2026.08	-10.18
31/08/2023	1967.10	2015.49	-48.39	17/01/2024	2031.10	2046.38	-15.28
01/09/2023	1966.65	2022.56	-55.91	18/01/2024	2038.50	2050.14	-11.64
02/09/2023	1966.65	2007.44	-40.79	19/01/2024	2031.10	2031.95	-0.85
03/09/2023	1963.25	2035.85	-72.60	20/01/2024	2031.10	2026.31	4.79
04/09/2023	1952.60	2018.15	-65.55	21/01/2024	2031.10	2048.65	-17.55

05/09/2023	1944.20	1987.59	-43.39	22/01/2024	2035.20	2063.36	-28.16
06/09/2023	1942.50	1983.62	-41.12	23/01/2024	2025.40	2083.68	-58.28
07/09/2023	1942.70	1951.01	-8.31	24/01/2024	2027.10	2056.92	-29.82
08/09/2023	1947.20	1911.35	35.85	25/01/2024	2026.60	2084.88	-58.28
09/09/2023	1947.20	1903.71	43.49	26/01/2024	2034.90	2104.71	-69.81
10/09/2023	1947.20	1887.72	59.48	27/01/2024	2034.90	2094.15	-59.25
11/09/2023	1935.10	1863.42	71.68	28/01/2024	2034.90	2096.23	-61.33
12/09/2023	1932.50	1864.64	67.86	29/01/2024	2050.90	2101.92	-51.02
13/09/2023	1932.80	1887.72	45.08	30/01/2024	2067.40	2102.83	-35.43
14/09/2023	1946.20	1894.91	51.29	31/01/2024	2071.10	2102.24	-31.14
15/09/2023	1953.40	1915.02	38.38	01/02/2024	2053.70	2112.14	-58.44
16/09/2023	1953.40	1913.26	40.14	02/02/2024	2042.90	2065.06	-22.16
17/09/2023	1953.40	1940.95	12.45	03/02/2024	2042.90	2058.67	-15.77
18/09/2023	1953.70	1927.49	26.21	04/02/2024	2042.90	2059.85	-16.95
19/09/2023	1967.10	1922.81	44.29	05/02/2024	2051.40	2077.35	-25.95
20/09/2023	1939.60	1906.61	32.99	06/02/2024	2051.70	2063.47	-11.77
21/09/2023	1945.60	1909.92	35.68	07/02/2024	2047.90	2104.65	-56.75
22/09/2023	1936.60	1905.41	31.19	08/02/2024	2038.70	2112.65	-73.95
23/09/2023	1936.60	1909.28	27.32	09/02/2024	2281.40	2108.78	172.62
24/09/2023	1936.60	1921.77	14.83	10/02/2024	2281.40	2115.85	165.55
25/09/2023	1919.80	1938.95	-19.15	11/02/2024	2281.40	2094.55	186.85
26/09/2023	1890.90	1949.23	-58.33	12/02/2024	2007.20	2096.01	-88.81
27/09/2023	1878.60	1947.96	-69.36	13/02/2024	2004.30	2088.36	-84.06
28/09/2023	1866.10	1911.17	-45.07	14/02/2024	2014.90	2103.44	-88.54
29/09/2023	1847.20	1921.44	-74.24	15/02/2024	2024.10	2074.37	-50.27
30/09/2023	1847.20	1900.98	-53.78	16/02/2024	2025.75	2043.75	-18.00
01/10/2023	1847.20	1889.23	-42.03	17/02/2024	2025.75	2033.05	-7.30
02/10/2023	1841.50	1854.33	-12.83	18/02/2024	2027.50	2055.09	-27.59
03/10/2023	1834.80	1845.63	-10.83	19/02/2024	2039.80	2057.37	-17.57
04/10/2023	1831.80	1832.62	-0.82	20/02/2024	2034.30	2052.66	-18.36
05/10/2023	1845.20	1833.07	12.13	21/02/2024	2030.70	2055.75	-25.05
06/10/2023	1864.30	1837.30	27.00	22/02/2024	2039.40	2090.22	-50.82
07/10/2023	1864.30	1831.89	32.41	23/02/2024	2029.10	2088.04	-58.94
08/10/2023	1864.30	1841.12	23.18	24/02/2024	2029.10	2087.94	-58.84
09/10/2023	1875.30	1846.86	28.44	25/02/2024	2029.10	2077.17	-48.07
10/10/2023	1887.30	1847.69	39.61	26/02/2024	2034.40	2079.60	-45.20
11/10/2023	1883.00	1878.77	4.23	27/02/2024	2042.70	2069.48	-26.78
12/10/2023	1941.50	1928.13	13.37	28/02/2024	2054.70	2080.52	-25.82
13/10/2023	1941.50	1909.53	31.97	29/02/2024	2095.70	2077.54	18.16
14/10/2023	1941.50	1929.88	11.62	01/03/2024	2095.70	2052.70	43.00
15/10/2023	1934.30	1945.24	-10.94	02/03/2024	2126.30	2031.26	95.04
16/10/2023	1935.70	1943.55	-7.85	03/03/2024	2126.30	2019.35	106.95
17/10/2023	1968.30	1986.72	-18.42	04/03/2024	2126.30	2036.43	89.87
18/10/2023	1980.50	2015.96	-35.46	05/03/2024	2141.90	2049.20	92.70
19/10/2023	1994.40	2029.70	-35.30	06/03/2024	2158.20	2055.47	102.73
20/10/2023	1978.20	1996.78	-18.58	07/03/2024	2165.20	2082.84	82.36
21/10/2023	1978.20	1985.89	-7.69	08/03/2024	2185.50	2103.63	81.87
22/10/2023	1978.20	1989.30	-11.10	09/03/2024	2188.60	2119.36	69.24
23/10/2023	1976.80	1965.02	11.78	10/03/2024	2188.60	2129.37	59.23
24/10/2023	1985.50	1953.69	31.81	11/03/2024	2188.60	2135.89	52.71
25/10/2023	1987.90	1974.38	13.52	12/03/2024	2166.10	2119.30	46.80
26/10/2023	1989.00	1984.93	4.07	13/03/2024	2180.80	2137.41	43.39
27/10/2023	2005.60	1965.83	39.77	14/03/2024	2166.10	2144.52	21.58