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Estimating the survival function of three parameters Lindley distribution for patients with COVID-19 by using MLE and Bayesian estimates

تقدير دالة البقاء لتوزيع لندلي بثلاث معلمات للمرضى المصابين بفيروس COVID-19 باستخدام تقديرات MLEو Bayesian.

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Abstract:

The aim of this paper is to estimate the survival function for patients infected with the COVID-19 virus in Al Nasiriya city by using a three-parameter Lindley distribution (TPLD) and suggested three-parameter Lindley distribution (STPLD). We used two methods of estimation: Maximum likelihood estimation (MLE) and standard Bayesian estimation to estimate the parameters and the survival functions. We compared between the two methods of survival function estimates for the two distributions through some criterions of accuracy such as IMSE, -2lnL, AIC, CAIC, and BIC. The simulation results shows that the standard Bayesian method was superior to the sample size (10) and the maximum likelihood method was superior to the sample size (50,100).The application shows that the survival function estimation of the (STPLD) is the best than (TPLD).

Keywords: three-parameter Lindley distribution, Suggested three-parameter Lindley distribution, survival functions, MLE, the standard Bayesian.

المستخلص
الهدف من هذه الدراسة هو تقدير دالة البقاء على قيد الحياة للمرضى المصابين بفيروس COVID-19 في مدينة الناصرية عن طريق استخدام توزيع لندلي بثلاث معلمات (TPLD) وصيغة مقترحة جديدة لتوزيع لندلي بثلاث معلمات . (STPLD) نحن نستخدم طريقتين للتقدير: طريقة الامكان الاعظم (MLE) وطريقة بيز القياسية لتقدير المعلمات ودوال البقاء. نحن ندرس بعض خصائص التوزيع العمري لـ TPLD و .STPLD وأخيراً، قمنا بمقارنة طريقتي التقدير لدالة البقاء على قيد الحياة للتوزيعين (TPLD) و (STPLD) من خلال بعض معايير الدقة مثل IMSE, -2lnL, AIC, CAIC, BIC. وكانت نتيجة المحاكاة هي أن طريقة بيز القياسية كانت أفضل عند حجم العينة (10) لأنها حققت أدنى المعايير وأن طريقة الامكان الاعظم كانت أفضل عند حجم العينة (50,100) . يوضح الجانب التطبيقي أن تقدير دالة البقاء لـ (STPLD) هو الأفضل من.(TPLD)

الكلمات المفتاحية : توزيع ليندلي ذو ثلاثة معلمات، توزيع ليندلي ثلاثي المعلمات المقترح، دوال البقاء، تقدير ,MLH التقدير بطريقة بيز القياسية.

1. Introduction:

People have been interested in finding ways to survive more time. In recent this curiosity has led to a greater focus on the research of the survival function, as this information is crucial for determining how long a person may survive an illness by studying the survival time from the initial stage of disease infection to the final stages of recovery or death. The survival function is utilized in research pertaining to the patient's health, since human health is the most significant topic of study in statistical analysis applications.

The probability distributions used in reliability and survival functions in previous studies did not achieve optimality in estimating the survival function, this led the researcher to look into possible distributions or develop existing probability distributions to find a probability distribution that would better estimate the survival function. This paper's aim is to estimate the survival function for COVID-19-infected people using the Lindley distribution with three parameters (TPLD) and suggested a new formula (STPLD) by assuming new mixing weights of the mixing function of TPLD.

A single parameter distribution was first presented by Lindley in 1958[7], Ghitany et al. (2008) [4] study a few features of classical statistics over reliability assessment using maximum likelihood and a Bayesian method, Krishna and Kumar (2011) [6] examined the Lindley distribution under progressive type II censoring, however they did not evaluate it over the entire data set using different loss functions. A two-parameter weighted Lindley distribution was presented by Ghitany et al. (2011) [5], who also noted that modeling biological data from mortality studies is a particularly good use for the Lindley distribution. And two-parameter Lindley distribution is the one by Shanker and Mishra (2013a, 2013b) [10, 11] presented in 2013 by Shanker, Sharma, and Shanker. Abd El-Monsef (2015) [3] presents the Lindley distribution with a location parameter as a three-parameter distribution. The inclusion of the location option provides greater flexibility in corresponding real-life data with other current distributions. The text discusses many statistical and reliability features. A simulation research was conducted to analyze the mean squared error (MSE), bias, and the coverage probability of the parameters. Rama Shanker & et al. (2017) [9] A Three-Parameter Lindley Distribution. The introduction of the three-parameter Lindley distribution (ATPLD) provides a method for modeling lifetime data. They studied the statistical properties of distribution. The process of estimating the parameters of a statistical model have been explored using maximum likelihood estimation. The study discovered that the goodness of fit of ATPLD is superior to that of TPGLD. This suggests that ATPLD has a significant lifetime distribution for modeling lifetime data compared to TPGLD., Al-Abadi Karma N. Hussein studied the estimate of the Bayesian survival function of the three-parameter Lindley distribution and its practical application (2021) [1] .the results reached by the researcher that the use of the Lindley distribution function with three parameters showed more accurate results than the use of the Lindley function with two parameters as well as the estimation methods used in the applied side. Mathil K. Thamer and Raoudha Zine (2023) [13] both study the Lindley distribution's three-parameter features. The searchers estimated the three parameters by using the MPS method. and the consistency of the three estimators was shown through a simulation exercise, as shown by the decrease in mean squared errors (MSEs) with increasing sample size.

We will prove some properties of (STPLD) throughout the life time distribution , and we will compare between two methods of estimation MLE and Bayes estimation to estimate the parameters and the survival functions of (TPLD) and (STPLD) by applying some of specific criterions errors across different distributions to determine which is best.

2. Three Parameter Lindley Distribution (TPLD)[8][13] [11]

The Lindley distribution (TPLD) with three parameters was first proposed by Shanker et al. (2017). The (TPLD) is a probability density function which can be defined simply as the mixture of exponential distribution with parameter (η), $f_1(t)$ and gamma distribution with parameters (2, η), $f_2(t)$ with a mixture proportion:

$$U = \frac{\eta\zeta}{\eta\zeta+\beta} \qquad (1)$$

And the following mixing formula:

$$f(t;\eta,\zeta) = U * f1(t) + (1-U) * f2(t) \qquad (2)$$

And defining as follows:

$$f(t;\eta,\zeta,\beta) = \left(\frac{\eta^2}{\eta\zeta+\beta}\right)(\zeta+\beta t)e^{-\eta t}\ ;\ t,\eta,\zeta > 0\ ;\ \eta\zeta+\beta > 0 \qquad (3)$$

Whereas,

η: rate parameter

ζ, β: shape parameters

The corresponding cumulative distribution function is defined as follows:

$$F(t;\ \eta,\zeta,\beta) = 1 - \left[\frac{\eta\zeta+\beta+\eta\beta t}{(\eta\zeta+\beta)}\right]e^{-\eta t}\ ;\ t,\eta,\beta > 0\ ,\ \ \eta\zeta+\beta > 0 \qquad (4)$$

So, the survival function S (t) can be been given by:

$$S(t;\ \eta,\zeta,\beta) = \left[\frac{\eta\zeta+\beta+\eta\zeta\beta t}{(\eta\zeta+\beta)}\right];\ t,\eta,\beta\ >\ 0\ ,\ \ \eta\zeta+\beta > 0 \qquad (5)$$

And we can get the hazard function of (TPLD) as follows:

$$h(t;\ \eta,\zeta,\beta)\ =\ \frac{\eta^2(\zeta+\beta t)}{\eta\zeta+\beta+\eta\beta t}\ \ ;\ t,\eta,\beta\ >\ 0\ ,\ \ \eta\zeta+\beta > 0 \qquad (6)$$

3. Modified formula of Three-Parameter Lindley Distribution

We suggest a new formula for the (TPLD) based on the following assumptions:

a) Assume f_1 (t) as the exponential distribution: $f_1\ (t,\eta) = \ \eta\ e^{-\eta t}$, t > 0 (7)

And f_2 (t) the gamma distribution: $f_2(t,2,\eta)\ =\ \eta^2 t\ e^{-\eta t}$, t > 0 (8)

b) By using the mixing proportion in (1) And so it is: $1 - U = \frac{\beta}{\eta\zeta+\beta}$ (9)

c) Assume the mixing equation is : $f(t;\ \eta\ ,\zeta,\beta) = (1-U)f_1(t) + U\ f_2(t)$ (10)

The new formula can be denoted as the suggested three-parameter Lindley distribution (STPLD) and defined as follows:

$$f_{(\,STPLD)}(t,\eta,\zeta,\beta) = \frac{\eta}{(\eta\zeta+\beta)}(\beta +\ \eta^2\zeta t\ )\ e^{-\ \eta t}\ ;\ t,\eta,\zeta,\beta > 0 \qquad (11)$$

ζ,β : Shape parameters

η : Rate parameter

We can prove that: $\int_0^\infty \frac{\eta}{(\eta\zeta+\beta)}(\beta +\ \eta^2\zeta t\ )\ e^{-\ \eta t}\ \ dt = 1$, that means (STPLD) is a p.d.f

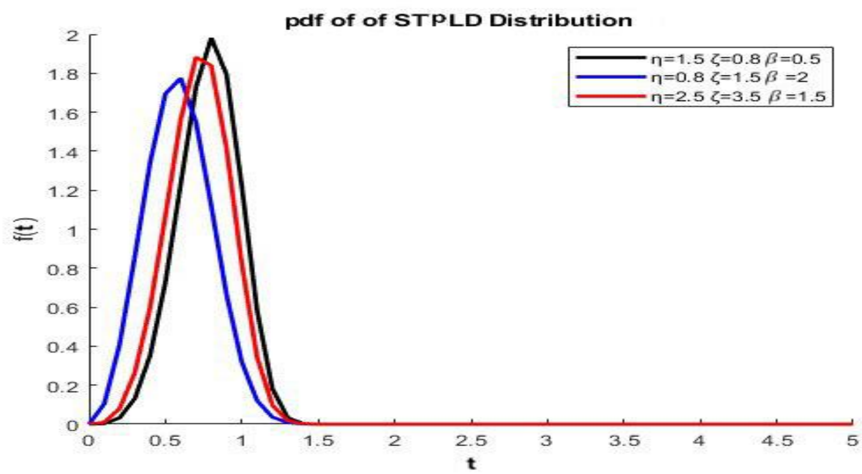

Figure (1) the behavior of the curve representing the probability density function of the distribution (STPLD)

The corresponding cumulative distribution function is**:**

$$F_{(t)} = 1 - \left[\frac{\beta+\ \eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right]e^{-\eta t}\ \ \ ,\ t \geq 0 \qquad (12)$$

4. The properties of (STPLD)

The survival function S(t) is provided as follows:

$$S(t,\eta,\zeta,\theta) = P(T > t) = \left[\frac{\beta + \eta^2\zeta t + \eta\zeta}{\eta\zeta + \beta}\right] e^{-\eta t};\ t,\eta,\beta > 0,\ \ \eta\zeta + \beta > 0 \qquad (13)$$

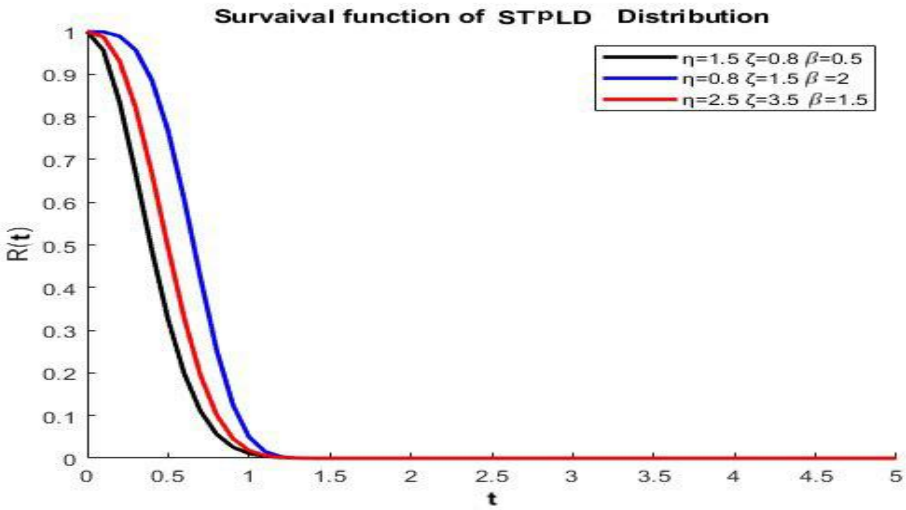

Figure (1) the behavior of the curve representing the survival function of the distribution (STPLD)

And the hazard function can be defined as follows:

$$h(t,\eta,\zeta,\beta) = \frac{\eta(\beta + \eta^2\zeta t)}{\beta + \eta\zeta(\eta t + 1)} \quad ; t,\zeta,\eta,\beta > 0 \qquad (14)$$

The mean and variance of STPLD are:

$$mean(t) = E(T) = \frac{\beta + 2\eta\zeta}{\eta(\eta\zeta + \beta)} \qquad (15)$$

$$var(t) = E(T^2) - (E(T))^2 = \frac{2\eta^2\zeta^2 + 4\eta\zeta\beta + \beta^2}{\eta^2(\eta\zeta + \beta)^2} \qquad (16)$$

The non-central moments about zero μ_r' of (STPLD) are:

$$E(T^r) = \mu_r' = \frac{r!\,[\beta + \eta\zeta(r+1)]}{\eta^r(\eta\zeta + \beta)} \quad ; \quad r=1,2,3,\ldots \qquad (17)$$

The four non-central moments about zero are:

$$\mu_1' = \frac{\beta + 2\eta\zeta}{\eta(\eta\zeta + \beta)} \qquad (18)$$

$$\mu_2' = \frac{2(\beta + 3\eta\zeta)}{\eta^2(\eta\zeta + \beta)} \qquad (19)$$

$$\mu_3' = \frac{6(\beta + 4\eta\zeta)}{\eta^3(\eta\zeta + \beta)} \qquad (20)$$

$$\mu_4' = \frac{24(\beta + 5\eta\zeta)}{\eta^4(\eta\zeta + \beta)} \qquad (21)$$

The central moments about the mean are:

$$E(t-\mu)^r = \frac{\eta}{\eta\zeta\beta + 1}\left[\sum_{i=0}^{r} \binom{r}{i}\left(-\frac{(\beta + 2\eta\zeta)^i (r-i)!}{\eta^{r+1}(\eta\zeta + \beta)^i} + \zeta\beta \sum_{i=0}^{r} \binom{r}{i}\left(-\frac{(\beta + 2\eta\zeta)^i (r-i+1)!}{\eta(\eta\zeta + \beta)^i}\right]\right.\right. \qquad (22)$$

The median and the mode are defined as follows:

$$Median = \int_0^{t_{med}} \frac{\eta}{(\eta\zeta + \beta)}(\beta + \eta^2\zeta t)\, e^{-\eta t} dt = 0.5$$

$$= \left[\frac{\beta + \eta^2\zeta t_{med} + \eta\zeta}{\eta\zeta + \beta}\right] e^{-\eta t_{med}} = 0.5$$

$$t_{median} = \left[\frac{(\eta\zeta + \beta)\ln(0.5)}{\eta(\beta + \eta^2\zeta t + \eta\zeta)}\right] \qquad (23)$$

$$Mo = Argmax[f(t)]$$

$$f(t) = \frac{\overset{t}{\eta}}{\eta\zeta + \beta}(\beta + \eta^2\zeta t)\, e^{-\eta t}$$

$$f'(t) = \frac{\eta}{\eta\zeta+\beta}\left[-\eta(\beta + \eta^2\zeta t\ )e^{-\eta t} + \eta^2\zeta\beta e^{-\eta t}\right] = 0$$

$$Mo = \begin{cases} \frac{\eta\zeta-\beta}{\eta^2\zeta}, |\eta\zeta| > \beta \\ zero, \quad otherwise \end{cases} \tag{24}$$

The standard deviation and coefficient of variation can be given as follows:

$$\sigma = \sqrt{\frac{2\eta^2\zeta^2+4\eta\zeta\beta+\beta^2}{\eta^2(\eta\zeta+\beta)^2}} \tag{25}$$

$$C.V = \frac{\sigma}{mean} = \sqrt{\frac{2\eta^2\zeta^2 + 4\eta\zeta\beta + \beta^2}{(\beta + 2\eta\zeta)^2}} \tag{26}$$

The Coefficient of Skewens (C.S) is defined as:

$$C.S = \frac{\mu_3'}{\mu_2'^{\,2}}$$

$$C.S = \frac{\eta(6\beta + 24\eta\zeta)(\eta\zeta + \beta)}{4(\beta + 3\eta\zeta)^2} \tag{27}$$

The Moment Generating Function $M_x(t)$

$$M_x(t) = Ee^{tx} = \int_0^\infty e^{tx}\frac{\eta}{\eta\zeta+\beta}(\beta + \eta^2\zeta x\ )\, e^{-\eta x}\, dx$$

$$= \frac{\eta}{\eta\zeta+\beta}\int_0^\infty (\beta + \eta^2\zeta x\ ) e^{-x(\eta-t)}\, dx$$

$$= \frac{\eta}{\eta\zeta+\beta}\left[\beta\int_0^\infty e^{-x(\eta-t)}\, dx + \eta^2\zeta\int_0^\infty x\, e^{-x(\eta-t)}\, dx\right]$$

$$= \frac{\eta}{\eta\zeta+\beta}\left[\frac{\beta}{(\eta-t)} + \frac{\eta^2\zeta}{(\eta-t)^2}\right]$$

Let $\frac{\beta}{(\eta-t)} = \frac{\beta}{\eta(1-\frac{t}{\eta})}$

Define: $Z = \frac{t}{\eta} \quad \Rightarrow \quad \frac{\beta}{(\eta-t)} = \frac{\beta}{\eta}\left(\frac{1}{1-Z}\right)$

$$\frac{\beta}{(\eta-t)} = \frac{\beta}{\eta}\sum_{k=0}^{\infty} Z^k = \frac{\beta}{\eta}\sum_{k=0}^{\infty}\left(\frac{t}{\eta}\right)^k$$

$$\frac{\eta^2\zeta}{(\eta-t)^2} = \eta^2\zeta\left[\frac{1}{[\eta(1-\frac{t}{\eta})]^2}\right] = \zeta\left[\frac{1}{(1-Z)^2}\right]$$

$$\frac{\eta^2\zeta}{(\eta-t)^2} = \zeta\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\eta}\right)^k$$

$$M_x(t) = \frac{\eta}{\eta\zeta+\beta}\left[\frac{\beta}{\eta}\sum_{k=0}^{\infty}\left(\frac{t}{\eta}\right)^k + \zeta\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\eta}\right)^k\right] \tag{28}$$

Distribution of Order Statistics for (STPLD)

Let $t_1, t_2, t_3, \ldots . t_n$ Obtain a random sample of size n from the (STPLD). Let $T_1 < T_2 < T_3 < \cdots < T_n$ order statistics. The (p.d.f.) of the $k^{\ th}$ order statistic, say T_k=t_k .Thus, the (p.d.f.) of $k^{\ th}$ order statistics for STPLD is defined as:

$$f_k(t_k) = \frac{n!\frac{\eta}{\eta\zeta+\beta}(\beta+\eta^2\zeta t\ )\ e^{-\eta t}}{(k-1)!(n-k)!} \sum_{s=0}^{n-k}\binom{n-k}{s}(-1)^s * [1-\left[\frac{\beta+\eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right]\ e^{-\eta t}\ ]^{k+s-1}$$

$$= \frac{n!\,\eta(\beta+\eta^2\zeta t\ )\ e^{-\eta x}}{(\eta\zeta+\beta)(k-1)!)(n-k)!} \sum_{s=0}^{n-k}\binom{n-k}{s}(-1)^s * [1-\left[\frac{\beta+\eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right]\ e^{-\eta t}\ ]^{k+s-1} \qquad (29)$$

And the (c.d.f) is:

$$F_k(t_k)\ = \sum_{l=k}^{n}\sum_{s=0}^{n-l}\binom{n}{l}\binom{n-l}{s}(-1)^s * [1-\left[\frac{\beta+\eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right]\ e^{-\eta t}]^{s+l} \qquad (30)$$

5. The Maximum Likelihood Estimation (M.L.E.)[2][10]

Let $t_1, t_2, t_3, \ldots . t_n$ form a random sample of size-n drawn from (STPLD), then the likelihood function has been given by:

$$L = \left(\frac{\eta}{\eta\zeta+\beta}\right)^n \prod_{i=1}^{n}(\beta+\ \eta^2\zeta t_i\ )\ e^{-n\eta\bar{t}}$$

$$\frac{\partial LnL}{\partial\hat{\eta}} = \frac{\mathrm{n}}{\hat{\eta}} - \frac{\mathrm{n}\hat{\zeta}}{(\hat{\eta}\hat{\zeta}+\hat{\beta})} + \sum_{i=1}^{n}[\frac{2\,\hat{\eta}\hat{\zeta}t_i}{\hat{\beta}+\widehat{\eta^2}\hat{\zeta}ti}] - n\bar{t} = 0 \qquad (31)$$

$$\frac{\partial LnL}{\partial\hat{\zeta}} = \sum_{i=1}^{n}[\frac{\hat{\eta}^2ti}{\hat{\beta}+\hat{\eta}^2\hat{\zeta}ti}] - \frac{n\,\hat{\eta}}{\hat{\eta}\hat{\zeta}+\hat{\beta}} = 0 \qquad (32)$$

$$\frac{\partial LnL}{\partial\hat{\beta}} \ = \sum_{i=1}^{n}[\frac{1}{\hat{\beta}+\hat{\eta}^2\hat{\zeta}ti}] - \frac{n}{\hat{\eta}\hat{\zeta}+\hat{\beta}} = 0 \qquad (33)$$

The initial derivatives of the likelihood function are in the context of η , ζ and β are non-linear equations that don't seem to have a direct solution. Thus; the following equations will be solved using the Fisher's scoring method:

$$\frac{\partial^2 LnL}{\partial\eta^2} = \frac{-\mathrm{n}}{\eta^2} + \frac{\mathrm{n}\zeta^2}{(\eta\zeta+\beta)^2} + \sum_{i=1}^{n}\frac{2\zeta\beta ti-2\,\eta^2\zeta^2\,ti^2}{(\beta+\eta^2\zeta t)^2}$$

$$\frac{\partial^2 LnL}{\partial\eta\partial\zeta} = \frac{-\mathrm{n}\beta}{(\eta\zeta+\beta)^2} + \sum_{i=1}^{n}\frac{2\eta\beta ti}{(\beta+\eta^2\zeta t)^2} = \frac{\partial^2 LnL}{\partial\zeta\partial\eta}$$

$$\frac{\partial^2 LnL}{\partial\eta\partial\beta} = \frac{\mathrm{n}\zeta}{(\eta\zeta+\beta)^2} - \sum_{i=1}^{n}\frac{2\eta\zeta ti}{(\beta+\eta^2\zeta t)^2} = \frac{\partial^2 LnL}{\partial\theta\partial\eta}$$

$$\frac{\partial^2 LnL}{\partial\zeta^2} = \frac{n\,\eta^2}{(\eta\zeta+\beta)^2} - \sum_{i=1}^{n}\frac{\eta^4\,ti^2}{(\beta+\eta^2\zeta t)^2}$$

$$\frac{\partial^2 LnL}{\partial\zeta\partial\beta} = \frac{n\,\eta}{(\eta\zeta+\beta)^2} - \sum_{i=1}^{n}\frac{\eta^2 ti}{(\beta+\eta^2\zeta t)^2} = \frac{\partial^2 LnL}{\partial\beta\partial\zeta}$$

$$\frac{\partial^2 LnL}{\partial\beta^2} = \frac{\mathrm{n}}{(\eta\zeta+\beta)^2} - \sum_{i=1}^{n}\frac{1}{(\beta+\eta^2\zeta t)^2}$$

The equations that follow can be solved to get the MLEs $\hat{\eta}$, $\hat{\zeta}$ and $\hat{\beta}$ of (STPLD):

$$\begin{bmatrix} \frac{\partial^2 LnL}{\partial\eta^2} & \frac{\partial^2 LnL}{\partial\eta\partial\zeta} & \frac{\partial^2 LnL}{\partial\eta\partial\beta} \\ \frac{\partial^2 LnL}{\partial\zeta\partial\eta} & \frac{\partial^2 LnL}{\partial\zeta^2} & \frac{\partial^2 LnL}{\partial\zeta\partial\beta} \\ \frac{\partial^2 LnL}{\partial\beta\partial\eta} & \frac{\partial^2 LnL}{\partial\beta\partial\zeta} & \frac{\partial^2 LnL}{\partial\beta^2} \end{bmatrix}_{\substack{\hat{\eta}=\eta_0 \\ \hat{\zeta}=\zeta_0 \\ \hat{\beta}=\beta_0}} \begin{bmatrix} \hat{\eta}-\eta_0 \\ \hat{\zeta}-\zeta_0 \\ \hat{\beta}-\beta_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial LnL}{\partial\eta} \\ \frac{\partial LnL}{\partial\zeta} \\ \frac{\partial LnL}{\partial\beta} \end{bmatrix}_{\substack{\hat{\eta}=\eta_0 \\ \hat{\zeta}=\zeta_0 \\ \hat{\beta}=\beta_0}}$$

Where, η_0, ζ_0 and β_0 are the initial values of η, ζ and β respectively. Iteratively solving these equations gives values of $\hat{\eta}$, $\hat{\zeta}$ and $\hat{\beta}$ that are properly approximately.

6. The Standard Bayesian Estimations [14]

The Bayes theorem may describe as the likelihood function of the observations based on the sample's current knowledge. We obtain the posterior probability distribution by integrating the density function for the parameters with the greatest possibility function for the current observation.

We employ a loss function in the Bayes method, which allows us to measure the loss that results from making decisions based on the value of (θ), while the resolution to choose depends on (θ), That is, there is a difference between the parameter and its estimate.

Now we need to give the initial distributions of the information to be estimated (η ,ζ and θ). According to the information available to the researcher about the initial distributions of the information, assume that the initial distributions of that information will be as follows:

$\eta \sim Gamma\ (a_1, b_1)$, rate parameter

$\zeta \sim Gamma\ (a_2, b_2)$, shape parameter

$\beta \sim Beta\ (c, d)$, shape parameter

The following defines the parameter's priority distribution functions:

$$\pi_1(\eta) \propto \frac{b_1^{a_1}}{\Gamma(a_1)}\ \eta^{a_1-1}\ e^{-b_1\eta}\quad ;\ \eta > 0 \qquad (34)$$

$$\pi_2(\zeta) \propto \frac{b_2^{a_2}}{\Gamma(a_2)}\ \zeta^{a_2-1}\ e^{-b_2\zeta}\quad ;\ \zeta > 0 \qquad (35)$$

$$\pi_3(\beta) \propto \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\ \beta^{\,c-1}\,(1-\beta)^{d-1}\quad ;\ 0 < \beta < 1 \qquad (36)$$

Equations (34), (35) and (36) shows the prior distribution of the three parameters (η, ζ and β). the parameter η has a gamma distribution with the hyper-parameters a_1 and b_1 ,the parameter ζ has a gamma distribution with the hyper-parameters a_2 and b_2, and the parameter β has a beta distribution with the hyper-parameters c and d, based on past experiences of the researchers .the joint prior distribution of η, ζ, and β as follows:

$$\pi_{(\eta,\zeta,\beta)} \propto \frac{b_1^{a_1} b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\,\frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\ \eta^{a_1-1}\ \zeta^{a_2-1}\ \beta^{\,c-1}(1-\beta)^{d-1} e^{-(b_1\eta+b_2\zeta)} \qquad (37)$$

$$\text{Log}\ \pi_{(\eta,\zeta,\beta)} = log\left(\frac{b_1^{a_1} b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\,\frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\ \eta^{a_1-1}\ \zeta^{a_2-1}\ \beta^{\,c-1}(1-\beta)^{d-1} e^{-(b_1\eta+b_2\zeta)}\right)$$

The conditional probability function of $t_1, t_2, \dots\dots, t_n$ can be given as follows:

$$L(t_1, t_2, \dots\dots, t_n|\eta) = \prod_{i=1}^{n} f(x,\ \eta, \zeta, \beta)$$

$$L(t_1, t_2, \dots\dots, t_n|\eta) = \left(\frac{\eta}{\eta\zeta+\beta}\right)^n \prod_{i=1}^{n}(\beta + \eta^2\zeta t_i)\ e^{-n\eta\bar{t}} \qquad (38)$$

The joint posterior distribution at the observed data and for parametersη, ζ andβ is as follows:

$$h(\eta, \zeta, \beta\,|t_1, t_2, \dots\dots, t_n) = \frac{L(t_1,t_2,\dots\dots,t_n|\eta)\pi_{(\eta,\zeta,\beta)}}{\iiint_{\eta,\zeta,\beta} L(t_1,t_2,\dots\dots,t_n|\eta)\pi_{(\eta,\zeta,\beta)}d\eta d\zeta d\beta}$$

$$= \frac{\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \prod_{i=1}^{n}(\beta+\eta^2\zeta t_i)\,e^{-n\eta\bar{t}}\,\frac{b_1^{a_1}b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\eta^{a_1-1}\ \zeta^{a_2-1}\ \beta^{\,c-1}(1-\beta)^{d-1}e^{-(b_1\eta+b_2\zeta)}}{\iiint_{\eta,\zeta,\beta}\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \prod_{i=1}^{n}(\beta+\eta^2\zeta t_i)\,e^{-n\eta\bar{t}}\frac{b_1^{a_1}b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\eta^{a_1-1}\ \zeta^{a_2-1}\beta^{\,c-1}(1-\beta)^{d-1}e^{-(b_1\eta+b_2\zeta)}d\eta d\zeta d\beta}$$

$$\therefore h(\eta,\zeta,\beta\,|t_1,t_2\ldots,t_n)=\frac{\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\,\beta^{\,c-1}(1-\beta\,)^{d-1}\prod_{i=1}^{n}(\beta+\eta^2\zeta t_i\,)\,e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}}{\iiint_{\eta,\zeta,\beta}\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\,\beta^{\,c-1}(1-\beta\,)^{d-1}\prod_{i=1}^{n}(\beta+\eta^2\zeta t_i\,)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\,d\eta d\zeta d\beta} \quad (39)$$

The Bayes estimate for the distribution's parameters (STPLD) can be obtained by using the squared loss function:

$$\hat{\theta}_{SBayes}(t_1,t_2,\ldots\ldots,t_n)=E(\eta,\zeta,\beta\,\,|t_1,t_2,\ldots\ldots,t_n)$$

$$=\frac{\iiint_{\eta,\zeta,\beta}^{\infty}(\underline{\hat{\theta}}-\underline{\theta})^2\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\,\beta^{\,c-1}(1-\beta\,)^{d-1}\prod_{i=1}^{n}(\beta+\eta^2\zeta t_i\,)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\,d\eta d\zeta d\beta}{\iiint_{\eta,\zeta,\beta}^{\infty}\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\,\beta^{\,c-1}(1-\beta\,)^{d-1}\prod_{i=1}^{n}(\beta+\eta^2\zeta t_i\,)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\,d\eta d\zeta d\beta} \quad (40)$$

Thus, the Bayes estimator of the survival function of the (STPLD) distribution is:

$$\hat{S}_{SBayes}=\;E(S|t_1,t_2,\ldots\ldots,t_n)$$

$$=\frac{\iiint_{\eta,\zeta,\beta}^{\infty}\left(\frac{\beta+\eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right)\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\,\beta^{\,c-1}(1-\beta\,)^{d-1}\prod_{i=1}^{n}(\beta+\eta^2\zeta t_i\,)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\,d\eta d\zeta d\beta}{\iiint_{\eta,\zeta,\beta}^{\infty}\left(\frac{\eta}{\eta\zeta+\beta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\,\beta^{\,c-1}(1-\beta\,)^{d-1}\prod_{i=1}^{n}(\beta+\eta^2\zeta t_i\,)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\,d\eta d\zeta d\beta} \quad (41)$$

Equation (41) represents the STPLD survival function and shows that it lacks a closed formula and is not conceptually complex. The estimation of the dependability function (survival) requires the numerical computation of these complex integrals using the Lindley Approximation.

7. Simulation

Simulation is defined as a technique of replicating and mimicking real reality by obtaining a precise representation of any system or model without using real data. In real life, we frequently come across processes that are difficult to comprehend, particularly in some statistical and technical issues or theories that are difficult to solve. Since it is very hard to use mathematical proofs in a logical way, these theories can be applied and gets the best answers by taking several random samples from such societies. In order to do this, it is necessary to explain the processes by using special models that are close to the true photographs by simulating the model; we may acquire knowledge of the original process or true reality using the Monte Carlo simulation.

Step one: theoretical value determination

Determining the theoretical values of the parameters of the distributions under study:

In this step we choose the theoretical value $(\boldsymbol{\eta}=\mathbf{1.5},\boldsymbol{\zeta}=\mathbf{0.8},\beta=\mathbf{0.5})$ of the parameters for the studied distributions.

Determining the theoretical values for the sample size (n = 10, 50, 100).

Determining the repeating of the experiment 1000 times.

Step two: The generating process

At this step, we will generate observations from distributions (**TPLD**) and (**STPLD**) as follows:

Generate three-parameter Lindley distribution data (TPLD) [12]

We generate the data that follows as three-parameter Lindley distribution defined in equation 3 by using Lambert W function as follows**:**

$$t=-\frac{\boldsymbol{\zeta}}{\boldsymbol{\theta}}-\frac{1}{\boldsymbol{\eta}}-\frac{1}{\boldsymbol{\eta}}W_{-1}\left(-\frac{(\boldsymbol{\eta\zeta}+\boldsymbol{\theta})(1-u)e^{-\frac{(\boldsymbol{\zeta\eta}+\boldsymbol{\theta})}{\boldsymbol{\theta}}}}{\boldsymbol{\theta}}\right) \quad (42)$$

Generate the suggesting three-parameter Lindley distribution data (STPLD)

At this step, the random observation (data) will be generated following to a suggesting three-Parameter Lindley distribution defined by equation 11 using Lambert W function as follows:

$$F(t,\eta,\zeta,\theta) = 1-\left[\frac{\beta+\eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right]e^{-\eta t}$$

Let $F(t) = u$, $\Rightarrow 1-u = \left[\frac{\beta+\eta^2\zeta t+\eta\zeta}{\eta\zeta+\beta}\right]e^{-\eta t}$; $0 < u < 1$

$$\Rightarrow \frac{(1-u)(\eta\zeta+\beta)}{\eta\zeta} = \left[\frac{\beta+\eta\zeta}{\eta\zeta}+\eta t\right]e^{-\eta t}$$

$$\Rightarrow -\frac{(1-u)(\eta\zeta+\beta)e^{-(\eta\zeta+\beta)/\eta\zeta}}{\eta\zeta} = -\left[\frac{(\eta\zeta+\beta)}{\eta\zeta}+\eta t\right]e^{-(\eta t+(\eta\zeta+\beta)/\eta\zeta)}$$

$$\Rightarrow W_{-1}\left[-\frac{(1-u)(\eta\zeta+\beta)e^{-(\eta\zeta+\beta)/\eta\zeta}}{\eta\zeta}\right] = -\left[\frac{\eta\zeta+\beta}{\eta\zeta}+\eta t\right]$$

$$t = Q(\eta,\zeta,\theta) = -\frac{\beta}{\eta^2\zeta}-\frac{1}{\eta\zeta}-\frac{1}{\eta}W_{-1}\left[-\frac{(1-u)(\eta\zeta+\beta)e^{-(\eta\zeta+\beta)/\eta\zeta}}{\eta\zeta}\right] \; ; 0 < u < 1 \qquad (43)$$

Step three: At this step, we estimate the survival functions for (TPLD) and (STPLD) by MLE and Bayesian estimation methods.

Step four: we used some criteria of accuracy (IMSE, -2lnL, AIC, CAIC, and BIC) to compare between two methods of estimation MLE and Bayes estimation to estimate the parameters and the survival functions.

Discussion the simulation results. The following shows the results of the simulation experiment

Table (1) shows the results of the simulation when (η=1.5, ζ=0.8, β =0.5).

n	Dist.	TPLD				STPLD			
	Parameters	method	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\beta}$	method	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\beta}$
		ML	0.899	1.977	2.777	ML	0.858	1.679	2.566
		Bayes	0.857	1.668	2.579	Bayes	0.836	1.566	2.526

n	Dist.	$S_{TPLDReal}$	$\hat{S}_{TPLDML}$	$\hat{S}_{TPLDBayes}$	$S_{STPLDReal}$	$\hat{S}_{STPLDML}$	$\hat{S}_{STPLDBayes}$
10	Survival	0.99798	0.81361	0.99472	0.98867	0.87454	0.98564
		0.95718	0.83804	0.95635	0.94235	0.82898	0.94729
		0.88582	0.68844	0.89027	0.87359	0.67938	0.88123
		0.79103	0.67618	0.80289	0.78197	0.66712	0.79384
		0.68167	0.56967	0.70187	0.67262	0.56058	0.69282

		0.60527	0.54508	0.62096	0.59621	0.53604	0.62190
		0.52906	0.46056	0.53986	0.52001	0.43152	0.55078
		0.45528	0.36824	0.45044	0.44624	0.35919	0.48138
		0.38581	0.27982	0.40132	0.37673	0.27074	0.39535
		0.35314	0.15754	0.34303	0.34406	0.14851	0.32395
IMSE			**0.25554**	**0.15115**		**0.13538**	**0.11165**
-2lnL		**11.33**	**11.24**	**11.03**	**11.09**	**11.09**	**11.08**
AIC		**12.45**	**12.35**	**12.23**	**12.23**	**12.09**	**12.06**
CAIC		**13.57**	**13.47**	**13.33**	**13.35**	**13.21**	**13.18**
BIC		**8.55**	**8.44**	**8.32**	**8.35**	**8.22**	**8.17**
Best Dist.		**2**			**1**		
Best Method		$\hat{S}_{TPLDBayes}$			$\hat{S}_{STPLDBayes}$		
n	**Dist.**	**TPLD**			**STPLD**		
	Parameters	**method**	$\hat{\eta}$	$\hat{\zeta}$ $\hat{\beta}$	**method**	$\hat{\eta}$	$\hat{\zeta}$ $\hat{\beta}$
		ML	**0.855**	**1.566 2.544**	**ML**	**0.827**	**1.565 2.535**
		Bayes	**0.878**	**1.655 2.877**	**Bayes**	**0.888**	**1.640 2.738**
50	**Survival**	$S_{TPLDReal}$	$\hat{S}_{TPLDML}$	$\hat{S}_{TPLDBayes}$	$S_{STPLDReal}$	$\hat{S}_{STPLDML}$	$\hat{S}_{STPLDBayes}$
		0.67522	0.67577	0.55883	0.66408	0.67464	0.54770
		0.63431	0.62847	0.51714	0.62315	0.61134	0.50600
		0.56293	0.47991	0.36855	0.45182	0.45877	0.35743
		0.46813	0.46762	0.35628	0.45702	0.45651	0.34515
		0.35878	0.36111	0.24975	0.34765	0.34295	0.23861
		0.28237	0.33653	0.22522	0.27124	0.27540	0.21406
		0.21617	0.23201	0.21967	0.19503	0.18088	0.00054
		0.13241	0.15971	0.18836	0.12127	0.12857	0.00041
		0.12292	0.15128	0.14744	0.11178	0.11415	0.00017
		0.11823	0.13502	0.13313	0.10711	0.11386	0.00002
IMSE			**0.0015**	**0.0441**		**0.0015**	**0.0038**
-2lnL		**13.1848**	**13.0869**	**13.8809**	**12.9229**	**13.0020**	**13.0022**
AIC		**15.3038**	**15.0339**	**15.0746**	**15.0029**	**15.1075**	**15.8864**
CAIC		**15.4285**	**15.1012**	**15.1766**	**15.1229**	**15.0299**	**15.0068**
BIC		**9.4078**	**9.1224**	**9.1612**	**9.1229**	**9.0025**	**9.9473**
Best Dist.		**2**			**1**		
Best Method		$\hat{S}_{TPLDML}$			$\hat{S}_{STPLDML}$		
n	**Dist.**	**TPLD**			**STPLD**		
	Parameters	**method**	$\hat{\eta}$	$\hat{\zeta}$ $\hat{\beta}$	**method**	$\hat{\eta}$	$\hat{\zeta}$ $\hat{\beta}$
		ML	**0.853**	**1.511 2.513**	**ML**	**0.817**	**1.54 2.513**
		Bayes	**0.969**	**1.645 2.880**	**Bayes**	**0.895**	**1.798 2.788**

n		$S_{TPLDReal}$	$\hat{S}_{TPLDML}$	$\hat{S}_{TPLDBayes}$	$S_{STPLDReal}$	$\hat{S}_{STPLDML}$	$\hat{S}_{STPLDBayes}$
100	Survival	0.56307	0.56362	0.44188	0.54234	0.54272	0.42045
		0.52215	0.52132	0.40018	0.50178	0.50142	0.37928
		0.45079	0.46774	0.25162	0.42988	0.42655	0.23072
		0.35599	0.35547	0.23933	0.33578	0.33157	0.21843
		0.24664	0.24894	0.13282	0.22575	0.22804	0.11192
		0.17023	0.17638	0.10824	0.14932	0.15348	0.00064
		0.09403	0.08986	0.00042	0.07312	0.06196	0.00002
		0.02027	0.02755	0.00042	0.00826	0.01756	0.00001
		0.01078	0.02915	0.00051	0.00177	0.00113	0.00001
		0.00951	0.00885	0.00021	0.00052	0.00045	0.00001
IMSE			**0.0011**	**0.0437**		**0.0007**	**0.0034**
-2lnL		**17.3198**	**16.0125**	**17.0559**	**17.0579**	**17.1368**	**17.1372**
AIC		**19.4388**	**19.1689**	**19.2096**	**19.1379**	**19.6827**	**19.0214**
CAIC		**18.6835**	**18.5697**	**18.5816**	**18.2579**	**18.0849**	**18.1418**
BIC		**13.6810**	**13.5697**	**13.5962**	**13.2576**	**13.7975**	**13.0823**
Best Dist.		**2**			**1**		
Best Method		$\hat{S}_{TPLDML}$			$\hat{S}_{STPLDML}$		

Results discuss

1. At all sample sizes, (STPLD) was shown better than (TPLD) according to the criterions of errors (IMSE, -2lnL, AIC, CAIC, BIC).
2. At a sample size of (10), the Bayes method was shown the better among other estimation methods for all models because it recorded the lowest criterions of errors, followed by the method of maximum likelihood.
3. At a sample size of (50), the maximum likelihood method was the best in the (TPLD), because it achieved the lowest criterions of errors, followed by the Bayes, and the Bayes method achieved an advantage over the rest of the estimation methods in the (STPLD) followed by the method of maximum likelihood.
4. For sample sizes of (100), the maximum likelihood method was the best of both distributions because it achieved the lowest comparison criteria, followed by the Bayes under both distributions.

8. The application

In the applied aspect, real data was relied upon, as data was taken for 100 patients hospitalized in Al-Hussein Teaching Hospital in Nasiriyah who were infected with the Corona virus. The data shows the length of time for the patients to survive until death due to Covid-19. The Chi-square test for goodness of fit test was used to confirm. The data follows the proposed distribution, where the following null hypothesis was tested, based on the MATLAB program:

H_0:The data have STPLD Distribution

H_1:The data don't have STPLD Distribution

Table (2): Outcomes of the STPLD distribution data fit test

Distribution	χ^2_c	χ^2_t	Sig.	Decision
STPLD	0.14856	123.226	0.46755	Accept the null hypothesis H_0

Table (2) shows the estimated value of χ_c^2 (0.14855) is less than the tabulated value of χ_t^2 (123.225) and that the value of Sig = 0.46754 is more than the significance threshold (0.05). Because of this, it is not possible to reject the null hypothesis, suggesting that the actual data are distributed according to the proposed (STPLD).

Table (3) Results of comparison and accuracy tests applied to real data

Distribution	Parameters estimation			-2 LnL	AIC	CAIC	BIC
TPLD	0.898	1.976	2.776	46.7784	46.9984	46.2484	46.2784
STPLD	0.857	1.678	2.565	43.7685	43.1086	43.2585	43.3786

Table (3) shows that the suggested Lindley distribution indicated lower criteria in the special tests. This suggests that it is more suitable for the real data compared to the original distributions.
The experimental findings showed the superiority of the (STPLD) and indicated that the maximum likelihood technique was optimal for large sample sizes. Hence, the data will be fitted to the (STPLD), and the MLH will be used to estimate the parameters of this distribution at the specified theoretical values (η=0.8, ζ=1.5, θ=2). The survival function was estimated using the (Mat Lab) program using the MLH, and the estimation results were as in table (4) below:

Table (4) Survival function estimated by the MLH using real data

t	The real Survival function S_Real	Survival function estimated by MLE S-ML	t	The real Survival function S_Real	Survival function estimated by MLE S-ML
0.51	0.85887	0.76050	0.18	0.99211	0.95155
0.97	0.61765	0.52408	0.87	0.68897	0.59404
0.51	0.73024	0.62721	2.14	0.09967	0.15575
0.18	0.97211	0.93153	0.92	0.65099	0.55889
0.87	0.66895	0.57401	0.77	0.79773	0.69595
2.05	0.09814	0.14471	0.13	0.99829	0.97187
0.89	0.65096	0.55881	0.56	0.86066	0.76886
0.77	0.72770	0.62589	0.33	0.95385	0.90081
0.13	0.98024	0.965181	0.89	0.67699	0.58387
0.56	0.84005	0.73883	1.1	0.49679	0.46271
0.35	0.95384	0.88085	0.97	0.63866	0.54419
4.1	0.00114	0.01667	0.59	0.89025	0.79778
1.98	0.12805	0.16821	0.18	0.99219	0.95158
2.34	0.03888	0.11784	0.17	0.99392	0.95579
0.73	0.75047	0.64711	0.77	0.77721	0.65591
0.12	0.98398	0.98279	0.1	0.99010	0.95722
0.88	0.68297	0.58894	3.4	0.00339	0.02303
0.18	0.97212	0.95153	1.12	0.53006	0.46260
0.52	0.86809	0.77135	0.7	0.73699	0.69979
0.22	0.99120	0.96241	0.77	0.74774	0.64594
0.46	0.91775	0.82885	0.42	0.92179	0.83418
0.78	0.98120	0.98240	0.42	0.92269	0.83515
0.82	0.66498	0.57389	0.61	0.83515	0.73179
1.55	0.39617	0.39576	0.54	0.86964	0.76968
2.12	0.11346	0.15949	0.85	0.70087	0.69420
0.66	0.79996	0.69564	0.22	0.98391	0.93393
0.5	0.98312	0.93991	0.43	0.93774	0.83883
0.66	0.86956	0.76546	0.51	0.88358	0.78598

0.29	0.98677	0.89517	6.2	0.00022	0.00023
0.12	0.98398	0.96277	1.54	0.31081	0.29988
0.78	0.79199	0.73062	0.3	0.93313	0.84995
0.34	0.94401	0.87489	0.12	0.98398	0.96277
7.01	0.00007	0.00005	0.32	0.95701	0.88587
0.68	0.79817	0.69387	0.97	0.62867	0.54407
0.33	0.96385	0.88684	2.6	0.03613	0.07959
0.93	0.65289	0.56386	2.7	0.02985	0.07029
0.27	0.97157	0.95057	0.7	0.73613	0.63549
0.16	0.98158	0.95555	0.43	0.91775	0.82886
1.8	0.17448	0.19976	0.75	0.75915	0.65648
2.5	0.04406	0.08992	0.42	0.92168	0.86416
0.43	0.91773	0.89881	0.21	0.98608	0.95845
0.8	0.68688	0.58886	0.5	0.85527	0.75255
1.1	0.58676	0.53276	0.77	0.74774	0.64594
2.5	0.05513	0.10400	0.55	0.86486	0.76426
0.8	0.68699	0.58388	0.92	0.65095	0.55885
0.74	0.75917	0.65649	1.5	0.26806	0.27277
2.11	0.12533	0.16602	1.7	0.19898	0.22557
1.7	0.19898	0.23438	0.23	0.97739	0.94844
0.22	0.97099	0.92787	0.47	0.89718	0.79848
6.4	0.00018	0.00019	0.34	0.95079	0.87577

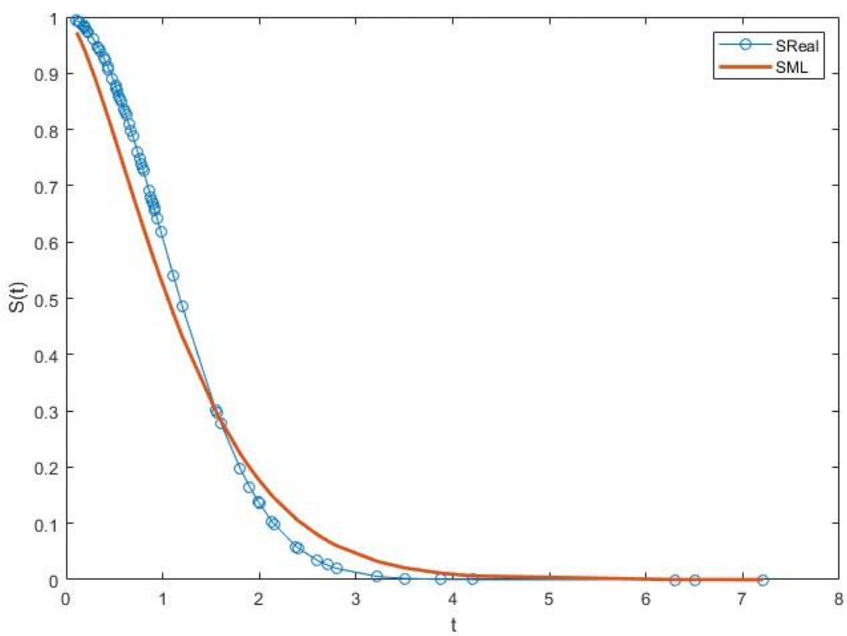

Figure (3) shows the graph of the theoretical survival function, which has been computed using the maximum likelihood method.

The blue color in the Figure (3) represents the plot of real data for patients' survival times.The red color represents the results of estimating the maximum likelihood of survival times based on the estimated values of the parameters from the research data using the maximum likelihood method.

Shorter hospital stays increase the likelihood of patient survival.The patient who stayed in the hospital for 10 minutes had a 97% probability of survival, whereas the patient who stayed for 7 days and one minutes had a 0.005% probability of survival.

9. Conclusions

We concluded that the suggested formula for the distribution is the best for all selected sample sizes and also superior to the Bayesian method at sample size n = 10 for all models because it recorded the lowest after analyzing the simulation and estimating the survival function for the original and proposed distribution using two estimation methods (MLE and Bayes method). The MLE recorded the lowest comparison criteria at a sample size of (n = 50, 100) for both distributions and followed the Bayes method. In the application aspect, the suggested Lindley distribution has lower special test requirements than the prior distribution, indicating that it fits the data more accurately. The results of our study showed that the survival function estimated using the Maximum Likelihood technique closely approximated the actual data function. Furthermore, the values of the survival function for the Maximum Likelihood method exhibited a declining trend with time. It has been observed that extended hospital stays are correlated with decreased survival rates.

References

1- Al-Abadi Karma Nasser, (2021)“ the Bayes estimate of the survival function for A Three-Parameter Lindley Distribution with practical application”, M.Sc. Thesis, Department of Statistics, Faculty of Management and Economics , University of Karbala , Karbala ,Iraq.

2- Al-Athari Zahraa Khallid, (2021) “Estimation of the Reliability Function of a two-parameter Lindley distribution For patients infected with COVID-19”, M.Sc. Thesis, Department of Statistics, College of Administration and Economics ,University of Al-Qadisiyah, AL-Diwaniyah,Iraq.

3- Abd El-Monsef M.M., (2015) “A new Lindley distribution with location parameter”, Communications in Statistics-Theory and Methods, Vol.45, No. (17), pp. 5204-5219.

4- Ghitany M. E., Atieh B., and Nadarajah S., (2008) “Lindley distribution and its application”, Mathematics and computers in simulation, Vol. 78, No.4, pp. 493-506.

5- Ghitany, M. E., Alqallaf, F., Al-Mutairi, D. K., & Husain, H.A., (2011). A two-parameter weighted Lindley distribution and its applications to survival data. Mathematics and Computers in simulation, Vol. 81, No.6, pp.1190-1201.

6- Krishna, H., & Kumar, K., (2011), “Reliability estimation in Lindley distribution with progressively type II right censored sample”. Mathematics and Computers in Simulation, Vol. 82, No.2, pp. 281-294.

7- Lindley, D. V., (1958), “Fiducial distributions and Bayes' theorem”, Journal of the Royal Statistical Society: Series B (Methodological), Vol.20, No.1, PP. 102-107.

8- Shanker R., Kamlesh K.K and Fesshaye H., (2017), “A Two Parameter Lindley Distribution: Its Properties and Applications”, Biostatistics and Biometrics Open Access Journal, Vol.1, No. 4, pp. 2573-2633.

9- Shanker R. & et al. (2017), “A Three-Parameter Lindley Distribution”, American Journal of Mathematics and Statistics, Vol. 1, No.7, pp. 15-26.

10- Shanker, R., & Mishra, A., (2013a). A two-parameter Lindley distribution. Statistics in Transition new series, Vol. 14, No.1, pp.45-56.

11- Shanker, R., Sharma, S., & Shanker, R., (2013b). A two parameter Lindley distribution for modeling waiting and survival times data.Vol.4, No.2.

12- Shanker, R., Sharma, S., & Shanker, R., (2013) “A two parameter Lindley distribution for modeling waiting and survival times data”, Vol.4, No.2, pp. 363-368.

13- Thamer Mathil k. and Zine Raoudha, (2023), “Statistical Properties and Estimation of the Three-Parameter Lindley Distribution with Application to COVID-19 Data”, Sains Malaysiana Vol. 52, No. 2, pp. 669-682**.**

14- Ming Han, (2006), “E-Bayesian Method to Estimate Failure Rate”, the Sixth International Symposium on Operations Research and Its Applications, ORSC & APORC, pp. 299–311.