Composition of paracompact and C-paracompact mappings

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تركيب الدوال فوق المتراصة والمتراصة- C

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المستخلص

في هذا البحث تمت دراسة تركيب أنواع معينة من التطبيقات المتراصة وتسمى التطبيقات فوق المتراصة والتطبيقات فوق المتراصة-C واكتشاف العلاقة بين هذه الأنواع.

الكلمات المفتاحية: التطبيقات المتراصة,التطبيقات فوق المتراصة , التطبيقات فوق المتراصة -c ,الغطاء المفتوح .

Abstract

In this paper, we study composition of certain types of compact mappings namely, paracompact and C-paracompact mappings and we investigate the relationship between these types.

Key words: Compact mapping. Paracompact mapping, C-paracompacct mapping, open cover

1- Introduction

Halfar in 1957 [4] introduced new types of mappings which said to be compact mapping and he gave some properties of these mappings and in 1969, G. Viglino, [2] studied the sensitivity of these types of mappings also introduce new class of compact space is called C-compact space and he discussed the relationship between these spaces and compact space. In 2003, G.maximilian [3] used preopen set to obtain in an analogous manner a new class of spaces namely PC-compact space, [3].

The aim of this work, is to study some classes of compact mapping which are paracompact and C-paracompact mappings and give certain properties of these types of mappings which show the relationship between these mappings and modify some theorems appear in [7], on these mappings by using same outline of its proof.

2-Preliminaries

In this section, we below list the definitions and results which are useful in the sequel .Through this paper, we denote X the topological space.

Definition (2.1): [5]

Let A be a subset of a topological space X then A is said to be compact if every open cover of A has a finite open sub cover.

Every finite set is compact set, but the converse is not necessary to be true, since [0,1] in usual topological space (R, T_u) is compact but infinite set, [7].

Remarks (2.2): [5][4]

let $F = \{X_\alpha : \alpha \in \Omega\}$ is a family of sets then $\prod_{\alpha \in \Omega} X_\alpha$ is compact if and only if X_α is compact for all $\alpha \in \Omega$..

(1) if A is closed set and B is compact set then $A \cap B$ is compact set.

Now, upon definition (2.1) G. Viglino give the definition of compact mapping as follow:

(2) if $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is compact mapping then $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are compact set for all $y_1 \in Y_1, y_2 \in Y_2$

(3) Let $\{X_\alpha\}$ be a family of spaces and $A_\alpha \subset X_\alpha$ for each α if A_α is closed in X_α then $\prod A_\alpha$ is closed in $\prod X_\alpha$.

Definition (2.3) : [4]

A mapping $f : (X, T) \longrightarrow (Y, \sigma)$ is said to be compact mapping if $f^{-1}(K) \subseteq X$ is compact set for every compact set $K \subseteq Y$.

Clearly, the compact mapping is not necessary to be continuous mapping. To illustrate that consider $f : (R, T_u) \longrightarrow (R, T_d)$ defined by $f(x) = x \quad \forall x \in X$, is compact mapping but not continuous.

Definition (2.4) : [1]

Let (X, T) be a topological space, A subset A of a topological space X is said to be paracompact if every open cover of A has a locally finite open subcover.

Notice that, general every compact set is paracompact but converse is not true

Remarks (2.5) : [5]

If A is closed set and B is paracompact set then $A \cap B$ is paracompact set.

Definition (2.6) : [7]

A mapping $f : (X, T) \longrightarrow (Y, \sigma)$ is said to be paracompact mapping if $f^{-1}(K) \subseteq X$ is paracompact set for every compact set $K \subseteq Y$.

From definition (2.6) and definition (2.1) we have, every compact mapping is paracompact mapping but converse is not true in general.

Now, we can give other types of paracompact mapping such paracompact* and paracompact** mappings as follow:

Definition (2.7) :

A mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be paracompact* mapping if $f^{-1}(K)\subseteq X$ is compact set for every paracompact set $K\subseteq Y$.

Thus from definitions (2.7) and (2.6), we get the paracompact* mapping stronger than paracompact mapping. Moreover, every paracompact* mapping is compact mapping.

Definition (2.8):

A mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be paracompact** mapping if $f^{-1}(K)\subseteq X$ is paracompact set for every paracompact set $K\subseteq Y$.

Examples (2.9):

1- Let $f:(R,T_d)\longrightarrow(R,T_u)$ be a mapping defined by $f(x)=x \quad \forall x\in R$, then f is paracompact* mapping but not compact mapping.

2- Let $f:(R,T_c)\longrightarrow(R,T_i)$ be a mapping defined by $f(x)=x \quad \forall x\in R$, then f is compact mapping but not paracompact mapping.

3- Let $f:(R,T_d)\longrightarrow(R,T_i)$ by a mapping defined by $f(x)=x \quad \forall x\in R$, then f is paracompact* mapping but not paracompact mapping.

4- Let $f:(R,T_u)\longrightarrow(R,T_u)$ be a mapping defined by $f(x)=x \quad \forall x\in R$, then f is paracompact** mapping but not paracompact* mapping.

5- Let $f:(R,T_d)\longrightarrow(R,T_i)$ by a mapping defined by $f(x)=x \quad \forall x\in R$, then f is paracompact** mapping but not paracompact mapping.

Remark (2.10) : [5]

Let $f:X\to X\times\{x\}$ be a function defined by $f(a)=(a,x)$ *for all* $a\in X$, then f is homeomorphism

Next, the following diagram shows the relationship between some types of paracompact mapping .

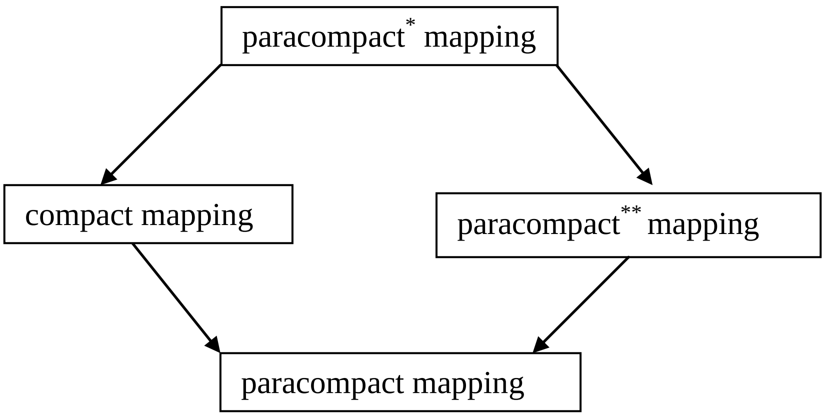

Diagram I: The relationship between some types of paracompact mapping

3- On paracompact mappings:

we study in this section certain properties of some types of paracompact mappings such as, restriction and composition and the product of these mappings. We start by recall that, the restriction of compact mapping is not necessary to be compact mapping. This is also true for paracompact mapping [7], to see that consider the following remark.

Remark (3.1) :

If a mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is paracompact then $f|A:A\longrightarrow Y$ is not necessary to be paracompact. To illustrate that, consider the following example:-

Example (3.2):

Let $f:(R,T_u)\longrightarrow(R,T_u)$ be a mapping defined by $f(x)=x$, then f is paracompact mapping and let $A=[0,3)$ then $f|A$ is not paracompact.

the following proposition provides a condition in order to make remark (3.1) is true.

Proposition (3.3):

Let $f:X\longrightarrow Y$ be a paracompact mapping. If $A\subseteq X$ is closed then $f|A:A\longrightarrow Y$ is a paracompact mapping.

Proof:

Let K be a compact subset of Y, thus $f^{-1}(K)$ is a paracompact in X and $(f|A)^{-1}(K)=A\cap f^{-1}(K)$, by using Remark (2.5), one can have $A\cap f^{-1}(K)$ is a paracompact set. From definition (2.6) we get, $f|A$ is a paracompact mapping.

From the above proposition we can get the following corollary :-

Corollary (3.4) :

Let $f:X\longrightarrow Y$ be a paracompact* (paracompact**) mapping. If $A\subseteq X$ is closed then $f|A:A\longrightarrow Y$ is a paracompact* (paracompact**) mapping.

We give the composition of paracompact mapping by the following theorem.

Theorem (3.5) :

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be paracompact mapping

1) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is compact mapping then $g\,o\,f$ is paracompact mapping.
2) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is paracompact* mapping $g\,o\,f$ is paracompact** mapping.

Proof:

1) Let K be a compact subset in Z, then $g^{-1}(K)$ is compact subset of Y and since f is paracompact mapping then $f^{-1}(g^{-1}(K))$ is paracompact set in X, but $(gof)^{-1}(K) = f^{-1}(g^{-1}(K))$, therefore; $g of$ is paracompact mapping.

2) Let K be a paracompact subset in Z, thus by using definition (2.7), one can have, $(g)^{-1}(K)$ is compact subset of Y, and since f is paracompact mapping we get, $(g of)^{-1}(K)$ is paracompact subset of X, therefore; gof is paracompact** mapping.

If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact or paracompact** mapping ,we cannot take any one of the types have studied .

Moreover, the following theorem give the composition of paracompact* mapping with other types.

Theorem (3.6) :

Let $f:(X,T) \longrightarrow (Y,\sigma)$ be paracompact* mapping

1) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is compact mapping then $g of$ is compact mapping.

2) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact mapping $g of$ is paracompact*mapping.

3) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact* mapping $g of$ is paracompact* mapping.

4) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact** mapping $g of$ is paracompact* mapping.

Proof:

1) Let K be a compact subset in Z, so $g^{-1}(K)$ is compact set in Y thus $g^{-1}(K)$ is paracompact subset of Y since f is paracompact* mapping then $f^{-1}(g^{-1}(K))$ is compact subset of X, but $(gof)^{-1}(K) = f^{-1}(g^{-1}(K))$, therefore; $g of$ is compact mapping.

2) Let K be a paracompact subset of Z, thus by using definition (2.7), one can have, $(g)^{-1}(K)$ is compact set in Y, so $(g)^{-1}(K)$ is paracompact set in Y and since f is paracompact* mapping we get, $(g of)^{-1}(K)$ is compact set in X therefore; gof is paracompact* mapping.

Similarly we can prove (3) and (4).

Theorem (3.7) :

Let $f:(X,T) \longrightarrow (Y,\sigma)$ be paracompact** mapping

1) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is compact mapping then $g of$ is paracompact mapping.

2) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact mapping $g of$ is paracompact mapping.

3) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact* mapping $g of$ is paracompact** mapping.

4) If $g:(Y,\sigma) \longrightarrow (Z,\gamma)$ is paracompact** mapping $g of$ is paracompact** mapping.

Next, the following propositions study the product of paracompact mapping.

Proposition (3.8) :

Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$ be two mappings. If $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is paracompact mapping then both f_1 and f_2 are paracompact mappings.

Proof:

Suppose K is a compact set in Y_1, $y_2 \in Y_2$, so $\{y_2\}$ is a compact set in Y_2 and by using Remark (2.2(1)) we get $K \times \{y_2\}$ is a compact subset of $Y_1 \times Y_2$ and since $f_1 \times f_2$ is paracompact mapping, thus $(f_1 \times f_2)^{-1}(K \times \{y_2\})$ is a paracompact set in $X_1 \times X_2$, but

$(f_1 \times f_2)^{-1}(K \times \{y_2\}) = f_1^{-1}(K) \times f_2^{-1}(\{y_2\})$ so $f_1^{-1}(K) \times f_2^{-1}(\{y_2\})$ is a paracompact set in $X_1 \times X_2$, hence by using remark (2.2(1)) we have $f_1^{-1}(K)$ is paracompact set in X_1, thus f_1 is a paracompact mapping and $f_2^{-1}(\{y_2\})$ is paracompact set in X_2, thus f_2 is a paracompact mapping.

Proposition (3.9) :

Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$ be two mappings. If $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is paracompact* mapping then both f_1 and f_2 are paracompact* mappings.

Proof:

Suppose K is a compact set in Y_1, $y_2 \in Y_2$, so $\{y_2\}$ is a compact set in Y_2 then K and $\{y_2\}$ are paracompact set and we get $K \times \{y_2\}$ is a compact subset of $Y_1 \times Y_2$ thus $K \times \{y_2\}$ is paracompact , and since $f_1 \times f_2$ is paracompact* mapping then by definition (2.7) , hence $(f_1 \times f_2)^{-1}(K \times \{y_2\})$ is a compact set in $X_1 \times X_2$, but

$(f_1 \times f_2)^{-1}(K \times \{y_2\}) = f_1^{-1}(K) \times f_2^{-1}(\{y_2\})$ so $f_1^{-1}(K) \times f_2^{-1}(\{y_2\})$ is a compact set in $X_1 \times X_2$, by Remark (2.2(3)) We have $f_1^{-1}(K)$ is compact set in X_1, thus f_1 is paracompact* mapping and $f_2^{-1}(\{y\})$ is paracompact* set in X_2, thus f_2 is a paracompact* mapping.

Proposition (3.10) :

Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$ be two mappings. If $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is paracompact** mapping then both f_1 and f_2 are paracompact** mappings.

Proof:

Suppose K is a compact set in Y_1, $y_2 \in Y_2$, so $\{y_2\}$ is a compact set in Y_2 then K and $\{y_2\}$ are paracompact set and we get $K \times \{y_2\}$ is a compact subset of $Y_1 \times Y_2$ thus $K \times \{y_2\}$ is paracompact , and since $f_1 \times f_2$ is paracompact** mapping then by definition (2.8) , hence $(f_1 \times f_2)^{-1}(K \times \{y_2\})$ is a paracompact set in $X_1 \times X_2$, but

$(f_1 \times f_2)^{-1}(K \times \{y_2\}) = f_1^{-1}(K) \times f_2^{-1}(\{y_2\})$ so $f_1^{-1}(K) \times f_2^{-1}(\{y_2\})$ is a paracompact set in $X_1 \times X_2$, by proposition(3.8) We have $f_1^{-1}(K)$ is paracompact set in X_1, thus f_1 is paracompact** mapping and $f_2^{-1}(\{y_2\})$ is paracompact set in X_2, thus f_2 is a paracompact** mapping

Theorem (3.11):

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a mapping then $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact mapping if and only if $I_W\times f$ is paracompact map, for each topological space W.

Proof:

Suppose $f\times I_W$ is paracompact mapping and we can say $I_W\times f = L_2 o(f\times I_w)oL_1$ such that $L_1 : W\times X\longrightarrow X\times W$ is mapping defined by $L_1(w,x)=(x,w)$ $\forall (w,x)\in W\times X$ and $L_2(y,w)=(w,y)$ $\forall (y,w)\in Y\times W$, also by Remark (2.10) we can get L_1 and L_2 are homeomorphism then L_1 and L_2 are paracompact mappings and by using theorem (3.5), we get $L_2 o(f\times I_w)oL_1$ is paracompact mapping. Therefore; $I_W\times f$ is paracompact mapping. By the same way we prove $f\times I_W$ is paracompact mapping.

4- On C-paracompact mappings:

In this section, we define and study some new types of paracompact mapping namely, C-paracompact mappings and study some properties of these mappings, we start by the following definition which appears in [7], [8].

Definition (4.1) : [7]

A mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be $C-$compact mapping if $f\times I_W : X\times W\longrightarrow Y\times W$ is compact map, for any topological space W.

Definition (4.2) :

A mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be $C-$paracompact mapping if $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact map, for any topological space W.

Now, the following lemma shows the relationship between paracompact and $C-$paracompact mappings.

Theorem (4.3) :

If $f:(X,T)\longrightarrow(Y,\sigma)$ is $C-$paracompact mapping then f paracompact mapping.

Proof:

Let $f:X\longrightarrow Y$ be $C-$paracompact mapping. Thus $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact compact, for any topological space W ,by proposition (3.8) one can have f is paracompact mapping.

We can gives anther type of C-paracompact mapping upon some types of paracompact mapping

Definition (4.4) :

A mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be C^* – paracompact mapping if $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact* mapping, for any topological space W.

For example:-

Let $f:([a,b],T_u)\longrightarrow(R,T_u)$ be a mapping defined by $f(x)=x \quad \forall x\in[a,b]$, then f is C^* – paracompact* mapping.

Next, the following proposition gives the relationship between C^* – paracompact and C – paracompact mappings.

Proposition (4.5):

If $f:(X,T)\longrightarrow(Y,\sigma)$ is C^* – paracompact mapping then f C – paracompact mapping.

Proof:

Let $f:X\longrightarrow Y$ be C^* – paracompact mapping, so from definition (4.4) we get $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact* mapping , for any topological space W, thus $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact mapping , for any topological space W, and by using definition (4.2), one can have f is C – paracompact mapping.

Definition (4.6) :

A mapping $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be C^{**} – paracompact mapping if $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact** mapping, for any topological space W.

Now, the following proposition shows the relation between C^{**} – paracompact and paracompact** mapping

Proposition (4.7) :

If $f:(X,T)\longrightarrow(Y,\sigma)$ is C^{**} – paracompact mapping then f paracompact** mapping.

Proof:

Let $f:X\longrightarrow Y$ be C^{**} – paracompact mapping, thus $f\times I_W : X\times W\longrightarrow Y\times W$ is paracompact** mapping , for any topological space W and by proposition (3.10) we have f is paracompact** mapping.

Now ,to illustrate the relationship between all types of paracompact and C-paracompact mapping have been studied ,see the following diagram:-

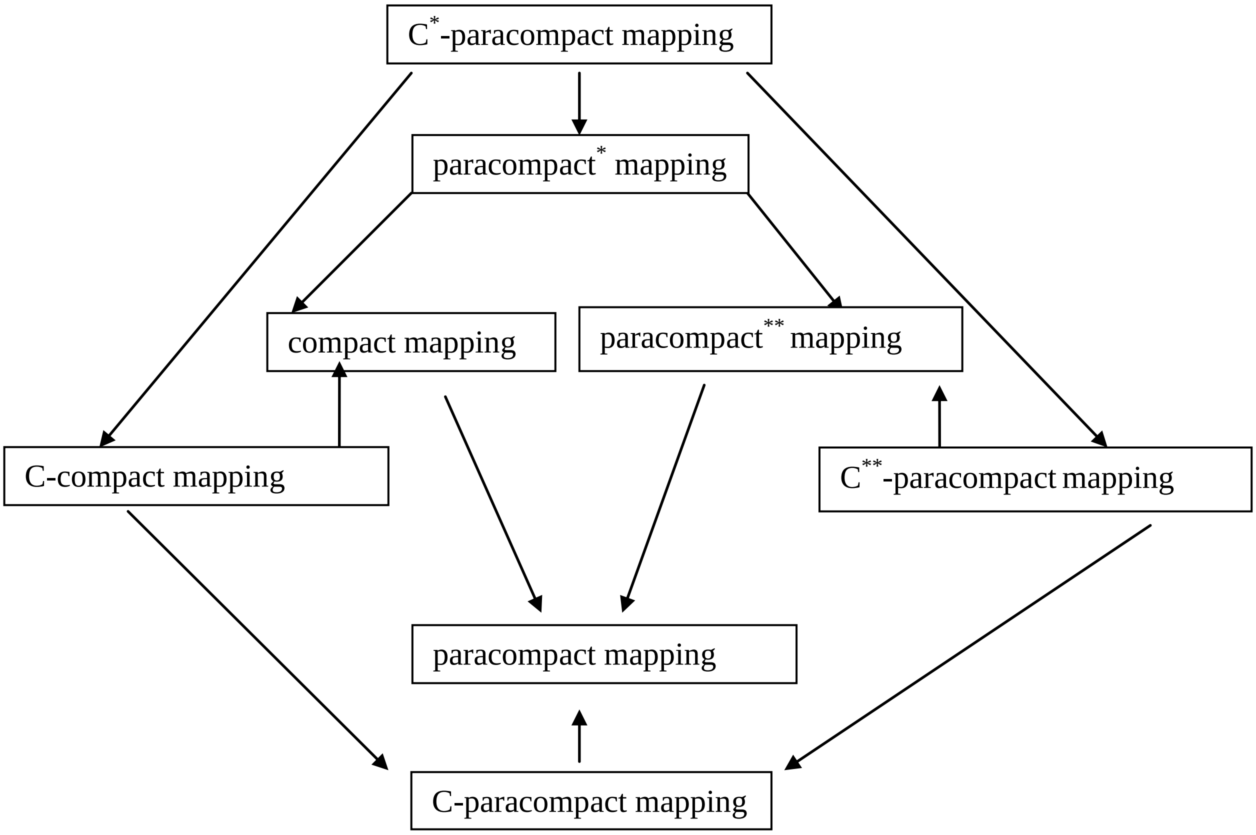

Diagram II: the relationship between all types of paracompact and C-paracompact mapping

Now, we study certain properties of C- paracompact mapping, and we start by the following remake

Remark (4.8):

If $f:(X,T)\longrightarrow(Y,\sigma)$ is C-paracompact mapping and $A\subseteq X$ then $f\big|A:(A,T_A)\longrightarrow(Y,\sigma)$ is not necessary to be C-paracompact. To illustrate that, consider the following example:-

Example (4.9):

Let $f:(R,T_u)\longrightarrow(R,T_u)$ be a mapping defined by $f(x)=x^2$, then f is C-paracompact mapping ,let A=[1,5) since A is not closed then $f\big|[1,5)$ is not C-paracompact.

Next, the following proposition give the condition to make the Remark (4.8) true.

Proposition (4.10) :

Let $f:X\longrightarrow Y$ be a C-paracompact mapping. If $A\subseteq X$ is closed then $f\big|A:A\longrightarrow Y$ is a C-paracompact mapping.

Proof:

Since $f : X \longrightarrow Y$ is C-paracompact mapping, so $f \times I_W$ is paracompact, for any topology W and since $A \subseteq X$ is closed thus, $A \times W$ is closed by Remark (2.2(4)) also, $(f|A) \times I_W$ is paracompact but $(f|A) \times I_W = (f \times I_W)_{A \times W}$, therefore $f|A$ is C-paracompact mapping.

Now, from above proposition we can get the following corollary

Corollary (4.11) :

Let $f : X \longrightarrow Y$ be a C*-paracompact (C**-paracompact) mapping. If $A \subseteq X$ is closed then $f|A : A \longrightarrow Y$ is a C*-paracompact (C**-paracompact) mapping.

Next, the following theorem give the composition of some types of C-paracompact mappings. Now, we need the following corollary:-

Corollary (4.12) :

Let $f : X \longrightarrow Y$ be $C-$paracompact mapping. Then $I_W \times f$ is paracompact for each space W.

Proof:

Let $f : X \longrightarrow Y$ is $C-$paracompact mapping, thus $f \times I_W$ is paracompact map and by using theorem (3.11) we get $I_W \times f$ is paracompact map.

Theorem (4.13) :

Let $f : (X, T) \longrightarrow (Y, \sigma)$ be C- paracompact mapping

1) If $g : (Y, \sigma) \longrightarrow (Z, \gamma)$ is C-compact mapping then $g o f$ is C-paracompact mapping.

2) If $g : (Y, \sigma) \longrightarrow (Z, \gamma)$ is C*-paracompact mapping $g o f$ is paracompact mapping.

Proof:

1) since f is C-paracompact mapping so by using definition (4.2), we get $f \times I_W$ is paracompact mapping for any topological space W , also since g is C-compact mapping thus by using definition (4.1), we get $g \times I_W$ is compact mapping, for any topological space W , then from theorem (3.5), one can have $(f \times I_W) o (g \times I_W) = (fog) \times I_W$ is paracompact mapping, therefore; from definition (4.2), we have ; $g o f$ is C-paracompact mapping.

2) since f is C-paracompact mapping so by using definition (4.2), we get $f \times I_W$ is paracompact mapping for any topological space W , also since g is C*-paracompact mapping , thus by using definition (4.4), we get $g \times I_W$ is paracompact* mapping, for any topological space W , then $g \times I_W$ is compact mapping and from theorem (3.5), one can have $(f \times I_W) o (g \times I_W) = (fog) \times I_W$ is paracompact mapping, therefore; from definition (4.2), we have ; $g o f$ is C-paracompact mapping.

When $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C**- paracompact mapping we cannot get any type of mapping have been studied. Moreover, the following theorem give the composition of C*-paracompact mappings with other types of C-paracompact .

Theorem (4.14):

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be C*- paracompact mapping

1) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C-compact mapping then $g\,o\,f$ is C-compact mapping.

2) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C-paracompact mapping $g\,o\,f$ is C-compact mapping.

3) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C*- paracompact mapping $g\,o\,f$ is C*- paracompact mapping.

4) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C**- paracompact mapping $g\,o\,f$ is C* -paracompact mapping.

Proof:

1) since f is C*-paracompact mapping so by using definition (4.4), we get $f\times I_W$ is paracompact* mapping for any topological space W , then $f\times I_W$ is compact mapping also since g is C-compact mapping , thus by using definition (4.1), we get $g\times I_W$ is compact mapping, for any topological space W , then from theorem (3.6), one can have $(f\times I_W)o(g\times I_W)=(fog)\times I_W$ is compact mapping, therefore from definition (4.1), we have , $g\,o\,f$ is C-compact mapping.

2) since f is C*-paracompact mapping so by using theorem (4.4), we get $f\times I_W$ is paracompact* mapping for any topological space W , also since g is C-paracompact mapping , thus by using definition (4.2), we get $g\times I_W$ is paracompact mapping, for any topological space W , then from theorem (3.6), we have $(f\times I_W)o(g\times I_W)=(fog)\times I_W$ is compact mapping, therefore from definition (4.1), we have , $g\,o\,f$ is C-compact mapping

similarly, we can get prove (3) and (4).

Now, the following theorem gives the composition of C**-paracompact mapping with other types of mapping

Theorem (4.15) :

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be C**- paracompact mapping

1) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C-compact mapping then $g\,o\,f$ is C-paracompact mapping.

2) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C-paracompact mapping $g\,o\,f$ is C-paracompact mapping.

3) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C*- paracompact mapping $g\,o\,f$ is C*- paracompact mapping.

4) If $g:(Y,\sigma)\longrightarrow(Z,\gamma)$ is C**- paracompact mapping $g\,o\,f$ is C** -paracompact mapping.

Proof: The proof is obvious.

Next, we study the product of some types of C- paracompact mapping and we start by the following proposition:-

Proposition (4.16) :

. Let $f_1:X_1\longrightarrow Y_1$ and $f_2:X_2\longrightarrow Y_2$ be two mappings ,then $f_1\times f_2$ is C-paracompact mapping if and only if $f_2\times f_1$ is C-paracompact mappings.

Proof:

Let $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ be C-paracompact mapping ,then $(f_1 \times f_2) \times I_W$ is paracompact for each space W

To prove that $(f_2 \times f_1) \times I_W$ is a paracompact mapping one can see ;

$(f_2 \times f_1) \times I_W = L_2 o (f_1 \times f_2 \times I_w) o L_1$

Such that $L_1 : X_2 \times X_1 \times W \rightarrow X_1 \times X_2 \times W$ defined by:

$L_1(x_2, x_1, w) = (x_1, x_2, w) \qquad \forall (x_2, x_1, w) \in X_2 \times X_1 \times W$

And $L_2 : Y_1 \times Y_2 \times W \rightarrow Y_2 \times Y_1 \times W$ defined by :

$L_2(y_1, y_2, w) = (y_2, y_1, w) \qquad \forall (y_1, y_2, w) \in Y_1 \times Y_2 \times W$

Since L_1 and L_2 is a homeomorphisms then L_1 and L_2 is paracompact

Thus $L_2 o (f_1 \times f_2 \times I_w) o L_1$ is paracompact ,therefore $(f_2 \times f_1) \times I_W$ is paracompact

Then $(f_2 \times f_1)$ is C-paracompact mapping .

Corollary (4.17):

Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$ be two mappings ,then $f_1 \times f_2$ is C^*-paracompact (C^{**}-paracompact) mapping if and only if $f_2 \times f_1$ is C^*-paracompact (C^{**} -paracomact) mappings.

Theorem (4.18) :

Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$ be two mappings. If $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is C-paracompact mapping then both f_1 and f_2 are C-paracompact mappings

Proof:

Since $f_1 \times f_2$ is C-paracompact mapping, thus $(f_1 \times f_2) \times I_W$ is a paracompact mapping , but $(f_1 \times f_2) \times I_W = f_1 \times (f_2 \times I_W)$ then by using proposition (3.8) , we get f_1 and $(f_2 \times I_W)$ are paracompact mappings, so f_2 is C- paracompact mapping and since $f_1 \times f_2$ is C-paracompact mapping then by using proposition (4.16) we get $f_2 \times f_1$ is C-paracompact mapping and $(f_2 \times f_1) \times I_W$ is paracompact mapping, but $(f_2 \times f_1) \times I_W = f_2 \times (f_1 \times I_W)$ thus by using proposition (3.8) , we get f_2 and $(f_1 \times I_W)$ are paracompact mappings, so f_1 is C- paracompact mapping.

Now, we can get the following conclusions:-

Conclusions (4.19) :

(1) If $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is C^*-paracompact mapping then both f_1 and f_2 are C^*-paracompact mappings.

(2) If $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is C^{**}-paracompact mapping then both f_1 and f_2 are C^{**}-paracompact mappings.

Proof

(1) Since $f_1 \times f_2$ is C*-paracompact mapping, thus $(f_1 \times f_2) \times I_W$ is a paracompact* mapping , but $(f_1 \times f_2) \times I_W = f_1 \times (f_2 \times I_W)$ then by using proposition (3.9) , we get f_1 and $(f_2 \times I_W)$ are paracompact* mappings, so f_2 is C*- paracompact mapping and since $f_1 \times f_2$ is C*-paracompact mapping then by using corollary (4.17) we get $f_2 \times f_1$ is C*-paracompact mapping and $(f_2 \times f_1) \times I_W$ is paracompact* mapping, but $(f_2 \times f_1) \times I_W = f_2 \times (f_1 \times I_W)$ thus by using proposition (3.9) , we get f_2 and $(f_1 \times I_W)$ are paracompact* mappings, so f_1 is C*- paracompact mapping

(2) Since $f_1 \times f_2$ is C**-paracompact mapping, thus $(f_1 \times f_2) \times I_W$ is a paracompact** mapping , but $(f_1 \times f_2) \times I_W = f_1 \times (f_2 \times I_W)$ then by using proposition (3.10) , we get f_1 and $(f_2 \times I_W)$ are paracompact** mappings, so f_2 is C**- paracompact mapping and since $f_1 \times f_2$ is C**-paracompact mapping then by using corollary (4.17) we get $f_2 \times f_1$ is C**-paracompact mapping and $(f_2 \times f_1) \times I_W$ is paracompact** mapping, but $(f_2 \times f_1) \times I_W = f_2 \times (f_1 \times I_W)$ thus by using proposition (3.10) , we get f_2 and $(f_1 \times I_W)$ are paracompact** mappings, so f_1 is C**-paracompact mapping

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