A study of the behaviorof the discharge parameters and their effects on the mechanical design of an electrothermal gun

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دراسة سلوك معلمات التفريغ وتأثيرها على التصميم الميكانيكي للمعجل الكهروحراري

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المستخلص

هذا البحث يعطي تغطيه عن إيجاد معادلات معلمات التفريغ و تأثيرها على تصميم معايير المعجل الكهروحراري بالاضافه إلى تحقيق حلول التصميم .الهدف من هذه الدراسة هو الحصول على معلومات لمعلمات التفريغ في المعجل الكهروحراري بتعزيز النماذج السابقة من أبحاث أخرى وتحليلها. حللت عمليات فيزيائيه مختلفة في تفريغ الضغط العالي. طول الأنبوب محدود لنسبة كثافة التفريغ و ضغط البلازما المعطى. تمتطوير نموذج تحليلي لتأثير تفريغ الضغط العالي في ألحاله شبه ألمستقره, التي فيها خصائص دفق البلازما مستقره عند تدرج الزمن الهايروديناميكي. هذا الموديل يسمح لحساب كل معلمات التفريغ , مثل : نسبة الاستئصال ; درجه حرارة البلازما; سرعة الصوت للبلازما; الكثافة; كثافة التيار و الضغط , كدالة ل "التغير البطيء" للتيار و أبعاد الأنبوب. حصلنا على توافق جيد للنتائج بين النموذج النظري الذي قمنا ببنائه و النتائج العملية للأبحاث العالمية .

Abstract

This paper gives an overview of finding the discharge parameters equations and their effects on the design criteria of ETA as well as the realized design solutions. The aim of this study is to obtain information about the discharge parameters in an electrothermal gun by extending the previous models of different researches and analyze their equations.

Different physical processes in confined high-pressure discharge analyzed. For a given discharge, ablation rate and plasma pressure,the tube length is limited. An analytical model is developed for confined high-pressure discharges operating in a quasi-steady state, in which the resulting plasma jet characteristics are steady at hydrodynamic timescales. This model allows the calculation of all the discharge parameters, such as ablation rate; plasma temperature; plasma sound velocity; density; current density and pressure, as a function of the "slowly varying" current and the tube dimensions. Good agreement for the results obtained between the theoreticalmodel, which is built, and the experimental results for global research.

Introduction

Various electro thermal propulsion systems comprise three major components. The first component is the power supply. The control system of the power supply changes the current pulse duration by consecutive time switching of the capacitor storage modules to the load. The capacitor storage was divided into a number of identical in dependent modules with each having its own switch, namely, an air triggatron [1].Figure (1) shows the scheme of ET GUN.

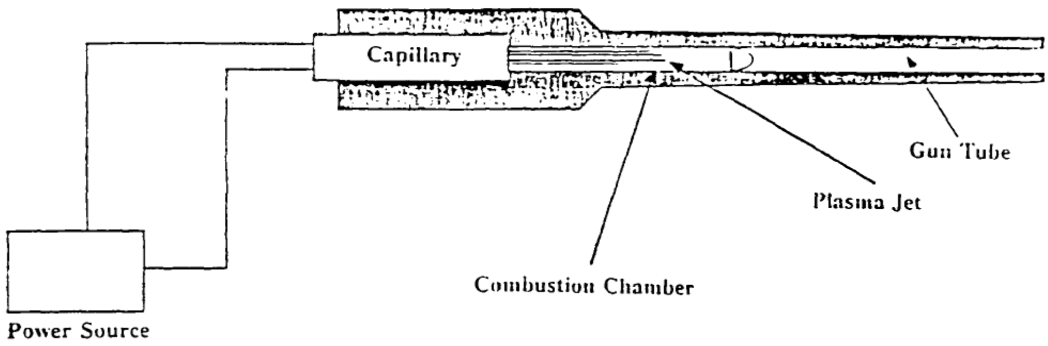

Figure 1: Schematic of Electrothermal Gun System

The second component is the high-pressure discharge capillary, which is a narrow tube, made of a plastic material (usually polyethylene), and placed between two electrodes, one of which is hollow [2]. The cylindrical discharge plasma formed in a capillary tube. The flow of this hot plasma jet out through the hollow electrode is balanced by appropriate ablation of the tube walls that are made of a low atomic weight material (PE). This ablation caused by yielding steady state operation conditions for a steady voltage current supply to the discharge [3].A tungsten anode inserted into one side of the tube, and a cylindrical stainless steel cathode inserted on the outside of the opposite end of the tube. They typically ignited by shunting current through a fine copper wire strung between the anode and cathode. The wire explodes in several μs and provides the initial plasma used to ignite the main discharge. Two concerns relating to the application of capillary discharges in spacecraft propulsion systems are immediately apparent: the single-use wire ignition system and the lack of a propellant feed mechanism [4]. The whole of the discharge process happens in capillary tube.

The third component is the gun tube (barrel) made typically from copperwhich the plasma accelerates the projectile under the influence of heat radiation from the plasma surface through it [5].

Theoretical model and the axial profile inside the discharge tube

Various ET Gun, makes use of a high-pressure discharge tube (capillary) to convert a pulse of electrical energy to thermal and kinetic energy of plasma jet. It is clear that it is strongly dependent on the flow properties of plasma, which on their part depend on the pressure outside tube, in addition to the dependence on the input electric power [2].

The set of one dimensional gas dynamics equation:[5]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = \frac{\partial \rho_a^*}{\partial t} \qquad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(P + \rho v^2)}{\partial x} = 0 \qquad (2)$$

$$\frac{\partial E}{\partial t} + \frac{\partial[(E + P)v]}{\partial x} = Q_j - Q_{out} \qquad (3)$$

Where $\rho, v, P$are the plasma density, velocity, pressurerespectively.$\rho_a^*$is the density of ablating matter, i.e. the density of polyethylene vapor inside the capillary tube, andQ_j is Joule heat

The first equation is respect to conservation of plasma mass was the replenishment of plasma with evaporated material from the walls taken into consideration. The second equation is respect to conservation low of impulse. Zero right hand of eq(2) means that vaporizing wall material enters into plasma with zero pulse. The third equation is respect to conservation low of energy and takes into account the radiation losses in the plasma as well as plasma replenishment with evaporated material(Q_{out}). In addition, it takes into consideration the resistive heating of plasma (Q_j) in the region of arc discharge ($0 \leq x \leq l$) , where l is the capillary tube length [5].

In ET- launcher model, we take two regions, namely the high- current discharge region ($o \leq z \leq l$) and the acceleration region ($z > l$) the axial behavior of the discharge characterized by the following equation:

1-The 1D continuity equations:[2]

$$\frac{\partial(\mathrm{\rho u})}{\partial \mathrm{z}} = \mathrm{\rho_a^*} \qquad (4)$$

Where u is the axial flow velocity.Due to ablation, the mass flux density has to increase along axis, i.e.$\rho_a^* > 0$.

2- Integrating the 2D momentum equation, applying gauss theorem and using (4), one gets:

$$\rho u \frac{\partial u}{\partial z} + \rho_a^*(u - u_s) = -\frac{\partial P}{\partial z} \qquad (5)$$

Where P is the plasma pressure and u_s is the axial flow velocity at the surface of plasma channel. Excluding the thin peripheral region from the integration, it seems reasonable to take u_s=u. Thus, it left with the familiar 1D Euler equation.

3-Axial homogeneity of the plasma temperature, i.e.:

$$T(z) \approx T_o \qquad (6)$$

This approximation, which is justified in [3] by the existence of the stabilizing feedback mechanisms of ohmic dissipation and radiant heat transport, replaces in fact the exact solution of the 1D energy equation:

$$\frac{\partial}{\partial z}\left[\rho u\left(\epsilon+\frac{P}{\rho}+\frac{u^2}{2}\right)\right]=\frac{J^2}{\sigma} \qquad (7)$$

Here, ϵ is the plasma internal energy while σ is the conductivity. Both are essentially function of the temperature only. J is the discharge current density. Using (4) and remembering that the radiated energy returned to the plasma by the ablated mass, we could write equation (7) as:

$$\rho u\frac{\partial}{\partial z}\left(\epsilon+\frac{P}{\rho}+\frac{u^2}{2}\right)+\frac{2}{a}\sigma_b T^4=\frac{J^2}{\sigma} \qquad (8)$$

Where σ_b is the Stefan – Boltzmann constant. One can see that indeed T(z)=T_o is a solution of (7), provided that the kinetic energy term $u^2/2$ is neglected.

4-Ideal gas equation of state:

$$P=\frac{1+\bar{Z}}{\bar{m}}\rho T \qquad (9)$$

Where$\bar{m}$ is the average mass and $\bar{z}$ is the average ionization of the plasma, which depends mainly on the temperature. We can find the velocity u_*

$$u_*=(\frac{1+\bar{Z}}{m}T)^{1/2} \qquad (10)$$

Where C_s is the plasma sound velocity equaled to $(\gamma\frac{1+\bar{Z}}{m}T)^{1/2}$, so $u_*=C_s/\gamma^{1/2}$ where γ is the adiabatic constant.Using (6) and (9), the 1D Euler equationscan be written as:

$$\frac{\partial}{\partial z}\left(\frac{u^2}{2}+u_*^2 ln\rho\right)=0 \qquad (11)$$

Integrating (11), we get the density and the pressure as functions of the axial velocity:

$$\frac{P}{P_o} = \frac{\rho}{\rho_o} = e^{-\frac{1}{2}(\frac{u}{u_*})^2} \qquad (12)$$

Where P_o and ρ_o are respectively the pressure and density at the back of the capillary (z=0) where the axial velocity vanishes, i.e., u (z=0) =0. Taking the derivative of the mass flux density with respect to z, we get:

$$\frac{\partial(\rho u)}{\partial z} = \rho\left(1 - \left(\frac{u}{u_*}\right)^2\right)\frac{\partial u}{\partial z} \qquad (13)$$

Since $\frac{\partial(\rho u)}{\partial z}$>0 and u (0) =0, solution exists only for axial velocity in the range$(o \leq u \leq u_*)$. As ρ_a^*depends on the ratio of the radiated energy to the plasma enthalpy

(The kinetic energy term was neglected) it is essentially temperature dependent, and so can be taken constant over the capillary length. Then (4) is easily solved to give:

$$\rho u = \rho_a^* z = \rho_l u_l \left(\frac{z}{l}\right) \qquad (14)$$

Where ρ_l , u_l are respectively the density and velocity at the open end. With the help of last equation, we get the axial dependence of u:

$$z = \frac{\rho_o u}{\rho_l u_l} e^{-\frac{1}{2}(\frac{u}{u_*})^2} l \qquad (15)$$

We conclude that the resistance formed in capillary tube of ETA is given by [6]

$$R = \frac{\ell}{\sigma A} = \frac{\ell}{\sigma \pi r^2} \qquad (16)$$

Where (ℓ), (r), are respectively length and radius of the capillary tube. $\boldsymbol{\sigma}$ is plasma conductivity. We also found the plasma temperature as:

$$T = \left(\frac{J^2 r}{2\sigma\sigma_s}\right)^{1/4} \qquad (17)$$

Where J is the current density,where$\sigma_s = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67x10^{-8}w^{-1}m^{-2}k^{-4}$ is the Stefan-Boltzmann constant [7].and we found the ablation density rate as:

$$\dot{\rho}_a = \frac{J^2}{\sigma\,(c_s^2 + \varepsilon)} \qquad (18)$$

Where ε is the internal energy and it given by the equation

$$\varepsilon = \frac{(1+\bar{z})T}{\bar{m}(\gamma - 1)} \qquad (19)$$

Where γ is the adiabatic constant. We can find the current equation and it is varying with time.

$$I(t) = V_o \sqrt{\frac{C}{L}} \sin\left[\sqrt{\frac{1}{LC}}\, t\right] \qquad (20)$$

Given that V_o, L, and C can be accurately measured, which is usually the case with capacitievly driven system.

Theoretical results

The peak value of the current flowing discharge circuit versus the discharge time that given by the equation (20) can be seenin figure (2), that we have by taking different values of time in the discharge process.The model depended on many experimental researches where they use different discharge time and we got a good agreement with them[8, 9, 10].

Table (1) The relationship between the time, discharge current, and current density at (c=0.133 f, and L=4*10^{-6}H) [10].

t(sec)	I(A)*10^5	J(A/m^2)*10^9
0	0	0
0.0002	0.2963	0.7698
0.0004	0.5704	1.4821
0.0006	0.8019	2.0836
0.0008	0.9735	2.5295
0.001	1.0723	2.7864
0.0012	1.091	2.835
0.0014	1.0283	2.6719
0.0016	0.8886	2.3091
0.0018	0.6826	1.7738
0.002	0.4256	1.1059
0.0022	0.1368	0.3554
0.0024	-0.1623	-0.4216
0.0026	-0.4492	-1.1672
0.0028	-0.7025	-1.8255
0.003	-0.9034	-2.3474
0.0032	-1.0368	-2.694
0.0034	-1.0927	-2.8392
0.0036	-1.0669	-2.7723
0.0038	-0.9615	-2.4983
0.004	-0.7842	-2.0376

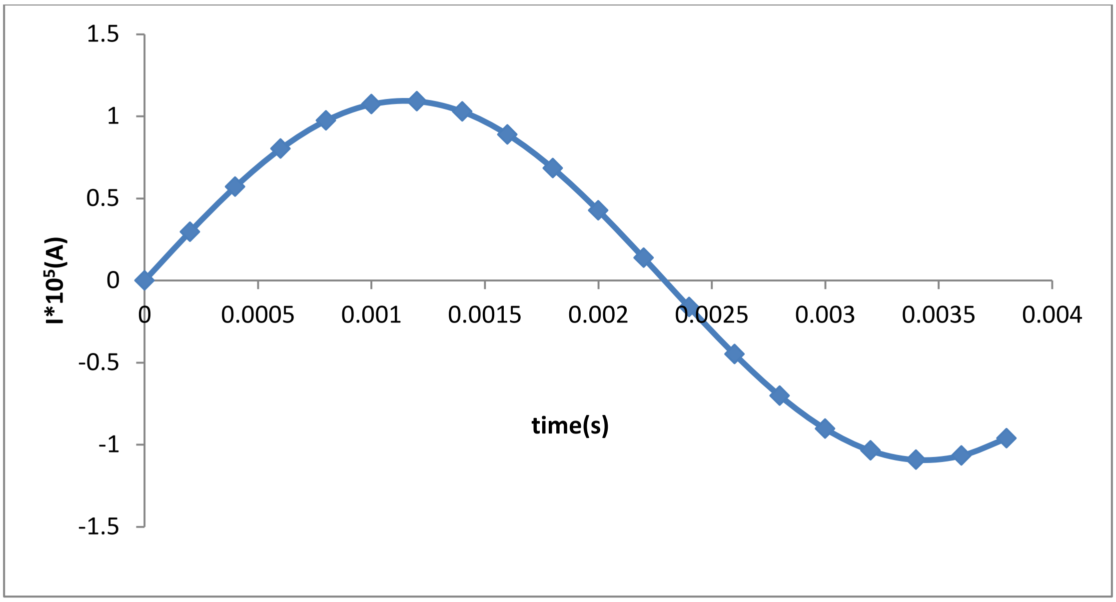

Figure (2): The discharge current with the time of discharge

From the figure (2) and table (1), we noticed that the discharge current peak is in the value of ($1.0941*10^5$ A) in (0.011 s) we get this from many values of time in (microsecond).Because discharge happens in a micro or milli second, so we must use a very short time to make an idealdischarge,therefore, when the time is very short we can save the gun and especially the electrode from erosion and the system from braking down. Experimentally the researcher used(spark gap switch), theoretically we used a program that has a condition to run (it is a step in program if the value of current the bigger value in the first pulse stop) do the workof the spark gap switchas shown in figure (3).

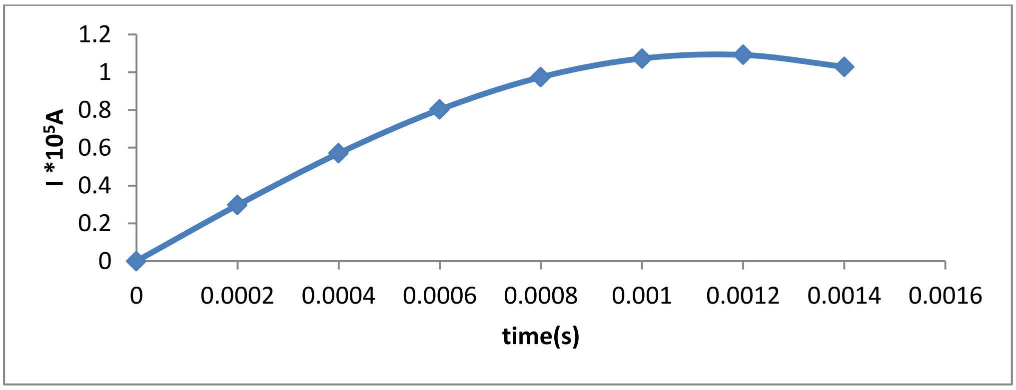

Figure (3): The relation between discharge current and time according to the discharge conditions

In the same table and figure (4), it can be noticed that the discharge current density peak is in the value of ($2.835(A/m^2)*10^9$) in (0.012 s) .We get this from many values of time in (microsecond).

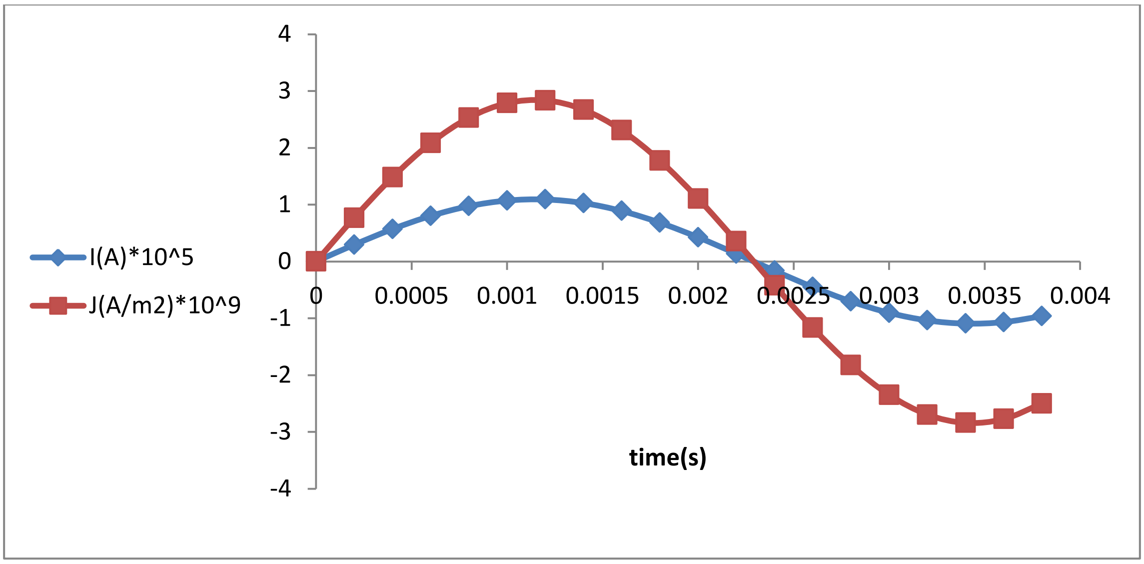

Figure (4): Discharge current and current density as a function of time

If takes the time in microseconds in table (2),it can be noticed changing in the parameters especially the current because the discharge lifetime will be very short but gives very high values of current. Making a change in the capacitance and inductance values.

Table (2): The relationship between the time and discharge current, at C= (1, 2, 3, 4, 5)* 10^{-6}F, L=4*10^{-6}H.

t*10^-6 s	IA[at c=1x10^6]	IA[at c=2x10^6]	IA[at c=3x10^6]	IA[at c=4x10^6]	IA[at c=5x10^6]
1	143.8277	146.8945	147.9253	148.4424	148.7531
3	299.2485	370.2459	395.8221	408.9833	417.0013
5	179.5416	416.0605	515.4027	569.3908	603.2300
7	-105.2350	262.3695	467.9019	590.3916	670.8101
9	-293.2590	-17.1305	268.7184	466.8439	606.4493
11	-211.6621	-288.4162	-17.5772	228.9966	422.8068
13	64.5360	-421.4032	-298.1748	-64.9171	156.0029
15	281.4000	-352.3229	-482.1111	-342.9368	-141.4850
17	239.5461	-114.2999	-509.7584	-536.9936	-411.1444
19	-22.5453	178.5311	-372.1541	-599.5757	-599.9363
21	-263.9087	385.7545	-113.9063	-515.3607	-670.7275
23	-262.6357	408.0045	181.2673	-304.9674	-609.5940
25	-19.8966	234.6119	417.6783	-19.9075	-428.5602
27	241.1353	-51.2797	518.6879	270.0264	-163.2335
29	280.4685	-312.5821	451.5511	493.8485	134.1995
31	61.9402	-423.9980	238.0322	596.7593	405.2369
33	-213.5356	-332.1023	-52.6512	553.5625	596.5688
35	-292.6878	-80.9600	-326.2663	374.8344	670.5622
37	-102.7442	209.0035	-494.1136	104.3337	612.6636
39	181.6620	398.7476	-501.7809	-191.7115	434.2609

Table (3): The relationship between the time and discharge current, at C= (1, 2, 3, 4, 5)* 10^{-6}F, L=6*10^{-6}H.

t*10^-6 s	IA[at c=1x10^6]	IA[at c=2x10^6]	IA[at c=3x10^6]	IA[at c=4x10^6]	IA[at c=5x10^6]
1	97.2453	98.6169	99.0766	99.3070	99.4454
3	230.4282	263.8814	275.6176	281.5984	285.2234
5	218.3393	343.6018	392.0361	417.6050	433.3923
7	68.5999	311.9346	422.9368	484.9721	524.4148
9	-124.3878	179.1456	361.5793	472.6269	546.2888
11	-238.9561	-11.7181	221.3478	382.5986	496.1300
13	-202.8763	-198.7832	32.8320	229.6845	380.5523
15	-38.8945	-321.4074	-162.8456	39.0184	214.7956
17	149.6081	-339.8390	-323.0005	-158.0609	20.7162
19	243.7912	-248.1028	-412.6969	-329.1606	-176.0947
21	184.2780	-75.9375	-412.3688	-446.1579	-349.6861
23	8.5879	120.8449	-322.0875	-489.8226	-477.1684
25	-172.5163	278.4522	-161.5470	-452.9778	-541.7321
27	-244.8588	345.7919	34.2330	-341.6795	-534.8638
29	-162.8319	301.0341	222.5455	-174.2211	-457.4692
31	21.8513	158.6881	362.3125	21.8731	-319.7534
33	192.7585	-35.1008	423.0456	214.3722	-139.8755
35	242.1424	-217.5109	391.4967	371.6360	58.4462
37	138.8694	-329.4090	274.5477	467.8160	249.0613
39	-51.9529	-334.5206	97.7096	487.1035	406.8355

Table (4): The relationship between the time and discharge current, at C= (1, 2, 3, 4, 5)*10^{-6}F, L=8*10^{-6}H.

t*10^-6 s	IA[at c=1x10^6]	IA[at c=2x10^6]	IA[at c=3x10^6]	IA[at c=4x10^6]	IA[at c=5x10^6]
1	73.4472	74.2212	74.4803	74.6100	74.6879
3	185.1230	204.4916	211.1988	214.6005	216.6569
5	208.0302	284.6954	313.2038	328.0441	337.1402
7	131.1848	295.1958	363.7291	400.9077	424.1895
9	-8.5652	233.4220	354.4702	424.1776	469.1721
11	-144.2081	114.4983	286.9489	394.9753	467.6272
13	-210.7016	-32.4585	172.2634	316.9133	419.7080
15	-176.1614	-171.4684	29.2638	199.6480	330.1666
17	-57.1499	-268.4968	-118.5457	57.6856	207.8827
19	89.2656	-299.7878	-246.8704	-91.4126	64.9832
21	192.8773	-257.6803	-334.6184	-229.2029	-84.3606
23	204.0022	-152.4837	-367.3670	-338.6400	-225.3384
25	117.3059	-9.9538	-339.7334	-406.1861	-343.9696
27	-25.6398	135.0132	-256.2596	-423.4857	-428.4894
29	-156.2910	246.9243	-130.6658	-388.3988	-470.5163
31	-211.9990	298.3796	16.4049	-305.2656	-465.8823
33	-166.0511	276.7813	160.7791	-184.3700	-415.0470
35	-40.4800	187.4172	278.7270	-40.6672	-323.0517
37	104.5018	52.1668	350.8620	108.0662	-199.0196
39	199.3738	-95.8558	365.3276	243.4315	-55.2508

From the codition we put in the program it can befound the best data for the discharge current if used the($C=5*10^{-6}$ F, $L=4*10^{-6}$H)gets the best maximum discharge current in table(5)

Table (5): The relationship between the time and discharge current, at$C=5*10^{-6}$ F, $L=4*10^{-6}$H

$t*10^{-6}$s	IA
1	148.7531
3	417.0013
5	603.2300
7	670.8101
9	606.4493
11	422.8068
13	156.0029
15	-141.4850
17	-411.1444
19	-599.9363
21	-670.7275
23	-609.5940
25	-428.5602
27	-163.2335
29	134.1995
31	405.2369
33	596.5688
35	670.5622
37	612.6636
39	434.2609

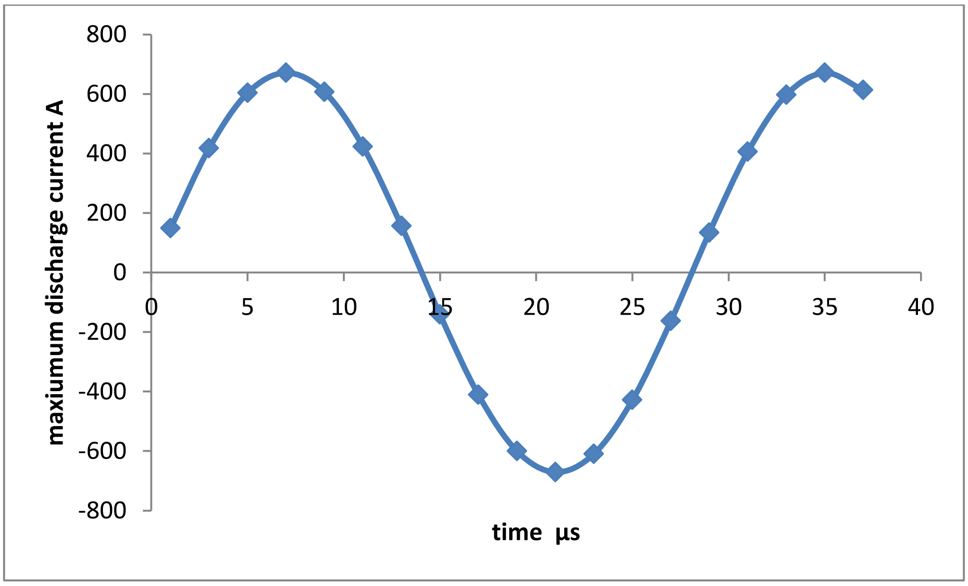

Figure (5):The maximum discharge current as a function of time agreed with [4]

After using the second condition, we explaedabove, it canchoose the best data from large amount of data (i.e. from200 values) to get figure (6).

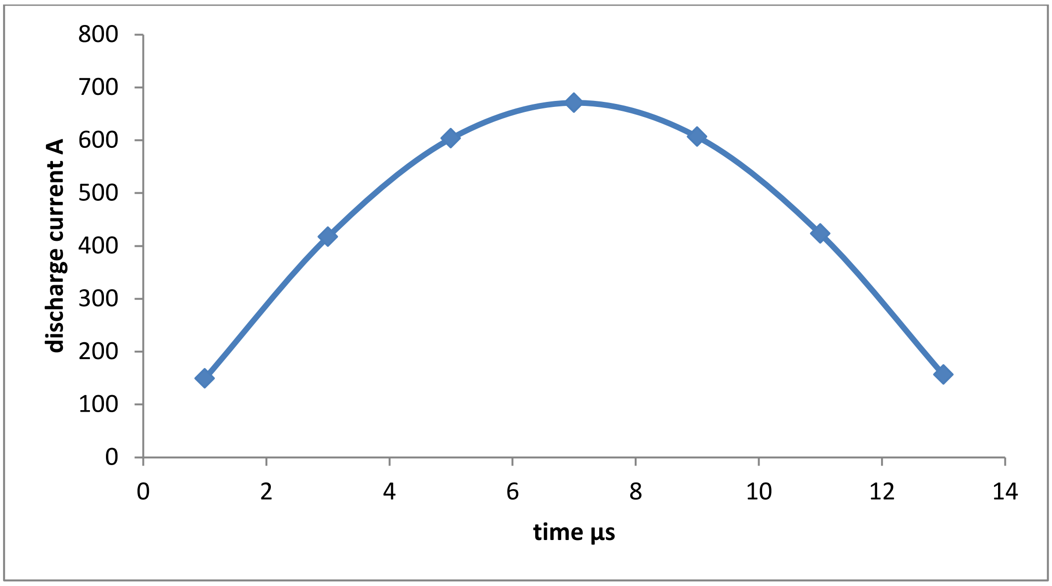

Figure (6): discharge current verses time in microsecond

So by using that data we improved and decreasing the discharge time from (ms) to (µs)and the value of capacitance what is proportional with time in(µs) and getting the discharge time (670.8101 (A) at seven (µs)). However, this current is not very good to make high discharge so it must be change the time to (ms) to get best data of the discharge current at (L=2µH, C=0.1F)then gets figure (7).

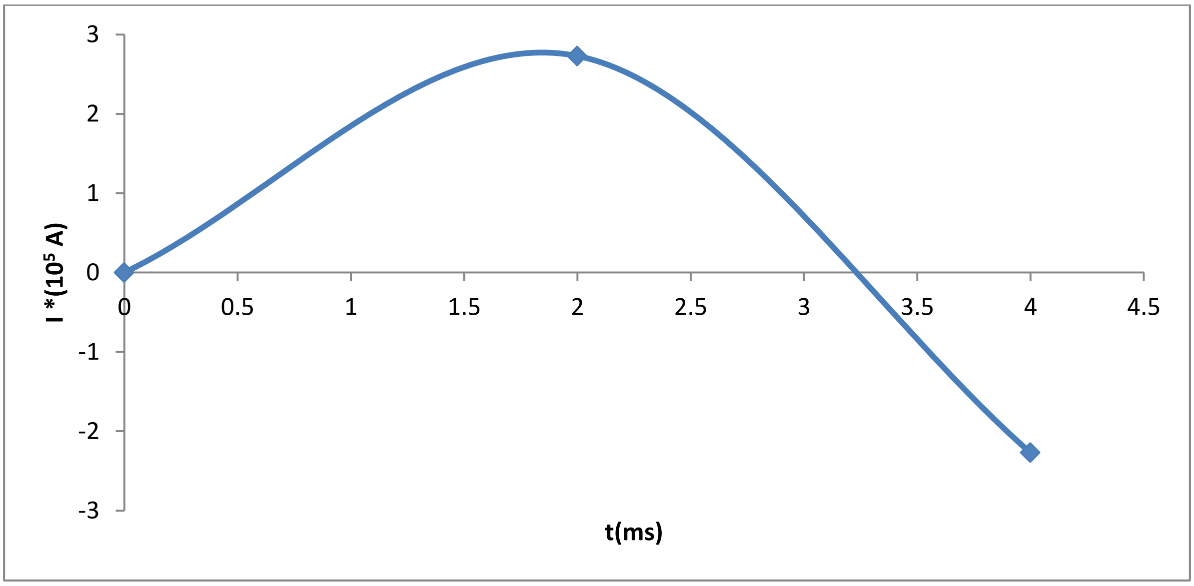

Figure (7):The current as a function of time with condition

From figure (7) get (272.79 kA) of discharge current and this is a good agreement with ref. [11]. This means if we want to get best or high discharge current, we must use small value of inductance and high value of capacitance and time in (ms) because high capacitance needs more time than microsecond to charge and create the discharge process.Therefore,the important parameters to get the high value of the discharge current are time, inductance, and capacitance selected from large data and they really affect on the electro thermal gun building.

Then evaluating the most important discharge parameters by changing the radius and length of the capillary tube we get the best length and radius (150mm, 3.5mm) respectively affected on the design of the electrothermal accelerator to get best discharge process subsequently high temperetuer plasma.

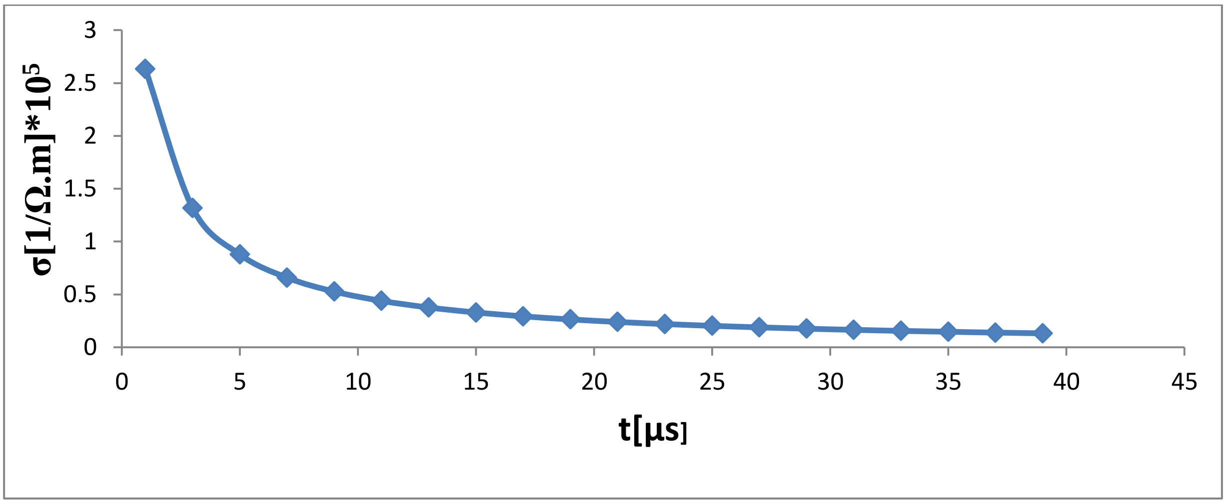

Figure (8):The conductivity verses with time

It isseen from figure (8) the plasma conductivity decreasing with time increasing because the conductivity for any gas must decrease after small time.

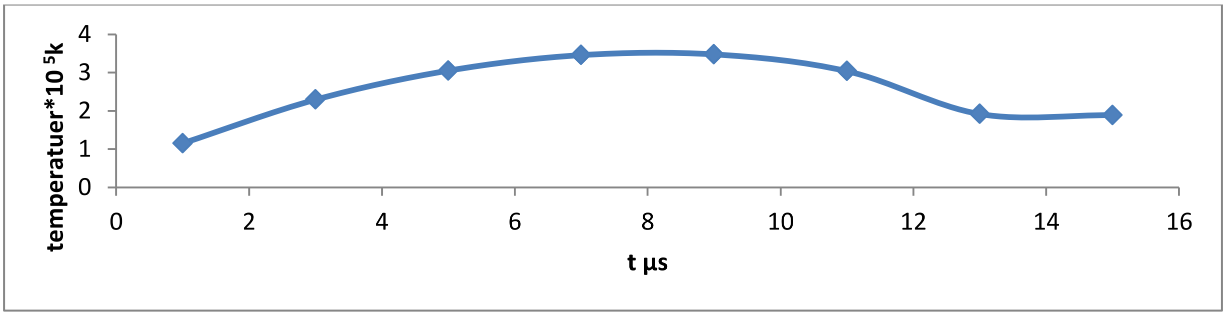

Figure (9): Temperature verses time

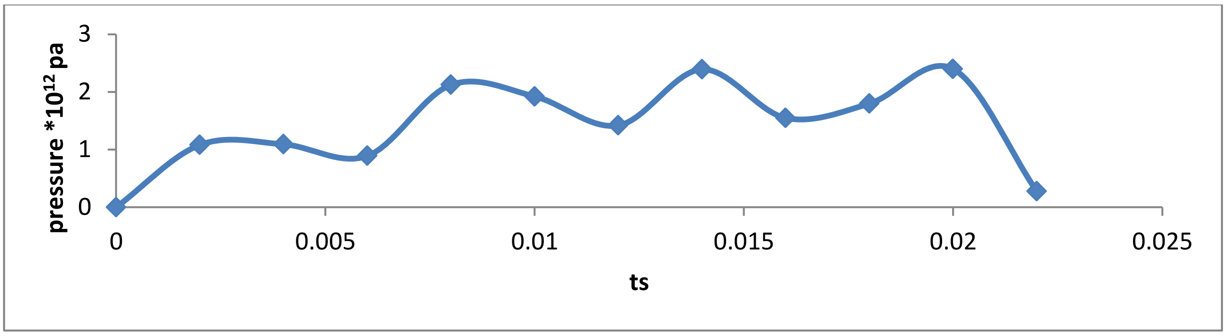

Figure (10):Pressure verses time agree with [11]

From figures (8, 9, 10) it can be noticed that the temperature and pressure in the first pulse increasing with time directly.

Conclusion

The theoretical study has been done for the discharge parameters inside the electrothermal accelerator. There are hydrodynamic limitations on the possibility to access a steady state in capillary discharges. At least as a first approximation, the axial pressure inhomogenity does not affect the calculation of the resistance, temperature, ablation and mass flow of the capillary at steady state. A number of simplifying assumption limits our results but they are valid within these limitations. Our major difference over previous treatment is the use of a more realistic equation of discharge parameters. We have shown how sensitive the electrothermal gun is to the discharge parameters and the length, radius of the capillary discharge tube. For assuming a constant temperature profile, a maximum flow velocity u_* exists for a particular capillary (length, radius and current) at steady state.If we want to get the best discharge current,we must usesmall value of inductance and high value of capacitance and time in (ms).We have established the validity of our calculation for large parameters in electrothermal gun, in order to show their general validity they should be applied in real world. Our model indicated that the capillary discharge does efficiently provide high-pressure plasma.

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