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An application of Wavelet Markov Chains Model to Study Earthquake Occurrence

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Abstract: The Arabian Plate, which is colliding with the Iranian (Eurasian) Plate, is where Iraq is located in the northeastern corner. An active Zagros seismic belt is formed by active seismicity at the interaction between the two plates. The research aims study the transition probability between the states of earthquake occurrence and estimate earthquake risk states. Use wavelet Markov chain model which is modern probability theory studies random processes for which the knowledge of previous outcomes influences predictions for future experiments. The data obtained from the Earth Scope website during the year (January 2013 to November 2022). The results show that after 115 months, the chance of an earthquake occurrence not being felt or being felt rarely is (0.009). While the chance of an earthquake occurrence being felt slightly by some people is (0.620), the chance of an earthquake occurrence being felt frequently by people is (0.124), and the chance of an earthquake occurrence being felt by the majority of people in the affected area is (0.237). Everyone believes that the last chance of an earthquake occurring (causing varying degrees of damage to poorly constructed buildings) is 0.008.

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تطبيق مويجة نموذج سلسلة ماركوف لدراسة حدوث الزلازل

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المستخلص

الصفيحة العربية التي تصطدم بالصفيحة الإيرانية (الأوراسية) هي المكان الذي يقع فيه العراق في الزاوية الشمالية الشرقية. يتكون حزام زاغروس النشط من النشاط الزلزالي النشط عند التفاعل بين الصفيحتين. يهدف البحث إلى دراسة احتمالية الانتقال بين حالات حدوث الزلازل وتقدير حالات خطورة الزلازل. وتم استخدام مويجة نموذج سلسلة ماركوف والذي يعتبر كونه نظرية احتمالية حديثة، يدرس العمليات العشوائية التي من أجلها تؤثر معرفة النتائج السابقة على تنبؤات التجارب المستقبلية. و تم الحصول على البيانات من موقع Earth Scope للفترة (يناير 2013 إلى نوفمبر 2022)، وتم التوصل الى أنه بعد 115 شهرًا، نادرًا ما تكون فرصة حدوث زلزال غير محسوس أو محسوس (0.009)، في حين أن فرصة حدوث زلزال الحدوث الذي يشعر به بعض الناس بشكل طفيف هو (0.620) ، واحتمال حدوث الزلزال الذي يشعر به الناس بشكل متكرر هو (0.124)، وفرصة حدوث الزلزال الذي يشعر به غالبية الناس في المنطقة المتضررة هي (0.237). يعتقد الجميع أن الفرصة الأخيرة لحدوث زلزال (يسبب درجات متفاوتة من الأضرار للمباني سيئة التشييد) هي 0.008.

الكلمات المفتاحية: سلاسل ماركوف، الزلزال، الموجة الصغيرة، الحالة الثابتة، مصفوفة احتمالية الانتقال (T.P.M.).

1-1. Introduction

An essential mathematical tool for stochastic processes is the Markov chain. The Markov property, or, to put it another way, the fact that some estimates about random processes can be made simpler by considering the future as no dependent on the past, given the current state of the process, is the fundamental idea. This is used to make future state forecasts for stochastic processes easier to understand. This study will start with a succinct introduction, move on to the analysis, and then offer some recommendations for further reading. The analysis will define Markov chains, describe the many forms of Markov chains, and give instances of how they are used in finance (Borucka, A. ,2018:3-10)

1-2. Markov chains (M.C.): Random process needs the use of Markov chains. They are extensively used throughout many academic fields. A random process known as a Markov chain satisfies the Markov property, which asserts that future states depend on the present rather than the past. So, without the need for further information about the process's earlier stages, the best prediction of the process' future can be made based just on

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

its current state. The complexity of such a process permits for a large reduction in the number of factors required for study. (Abba Auwalu, L. B. ,2013:1-10)

$$P_{ij} = P\{Y_{n+1} = j, |Y_0 = i_0, Y_1 = i_1, \dots, Y_n = i_n\}$$
$$= P\{Y_{n+1} = j, |Y_n = i_0\} \quad , i_0 \geq 0 \ , j \geq 0 \qquad (1)$$

$P\{Y_{n+1} = j|Y_n = i_0\}$

is called the 1-step transition probability matrix. The procedure is known as a homogeneous Markov chain if $P\{Y_{n+1} = j|Y_n = i_0\}$ is not dependent on time and the Markov chain has stationary transition probability. A nonhomogeneous Markov chain is the term for the process in the absence of this. (ÇELEBİOĞLU, S. Ü. ,2011: 263-274).

1-3. Markov property: Future states of the process are independent of any previous states and only depend on the current state.

1-4. Matrix of Transition Probability (T.P.M.): A homogeneous Markov chain, $\{Y(n), n \geq 0\}$ with discrete state space S Then

$$P_{ij} = P\{Y_{n+1} = j|Y_n = i_0\} \quad , i_0 \geq 0 \ , j \geq 0$$

Regardless of the value of n, the definition of a T.P.M. is

$$P_{ij} = \begin{bmatrix} P_{00} & P_{01} & P_{02} \cdots \\ P_{10} & P_{11} & P_{12} \cdots \\ P_{20} & P_{21} & P_{22} \cdots \\ \vdots & \vdots & \vdots \end{bmatrix} \qquad (2)$$

Where

$$P_{ij} \geq 0 \quad , \quad \sum_{j=0}^{\infty} P_{ij} = 1 \text{ (sum of each row } = 1), i_0 = 0,1,2, \dots.$$

(Borucka, A. ,2018:3-10)

1-5. Higher Order Markov chain: Let P_{ij} denote the T.P.M of the Markov chain, matrix power of $P_{ij}^{(2)}$ are defined by

$$P_{ij}^{(2)} = \sum_{k=0}^{\infty} P_{ik} P_{kj} \qquad (3)$$

Similarly, matrix power of $P_{ij}^{(3)}$ are defined by

$$P_{ij}^{(3)} = \sum_{k=0}^{\infty} P_{ik} P_{kj}^{(2)} \qquad (4)$$

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

We compute $P_{ij}^{(n+m)}$ by taking matrix powers.

$$p_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^{(n)} P_{kj}^{(m)} \qquad (5)$$

The Chapman-Kolmogorov equation is the name given to equation (5). (ÇELEBİOĞLU, S. Ü., 2011: 263-274)

1-6. The Markov chain's probability distribution:

Let $P_{i0}^{(n)} = P(Y_n = i_0)$ and

$$P_{i0}^{(n)} = [P_0^{(n)} \quad P_1^{(n)} \quad P_2^{(n)} \quad \ldots] \qquad (6)$$

Where

$$\sum_i P_{i0}^{(n)} = 1$$

$P_{i0}^{(n)}$ is known as the initial-state probability vector P^n , and after n transitions, it is known as the probability distribution. (D, L. F. ,2009)

$$P^n = P^0 P^n \qquad (7)$$

Which indicates that the probability distribution of a homogeneous Markov chain is completely determined by 1-step T.P.M. and the initial-state probability vector (Qurat-ul-Ain Sultan, K. F. ,2019:100-113)

1-7. Stationary Distributions: Let P_{ij} be the T.P.M. of a homogeneous Markov chain $\{Y(n)\,, n \geq 0\}$. If there exists a probability vector $\widehat{P}$ such that

$$\widehat{P} P_{ij} = \widehat{P} \qquad (8)$$

$\widehat{P}$ is called a stationary distribution for the Markov chain. Its components sum to one.

1-8. Markov's classification states: A state b is **accessible** from a ($a \to b$) if there is a walk in the graph from a to b.

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

Two distinct states a and b **communicate** ($a \leftrightarrow b$) if a is accessible from b and b is accessible from a.

A class C of states is a **non-empty set** of states such that each $a \in C$ communicates with every other state $b \in C$ and communicates with no $b \notin C$.

For finite-state Markov chains, a recurrent state is a state a that is accessible from all states that are accessible from a (a) is recurrent if $a \rightarrow b$ implies that $b \rightarrow a$.

A transient state is a state that is not recurrent.

For finite-state Markov chains, either all states in a class are transient or all are recurrent. The period of a state aa, denoted $d(a)$, is the **greatest common divisor** (gcd) of those values of n for which $P_{aa}^{n} > 0$. If the period is 1, the state is **aperiodic**, and if the period is 2 or more, the state is **periodic**. (Qurat-ul-Ain Sultan, K. F. ,2019:100-113)

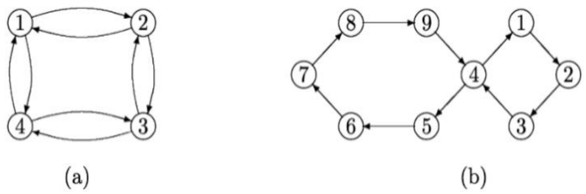

Figure (1): Periodic Markov Chains

All states in the same class of a Markov chain, whether it has a finite or countable infinite number of states, have the same period. (Mhamad et al., 2018:495-507)

1-9. Wavelets: A wavelet is a waveform with a non-zero norm and an average value of zero, which effectively limits its duration. (Ali, T. H., & Saleh, D. M., 2022: 920-939).

Many exciting signals and images have piecewise smooth behavior that is broken up by transients. Consonant-encoding brief bursts are followed by steady-state fluctuations that represent vowels in speech signals. Images in nature have limits. Transient behavior, which is characteristic of sudden changes in economic conditions such as ups and downs, can be seen in monetary time series. Wavelet bases, in contrast to the Fourier basis, are skilled at sparsely describing piecewise regular signals and pictures, including transient behavior. (C, S. B. et al., 1998)

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

Compare sine waves, the foundation of Fourier analysis, with wavelets. Sinusoids span the entire time range, from $(-\infty, \infty)$. Sinusoids are smooth and predictable, but wavelets are typically erratic and asymmetric.

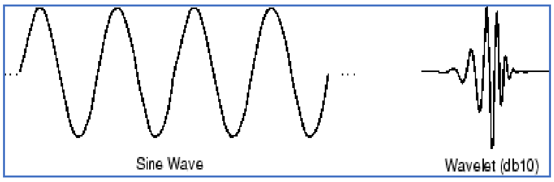

Figure (2): Show sine wave and Wavelet

In Fourier analysis, a signal is divided into sine waves with different frequencies. The splitting up of a signal into scaled and shifted variations of the original (or mother) wavelet is what is known as wavelet analysis. It is intuitively obvious from looking at images of sine waves and wavelets that signals with sharp changes may be better examined using an irregular wavelet rather than a smooth sinusoid. It also makes logical that wavelets with a local extent can better describe local features. This is demonstrated in the example that follows for a straightforward signal that consists of a sine wave with a discontinuity.

1-10. Wavelet Properties: The following is a description of wavelets' key characteristics: (Hamad, A. S. ,2010)

1. The wavelet function $\varphi(.)$has a zero average on interval $(-\infty,\infty)$, which means that

$$\int_{-\infty}^{\infty} \varphi(x)dx = 0 \qquad (9)$$

This means that the wavelet should have an oscillatory waveform. $\varphi(x)$ should be localized.

2. $\varphi(.)$ is square integrates to unity.:

$$\int_{-\infty}^{\infty} \varphi^2(x)dx = 1 \qquad (10)$$

This requirement guarantees either compact support for the wavelet function or that the function has a finite length.

3. n-vanishing moments be in the wavelet function.

$$\int_{-\infty}^{\infty} x^k \varphi(x)dx = 0 \qquad k = 0,1,\dots,N \qquad (11)$$

The signal was accurately represented by the wavelet finite sum.

4. The integral of $\phi(.)$ one interval $(-\infty,\infty)$ is one:

$$\int_{-\infty}^{\infty} \emptyset(x)dx = 1 \qquad (12)$$

The scaling function's area under the curve is normalized to one.

1-11. Haar Wavelet: Haar wavelet is the simplest wavelet, which was first presented by Alfred Haar in 1909, is frequently wavelet of choice for people learning about wavelet. Reason about the restrictions on v_k for N = 2 to produce a Haar wavelet. (K, W., & Xu, D. Z., 2005: 1973-1975).

The stability condition applies $v_0 + v_1 = 2$, while the accuracy condition implies $v_0 - v_1 = 0$, and the orthogonality gives $v_0^2 + v_1^2 = 2$. Then a unique solution that exists is $v_0 = v_1 = 1$, using equation $\sum_{k=0}^{N-1} v_k^2 = 2$, then

$$\emptyset(x) = \emptyset(2x) + \emptyset(2x - 1) \qquad (13)$$

The scaling function is satisfied by a box function

$$B(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & O.W. \end{cases} \qquad (14)$$

If define the function φ as $\varphi(x) = \varphi(2x) - \varphi(2x - 1)$ Then the Haar wavelet obtained,

$$\varphi(x) = \begin{cases} 1 & 0 < x \leq \frac{1}{2} \\ -1 & \frac{1}{2} < x \leq 1 \\ 0 & O.W \end{cases} \qquad (15)$$

$\varphi(x)$ is the Haar Wavelet, and the function is $\emptyset(x)$ the Haar scaling function. Figure (3) provides an explanation.

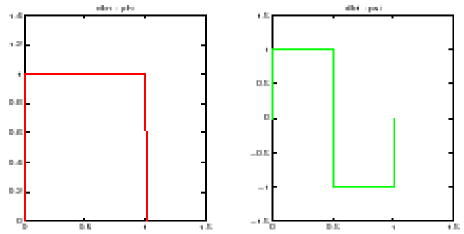

Figure (3): The Haar Wavelet and the Scaling Function

1-13. Daubechies Wavelet: Daubechies proposed two varieties of wavelets that are compactly supported and have a specified degree of smoothness (1988, 1992). These wavelets are also known as Daubechies very phase wavelets and least-symmetric wavelets. The compactly supported wavelets of the Daubechies are constructed. The equations for the masks are if N = 4.

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

$$v_0 + v_1 + v_2 + v_3 = 2$$
$$v_0 - v_1 + v_2 - v_3 = 0$$
$$-v_1 + 2v_2 - 3v_3 = 0$$
$$v_0 v_2 + v_1 v_3 = 0$$
$$v_o^2 + v_1^2 + v_2^2 + v_3^2 = 2 \quad (16)$$

Then a unique solution for the equations is exists:

$$v_0 = \frac{1+\sqrt{3}}{4}, v_1 = \frac{3+\sqrt{3}}{4}, \; v_2 = \frac{3-\sqrt{3}}{4}, \; v_3 = \frac{1-\sqrt{3}}{4} \quad (17)$$

As illustrated in figure (4), the appropriate wavelet is the Daubechies (db2) wavelet, which is supported on the interval [0, 3]. (Hamad, A. S. ,2010)

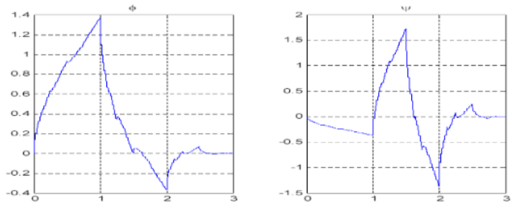

Figure (4): The db2 wavelet function

1-14. Thresholding: Thresholding, which divides the wavelet coefficient into two sets, one of which represents signal and the other noise, is the most basic non-linear wavelet de-noising approach. The wavelet coefficient thresholds can be applied in a variety of ways and according to a variety of rules, including:

1-14-1. Hard Thresholding: Donoho and Johnstone's simplest thresholding method, known as "hard thresholding," translates the phrase "keep or kill." Hard thresholding is a straightforward technique that was used to implement wavelet denoising. The wavelet coefficient is a vector $\omega_n^{H(t)}$ containing element.

$$\omega_n^{H(t)} = \begin{bmatrix} 0 & \text{if} & |\omega_n| \leq \delta \\ \omega_n & \text{if} & |\omega_n| > \delta \end{bmatrix} \quad (18)$$

In this case, coefficients more than are preserved, whilst those less than or equal to are removed or set to zero. Thus, hard thresholding is not continuous mapping in operation. (Hussein, M. M. ,2022:32-46)

1-14-2. Soft Thresholding: Donoho and Johnstone also proposed "soft thresholding of the wavelet coefficient," which is another generally used technique for wavelet de-noising.

$$\omega_n^{(st.)} = \text{sign}\{\omega_n\}(|\omega_n| - \delta) + \quad (19)$$

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

Where

$$\text{Sign}\{\omega_n\} = \begin{bmatrix} +1 & \text{if} & \omega_n > 0 \\ 0 & \text{if} & \omega_n = 0 \\ -1 & \text{if} & \omega_n < 0 \end{bmatrix} \qquad (20)$$

and

$$(|\omega_n| - \eta)_+ = \begin{bmatrix} (|\omega_n| - \delta) & \text{if } (|\omega_n| - \delta) \geq 0 \\ 0 & \text{if } (|\omega_n| - \delta) < 0 \end{bmatrix} \qquad (21)$$

All coefficients greater than threshold are reduced by the same amount as threshold, and coefficients less than threshold are set to zero. Hence, the continuous mapping of the soft threshold is shown in figure 5. Which of these two threshold rules should be applied in small samples depends on the desired characteristics of the generated estimates. Although the hard threshold estimates are more likely to have a lesser bias and mean square error, the soft threshold estimates typically have a smaller bias. (Hussein, M. M. ,2022:32-46)

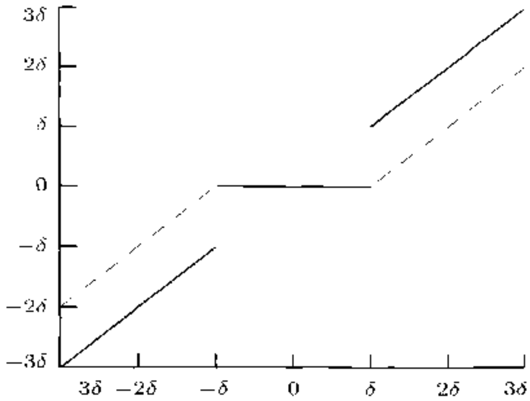

Figure (5): The Hard and Soft Thresholding Functions

Figure (5) shows "hard thresholding" as solid lines and "soft thresholding" as dashed lines. Figure (5) depicts "coefficients before thresholding" as a horizontal line and "threshold coefficients" as a vertical line (soft threshold).

1-15. Result and Discussion

1-15-1. Data and Methodology: In this paper, we used the time series data of the earthquake occurrence in Iraq during the period (January 2013– November, 2022) per month. The data were obtained from the website (Earth Scope) https://ds.iris.edu/ieb/index.html, and wavelet Markov chains will be relied upon to predict the probabilities and transitions of earthquake occurrence.

Table (1): Represent Original data / Richter Scale

Year	Months	Degrees of earthquake /Richter Scale
2013	1	3.9
.	2	3.7
.	3	4.6
.	.	.
.	.	.
.	.	.
2022	9	3.078
.	10	2.783
.	11	2.713

In the beginning, we use the wavelet to detect the incidence of noise in the data,

Table (2): Represent data transform / Wavelet(db1-1)

Year	Months	Degrees of earthquake /Richter Scale
2013	1	3.80
.	2	3.80
.	3	4.39
.	.	.
.	.	.
.	.	.
2022	9	2.9305
.	10	2.9305
.	11	2.7130

The Daubechies wavelet (db1-1) is used to reduce noise from the data, after which the converted data is examined.

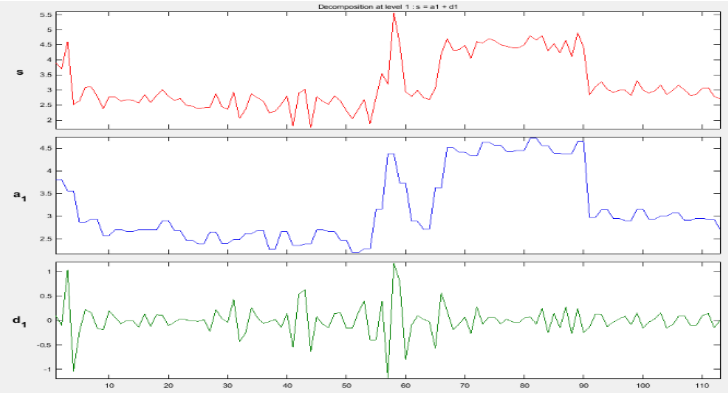

Figure (6): Wavelet 1-D Decomposition

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

Figures 6 and 7 show noise detection using wavelets, with black indicating no noise and white indicating noise in the earthquake occurrence data.

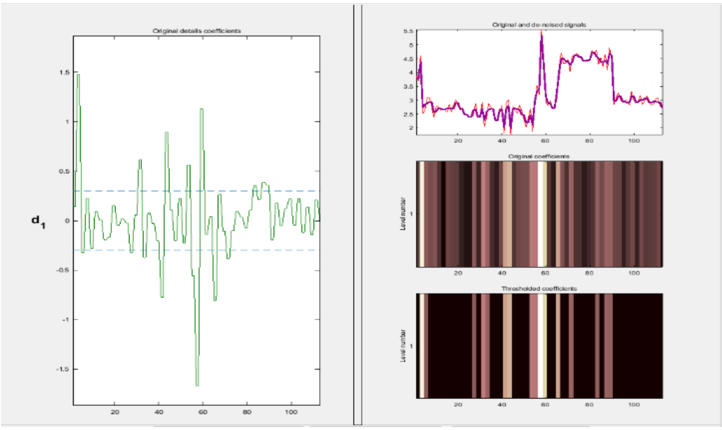

Figure (7) Daubechies1-level-1 for earthquake occurrence

and we find the mean square error of the data before and after using the wavelet, and the results are as follows:

Table (3): Comparison between Markov chain before and After using Wavelet

Measure	Markov Chain Before Using wavelet	Markov Chain After Using Wavelet (db1-1)	% Noise
Mean Square Error (MSE)	0.332	0.278	0.054

The table above shows that the Mean Square Error (MSE) in the Markov chain after applying the wavelet is lower (0.278) than it was in the Markov chain without using the wavelet.

1-15-2. Construction of Markov chain Model using Wavelet: To create the Markov chain model's reliability as a prediction model and satisfy other study objectives, a Markov chain model for forecasting the trend of earthquake occurrence must be constructed. The following stages are used to construct a Markov chain model: After handling and conducting an initial analysis on the data set for the specified time period, it is assumed that the earthquake occurred in one of five states:

I: Micro earthquakes, not felt, or felt rarely by sensitive people

II: Slightly felt by a few to many people,

III: often felt by at least some people, but very rarely causing damage,

IV: The earthquake was felt by a large number of people.

V: Can cause moderate to major damage to poorly constructed buildings.

The Markov model's state space can be expressed as S. (I, II, III, IV, V). The initial state vector, sometimes referred to as the initial state distribution, can be described as follows: $P_0 = [P_1(i_1),\ P_2(i_2),\ P_3(i_3),\ P_4(i_4),\ P_5(i_5)]$

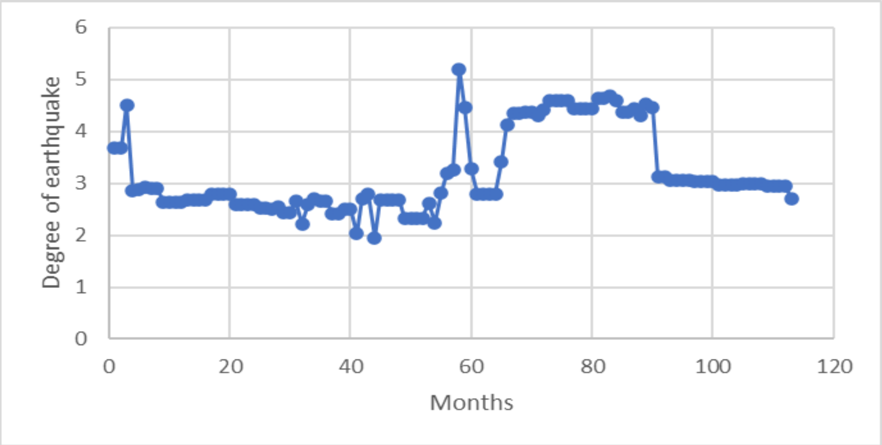

Figure (8): Data transform and Wavelet-Richter scale for earthquake occurrence

Through the figure, we notice that the degree of the earthquake's occurrence ranged between (2–5) degrees on the Richter scale during the period 2013–2022. The degree 2 means that it is felt slightly by a few people, and the degree 5 means that it causes moderate to major damage to poorly constructed.

Table (4): Data-Transform Wavelet of earthquake occurrence

States	I	II	III	IV	V
Frequencies	1	67	16	27	1

The initial state probability can be calculated by finding out the probability of each state in the following manner:

Using Wavelet:

$P_1(i_1) = \frac{1}{112} = 0.009$, $P_2(i_2) = \frac{67}{112} = 0.598$, $P_3(i_3) = \frac{16}{112} = 0.143$,

$P_4(i_4) = \frac{27}{112} = 0.241$ $P_5(i_5) = \frac{1}{112} = 0.009$

Thus, the initial state vector for the earthquake occurrence is

Using db 1-1 wavelet: $P_0 = [0.009\ \ 0.598\ \ 0.143\ \ 0.241\ \ 0.009]$

1-15-3. Derivation of Transition Probability Matrix: There are various methods for obtaining a Markov chain's transition probability matrix. It is derived primarily empirically, but in some instances, such as this study, it is also derived by examining and interpreting historical data that is thought to behave like a Markov chain. For each of our three states, a transition probability matrix will be mathematically expressed as follows:

Table (5): States Transition Using Wavelet

0	1	0	0	0
1	64	2	0	0
0	2	11	2	1
0	1	2	24	0
0	0	0	1	0

Here is how the transition probabilities matrix will be determined:

Table (6): Wavelet-based Transition Probability Matrix

0.000	1.000	0.000	0.000	0.000
0.015	0.955	0.030	0.000	0.000
0.000	0.125	0.688	0.125	0.063
0.000	0.037	0.074	0.889	0.000
0.000	0.000	0.000	1.000	0.000

Given that there is zero probability, the transition probability found above indicates that there exist multiple classes of states. As the Markov chain contains several class states that are members of a single class, the transition probability demonstrates that it is not irreducible.

1-15-4. Chi-Square Test: A chi-square test was used to determine the transition matrix's goodness-of-fit. We derived the following observed and estimated frequencies from the data:

H_0: The estimated transition matrix fits the data.

H_1: The estimated transition matrix does not fit the data.

Table (7): Expected Frequency before and after using Wavelet

0.0	0.6	0.1	0.2	0.0
0.6	40.7	9.0	16.2	0.6
0.1	9.7	2.1	3.9	0.1
0.2	16.4	3.6	6.5	0.2
0.0	0.6	0.1	0.2	0.0

From the chi-square analysis, we have

Table (8): Result of chi-square test before and after using Wavelet

After using Wavelet	Value	d.f.	Asymptotic Significance (2-sided)
Pearson Chi-Square	151.181	16	0.000

Doi: www.doi.org/10.25130/tjaes.19.61.2.18

We can see from the table above that the p-value of the chi-square test is (0.000) compared to the value of (α=0.05), We accept the H_0 because the P-value of the test is less than the level of significance, which means that the transition probability matrix fits the data.

1-15-5. Calculating state probabilities for forecasting the next month earthquake occurrence: The initial probability matrix and the transition probability matrix can be multiplied to obtain the state probabilities for the Markov chain model. It can be written as follows in mathematical notation:

$$P_1 = P_0 \times P_{T.P.M.}$$

.

.

.

$$P_{n+1} = P_n \times P_{T.P.M.}$$

The state probabilities of earthquake occurrence for the 113 moths will be:

$$P_{113} = P_0 \times P_{T.P.M.}$$

$$= [0.009 \quad 0.598 \quad 0.143 \quad 0.241 \quad 0.009] \begin{bmatrix} 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.015 & 0.955 & 0.030 & 0.000 & 0.000 \\ 0.000 & 0.125 & 0.688 & 0.125 & 0.063 \\ 0.000 & 0.037 & 0.074 & 0.889 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix}$$

$$= [0.009 \quad 0.607 \quad 0.134 \quad 0.241 \quad 0.009]$$

Which indicates:

- The probability that the earthquake occurrence in the 113 months will remain unchanged is 0.009 (Micro earthquakes, not felt, or felt rarely by sensitive people)
- The probability that the earthquake occurrence in the 113 months will increase is 0.607(Felt slightly by few to many people)
- The probability that the earthquake occurrence in the 113. months will reduce is 0.134(often felt by at least some people, but very rarely causes damage)
- The probability that the earthquake occurrence in the 113 months will remain same is 0.241 (Many people to everyone feel the earthquake)
- The probability that the earthquake occurrence in the 113 months will remain unchanged is 0.009 (it can cause moderate to major damage to poorly constructed buildings).

The state probabilities of earthquake occurrence for the 114· month will be:

$P_{113} = P_{112} \times P_{T.P.M.}$

$$= [0.009\ \ 0.607\ \ 0.134\ \ 0.241\ \ 0.009]\begin{bmatrix} 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.015 & 0.955 & 0.030 & 0.000 & 0.000 \\ 0.000 & 0.125 & 0.688 & 0.125 & 0.063 \\ 0.000 & 0.037 & 0.074 & 0.889 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix}$$

$$= [0.009\ \ 0.614\ \ 0.128\ \ 0.240\ \ 0.008]$$

The state probabilities of earthquake occurrence for 115 months will be:

$P_{113} = P_{112} \times P_{T.P.M.}$

$$= [0.009\ \ 0.614\ \ 0.128\ \ 0.240\ \ 0.008]\begin{bmatrix} 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.015 & 0.955 & 0.030 & 0.000 & 0.000 \\ 0.000 & 0.125 & 0.688 & 0.125 & 0.063 \\ 0.000 & 0.037 & 0.074 & 0.889 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix}$$

$$= [0.009\ \ 0.620\ \ 0.124\ \ 0.237\ \ 0.008]$$

Which indicates:

- ❖ The probability that the earthquake occurrence in the 115 months will remain same is 0.009 (Micro earthquakes, not felt, or felt rarely by sensitive people)
- ❖ The probability that the earthquake occurrence in the 115 months will increase is 0.620(Felt slightly by few to many people)
- ❖ The probability that the earthquake occurrence in the 115 months will reduce is 0.124(often felt by at least some people, but very rarely causes damage)
- ❖ The probability that the earthquake occurrence in the 115 months will reduce is 0.237 (Many people to everyone feel the earthquake)
- ❖ The probability that the earthquake occurrence in the 115. months will remain the same is 0.008 (Can cause moderate to major damage to poorly constructed buildings)

1-15-6. Long Run Behavior of Earthquake Occurrence: The higher-order transition probability matrix given below determines the prediction of long-run behavior of earthquake occurrence:

$$P_{EO}^{2}=\begin{bmatrix}0.015 & 0.955 & 0.030 & 0.000 & 0.000\\ 0.014 & 0.931 & 0.049 & 0.004 & 0.002\\ 0.002 & 0.210 & 0.486 & 0.260 & 0.043\\ 0.001 & 0.077 & 0.018 & 0.800 & 0.005\\ 0.000 & 0.037 & 0.074 & 0.889 & 0.000\end{bmatrix}$$

$$P_{EO}^{3}=\begin{bmatrix}0.014 & 0.931 & 0.049 & 0.004 & 0.002\\ 0.014 & 0.910 & 0.062 & 0.011 & 0.003\\ 0.003 & 0.273 & 0.360 & 0.335 & 0.031\\ 0.001 & 0.119 & 0.143 & 0.730 & 0.007\\ 0.001 & 0.077 & 0.118 & 0.800 & 0.005\end{bmatrix}$$

$$P_{EO}^{4}=\begin{bmatrix}0.014 & 0.910 & 0.062 & 0.011 & 0.003\\ 0.014 & 0.891 & 0.071 & 0.021 & 0.004\\ 0.004 & 0.321 & 0.281 & 0.374 & 0.023\\ 0.002 & 0.160 & 0.156 & 0.674 & 0.009\\ 0.001 & 0.119 & 0.143 & 0.730 & 0.007\end{bmatrix}$$

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$$P_{EO}^{32}=\begin{bmatrix}0.011 & 0.710 & 0.107 & 0.168 & 0.007\\ 0.011 & 0.709 & 0.108 & 0.169 & 0.007\\ 0.010 & 0.657 & 0.116 & 0.217 & 0.007\\ 0.009 & 0.635 & 0.118 & 0.234 & 0.008\\ 0.009 & 0.630 & 0.119 & 0.238 & 0.008\end{bmatrix}$$

A matrix with all of its rows convergent to the near same probabilities is revealed by raising the transition probability matrix to a higher power, i.e., 32.

$$\lim_{n\to\infty} p^{n} = P = [0.009 \quad 0.630 \quad 0.119 \quad 0.238 \quad 0.008].$$

This is the stationary distribution that shows that after 32 months, for 112 months of earthquake occurrence, it converges to its steady state distribution. The steady state distribution shows the following information about the earthquake occurrence months coming up in the future:

- ❖ The chances that the earthquakes, not felt, or felt rarely by sensitive people, will approximately remain unchanged in the future are 0.009.
- ❖ The chances of the earthquake, which was felt slightly by a few to many people, increasing in the future are 0.630.
- ❖ The chances that earthquakes, which are often felt by at least some people, but very rarely cause damage, will decrease in frequency in the future are 0.119.
- ❖ The likelihood of more people feeling the earthquake in the future is 0.238.
- ❖ The chances that the earthquake can cause moderate to major damage to poorly constructed buildings, will decreasing in the future at 0.008

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If we assume that our chain begins with its initial vector, or that the distribution shown above is the steady state distribution, then we may verify that it is.

P_0=[0.009 0.598 0.143 0.241 0.009]

And we want to know the distribution for the 33 months, which we will see.

P_{33}=[

$$[0.009\ \ 0.598\ \ 0.143\ \ 0.241\ \ 0.009]\begin{bmatrix} 0.011 & 0.710 & 0.107 & 0.168 & 0.007 \\ 0.011 & 0.709 & 0.108 & 0.169 & 0.007 \\ 0.010 & 0.657 & 0.116 & 0.217 & 0.007 \\ 0.009 & 0.635 & 0.118 & 0.234 & 0.008 \\ 0.009 & 0.630 & 0.119 & 0.238 & 0.008 \end{bmatrix}$$

P_{33}=[0.010 0.683 0.112 0.192 0.007]

is the near same as the steady state distribution i.e., after the chain has reached its steady distribution, despite its initial vector, the distribution for the chain remains the same, which is the steady state distribution of the chain. This also validates our assumption of taking the Markov chain for earthquakes as ergodic, i.e., an irreducible, positive recurrent, aperiodic, time homogenous Markov chain is called an ergodic Markov chain.

1-15-7. Determination of expected recurrence time: Steady state distribution can also help us compute the expected return time μ_{jj} i.e., the time the chains take to visit j once it left j. The relation between limiting probabilities and expected return time is given by $\mu_{jj} = \frac{1}{P_j}$

Earthquakes is ergodic Markov chain, i.e., it has a limiting distribution, so we can calculate a expected recurrence time for the chain. Expected return times give information about the expected stay time of an earthquake in all five states:

Table (9): The expected return time

States	P_j	Expected return time
I	0.010	100.0
II	0.683	1.5
III	0.112	8.9
IV	0.192	5.2
V	0.007	142.8

The expected return time indicates that the earthquakes will not be felt, or will be felt only infrequently by sensitive people, for approximately 100 months. The expected return time indicates that the earthquake's chain will be felt slightly by a few to many people in approximately two months. The expected time signifies that the earthquakes, often felt by at least some people, but very rarely causing damage, will occur in approximately 9 months. The expected time indicates that many people will feel the earthquake, but it will very rarely cause damage state in 5 months. The expected time indicates that the earthquake causes moderate to major damage to poorly constructed buildings, but that damage states are extremely rare in 143 months. This also shows that the earthquake is very volatile and there is no stationary.

1-16. Conclusions and Recommendations:

1-16-1 Conclusions: The researcher has the following conclusions and recommendations based on the analysis:

1. In this study, a Markov chain model of first order was applied to earthquake occurrence and proposed to assess the accuracy of Markov chain model prediction
2. During this time period, there were 1(0.9%) earthquake occurrence of the first type, 67(59.8%) of the second type, 16(14.3%) of the third type, 27(24.1%) of the fourth type, and 1(0.9%) of the fifth type.
3. We can see that the first and fifth states are closed state according to the transition probability matrix. The probability of switching from the second to the first, second, and third states are (0.015), (0.955), and (0.030), respectively. The probability of switching from the third to the second, third, fourth, and fifth states are (0.125), (0.688), (0.125), and (0.063), respectively. The probability of switching from the fourth state to the second, third, and fourth states are (0.037), (0.074), and (0.889), respectively.
4. Wavelet filters could be used to solve the problem of noise or outliers while calculating the Markov chain model.
5. When it came to estimating Markov chain Model, the wavelet filter (Daubechies Wavelets db1-1)
   was the best in terms of types and levels of wavelet.
6. The results verified that the dataset for earthquake occurrence had Markovian properties and displayed ergodic, which was supported by the

convergence of the transition probability matrix to a steady state distribution. This verified the capability of using the Markov chain model to model earthquake occurrence.

7. The probability of micro earthquake occurrence, not felt, or felt rarely by sensitive people in the 115 months will remain same is 0.009. While the probability that the earthquake occurrence felt slightly by few to many people in the 115 months will increase is 0.620. The probability that the earthquake occurrence often felt by at least some people. But very rarely causes damage in the 115months will reduce is 0.124, the probability that the many people to everyone feel the earthquake occurrence in the 115 months will remain same is 0.237, The probability that the earthquake occurrence causes moderate to major damage to poorly constructed buildings in the 114 months will reduce is 0.008.
8. The expected time to reach the (first, second, third, fourth, and fifth) states is approximately (100, 1.5, 8.9, 5.2, and 142.8) months, respectively.

1-16-2. Recommendations: Based on the conclusions, the following are some recommendations for this study.

1. Markov chains are the only predictive tool that does not require knowledge of the elements influencing the phenomenon under study; thus, researchers must focus on the topic of using them to make predictions.
2. In this type of research, other statistical techniques such as the hidden Markov model and the gray model are used.

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