CHAOTIC COMPRESSIVE SENSING OF TV-UHF BAND IN IRAQ USING CHEBYSHEV GRAM SCHMIDT SENSING MATRIX

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Abstract- Cognitive radio (CR) is a promising technology for solving spectrum sacristy problems. Spectrum sensing is the main step of CR. Traditional sensing techniques sense one channel at a time while sensing the wide band spectrum which is required in 5G communications produces more challenges. Compressive sensing (CS) is a technology used as a spectrum sensing technique in CR to solve these challenges. CS consists of three stages: sparse representation, encoding and decoding. In encoding stage measurement (sensing) matrix are required, and it plays an important role in the performance of CS. Selecting the best sensing matrix leads to low reconstruction error with high compression. The design of an efficient sensing matrix requires achieving low mutual coherence or satisfying the Restricted Isometry Property (RIP) with high probability. In the decoding stage, the recovery algorithm is applied to reconstruct a sparse signal from a few measurements. In this paper a new chaotic matrix is proposed based on the Chebyshev map and modified gram Schmidt (MGS). The CS-based proposed matrix is applied for sensing real TV signals as a primary user (PU). The proposed system is tested under two types of recovery algorithms: CoSaMP and SWOMP. The performance of the CS-based proposed matrix is measured using recovery error, mean square error (MSE) and probability of detection and evaluated by comparing it with Gaussian, Bernoulli and chaotic matrix in the literature. The simulation results show that the CS-based proposed matrix has low recovery error and high performance detection under low SNR values and has low MSE with high compression.

keywords: Cognitive radio, Compressive sensing, TVWS, CoSaMP, SWOMP, Chaotic matrix, Chebyshev map, Modified gram Schmidt.

I. Introduction

Cognitive radio (CR) is a promising technology for wireless communications. It needs the design of novel spectrum sensing techniques which have a high degree of reliability, even at low SNR [1]. Spectrum sensing can be defined as the process of primary user (PU) detection in the specific channel by the secondary user (SU) or unlicensed user to decide the channel status that is either occupied or free. The spectrum sensing process should be accurate enough to avoid harmful interference to the PU [2]. The traditional sensing techniques are energy detector, cyclostationary and matched filter. These techniques are called narrow band sensing which senses only one channel at a time. For sensing a wideband spectrum using narrow band sensing techniques, the spectrum is usually divided into multiple sub band and then they are sensed sequentially or in parallel and both ways are costly and impractical [3]. Compressive sensing (CS) is proposed to solve the challenges of wideband spectrum sensing in CR. CS is an emerging technology through which sub-Nyquist sampling rates can be achieved without the loss of important information. The signal must be sparse in some domain, then CS ensures that such a signal can be sampled at a rate lower than the Nyquist rate and still be successfully recovered [4]. CS reduces the processing time and accelerates the sensing process [5] , [6]. CS consists of three stages: sparse representation, encoding, and decoding [6] [3]. In the encoding process, a measurement (sensing) matrix is used to sample the sparse signal. The selection of the sensing matrix plays an important role in the accuracy and the processing time of the sparse reconstruction process. Hence, the design of accurate measurement matrices is of vital importance in compressive sensing.

The sensing matrix can be classified into two categories: random and deterministic [7]. In the decoding stage, the recovery (reconstruction) algorithms are applied to reconstruct a sparse signal from few measurements [8]. Recovery algorithms can be classified into three classes: convex relaxation iterative, Greedy, and Bayesian recovery [9]. Iterative greedy algorithms received significant attention due to their low complexity [10]. The most important works that used chaotic matrix are [4] and [11] where different types of 1-D chaotic map are used, although this matrix provides security from PU attacks and low complexity as compared to the random matrix but have a challenge that large sample distance is required for low coherence which requires large resources and long sensing time. In [12] the Chebyshev map is used as a chaotic matrix but also used large sample distance. In [13] measurement matrix is created using a Chebyshev map with reduced sample distance. Also, all these works do not apply to sensing real signals. The main contributions in this paper are:

1) We proposed a chaotic matrix based on Chebyshev and Gram Schmidt without using sample distance as compared with the above references in the literature.

2) applying the proposed system for sensing real TV signals, while this is not considered in the previous works.

3) The proposed matrix has low complexity since it requires storage only maps, parameters and equations as compared with the random matrix that is required to store all matrices.

4) The proposed system provides high security for sensing information from an attacker outside the network.

5) The proposed matrix has low mutual coherence and this produce low reconstruction error and gives the same performance of random matrix and better performance than matrix based logistic map in [4].

II. SENSING TV SIGNALS IN COGNITIVE RADIO

The TV white space (TVWS) technology was considered after the transition from analogue to digital to increase the capacity, without causing interference to licensed users (PU) of television channels transmitting at low-power levels and low-cost equipment. From the transition of analogue television services towards digital terrestrial television (DTT), it was identified that there are many unused frequencies (the very high frequency (VHF)/ultra high-frequency (UHF) bands) in certain areas of the world [14]. TVWS has come to be the primary spectrum for cognitive radio network (CRN) services and applications. The emergence of CRN technology and TVWS provides new access opportunities for unlicensed secondary users (SU), and since TVWS exists in the low UHF band, it offers greater coverage and increased throughput so satisfying a major 5G requirement [15].

III. SENSING MATRICES FOR COMPRESSIVE SENSING

Compressive sensing is a spectrum sensing technique in cognitive radio and it works when a signal is sparse in one domain. Since most channels in the wideband spectrum (PU signals) are free at time t, so the wideband spectrum is sparse in the frequency domain and this makes the CS is applicable for CR [3]. The encoding part of CS can be represented as:

$$Z_f = Ay_f \tag{1}$$

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where z_f denotes the $M \times 1$ measurements vector, A denotes an $M \times N$ sensing matrix, and y_f is $N \times 1$ vector represents the PU signal which is sparse in the frequency domain, i.e. have a k occupied channels, where $M < N$ and k is the number of non-zeros in the sparse signal y_f [16]. The sparse recovery problem can be stated as [7]:

$$min||y_f||_1 \, subject \, to \, z_f = Ay_f \tag{2}$$

Where $||y_f||_1$ is the ι_1 - norm, and there are two ways to solve this problem, ι_1 - minimization problem and greedy algorithms [17]. The conditions of sensing matrix that must be achieved for small reconstruction error are: low mutual coherence or satisfies the restricted isometry property (RIP) [18] [7].

1) **Mutual Coherence**

It examines the sensing matrix quality and evaluates its efficiency [9]. The mutual coherence $\mu(A)$ measures the maximum correlation between any two columns of sensing matrix A. If A is an $M \times N$ matrix with normalized column vector a_1, a_2, $a_3...a_N$. each ai,(i =1,...N) is of unit length. The mutual coherence is defined as [7] [17]:

$$\mu(A) = \frac{|(a_i, a_j)|}{||a_i||_2 ||a_j||_2}, \, i, j = 1, ...N \tag{3}$$

Smaller coherence value produced better reconstruction which means that perfect recovery of the sparse signal is possible with fewer samples (high compression) [18] [7].

2) **Restricted Isometry Property (RIP)**

The existence and uniqueness of the solution can be guaranteed as soon as the measurement matrix A satisfies the RIP [17]. A sensing matrix satisfies the restricted isometry property if there exists a constant δ_k such as [7]

$$(1 - \delta_k) ||y_f||_2^2 \leqslant ||Ay_f||_2^2 \leqslant (1 + \delta_k) ||y_f||_2^2 \tag{4}$$

Where $|| \cdot ||_2$ is the ι_2 - norm and $\delta_k \in [0, 1]$ is called the restricted isometry constant (RIC) of A that should be much smaller than 1. For deterministic matrix the computation of RIP is hard. So, instead of using RIP, we compute the mutual coherence to verify whether a sensing matrix is suitable for the reconstruction of the sparse signal [19]. Many types of sensing matrices are design to achieve the requirement of CS of low mutual coherence and satisfying the RIP. The well known types of these matrices are Gaussian and Bernoulli

- **Gaussian Matrix**

Gaussian Matrix is generated by selecting all entries randomly and independently and follow a normal distribution with expectation $\mu = 0$ and variance 1/M (where M is the size of rows of sensing matrix), the Gaussian matrix satisfies the RIP with high probability [7] [20] [21]. The main advantage of this matrix is simple to implement, but its hardware implementation is expensive. It requires high memory space with O(MN) and it is slow [9].

- **Bernoulli Matrix**

Is a matrix whose entries take the value $\frac{+1}{\sqrt{M}}$ " or " $\frac{-1}{\sqrt{M}}$, with equal probabilities. It, therefore, follows a Bernoulli

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distribution, which has 2 possible outcomes, the probability density function [7] [22].

$$f(n) = \begin{cases} 1/2; \ for \ n = 0 \\ 1/2; \ for \ n = 1 \end{cases} \tag{5}$$

IV. RECOVERY ALGORITHMS

Reconstruction algorithms allow recovering a sparse signal from a few measurements vector [8]. Greedy algorithms select one position of a non-zero element of the signal, which corresponds to picking one column from the sensing matrix [9]. The greedy iterative algorithm searches the support of the sparse signal. In each greedy iteration search, the correlation between each column of measurement matrix and residual are compared to identify the elements of the support [10]. Once the support of the signal is computed correctly, the pseudo-inverse of the measurement matrix restricted to the corresponding columns can be used to reconstruct the actual signal y_f. The clear advantage to this approach is speed, but the approach also presents new challenges [23].

- **Stagewise Weak Orthogonal Matching Pursuit (SWOMP)**

  The stagewise weak OMP (SWOMP) has an intuitive support estimation criterion. In every iteration, it selects all indices that are within some limits of the highest value [24]. Instead of using the norm of the residual to define a threshold (Thr) for element selection as done in nthe StOMP algorithm, the threshold is computed based on the maximum correlation value (dot product between atom and residual) [25]. The procedures of the algorithm are shown in Fig. 1.

- **Compressive sampling matching pursuit (CoSaMP)**

  Differently, from (orthogonal matching pursuit) OMP, the CoSaMP algorithm tries to find the k columns of A that are the most strongly correlated with the residual, thus correcting the reconstruction based on residual achieved at each iteration [27]. In CoSamp, positions of the top 2k coefficients of the correlate (Ct) are chosen as the intermediate values. The values at the support are updated accordingly; this is an intermediate variable FF. The solution at the current iteration is updated by pruning FF and keeping the top k coefficients in it [24]. The advantage of the CoSaMP over other types of the greedy algorithm is that the CoSaMP can rectify/prune an incorrectly selected index while in other types if an index is incorrectly selected, it stays in the solution for future iterations. When one index was incorrectly selected in the first iteration, it was corrected in the next iteration. The pruning step in CoSaMP allows for correction [24]. The procedures of this algorithm are shown in Fig. 2.

V. THE PROPOSED CHAOTIC MATRIX

The proposed chaotic matrix is based on the Chebyshev map and Modified Gram Schmidt shown in the following steps:

1) A chaotic sequence is created using Chebyshev map as shown in the following [4]

$$x_{n+1} = cos(rcos^{-1}(x_n)) \tag{6}$$

Where $r \in [2, \infty]$

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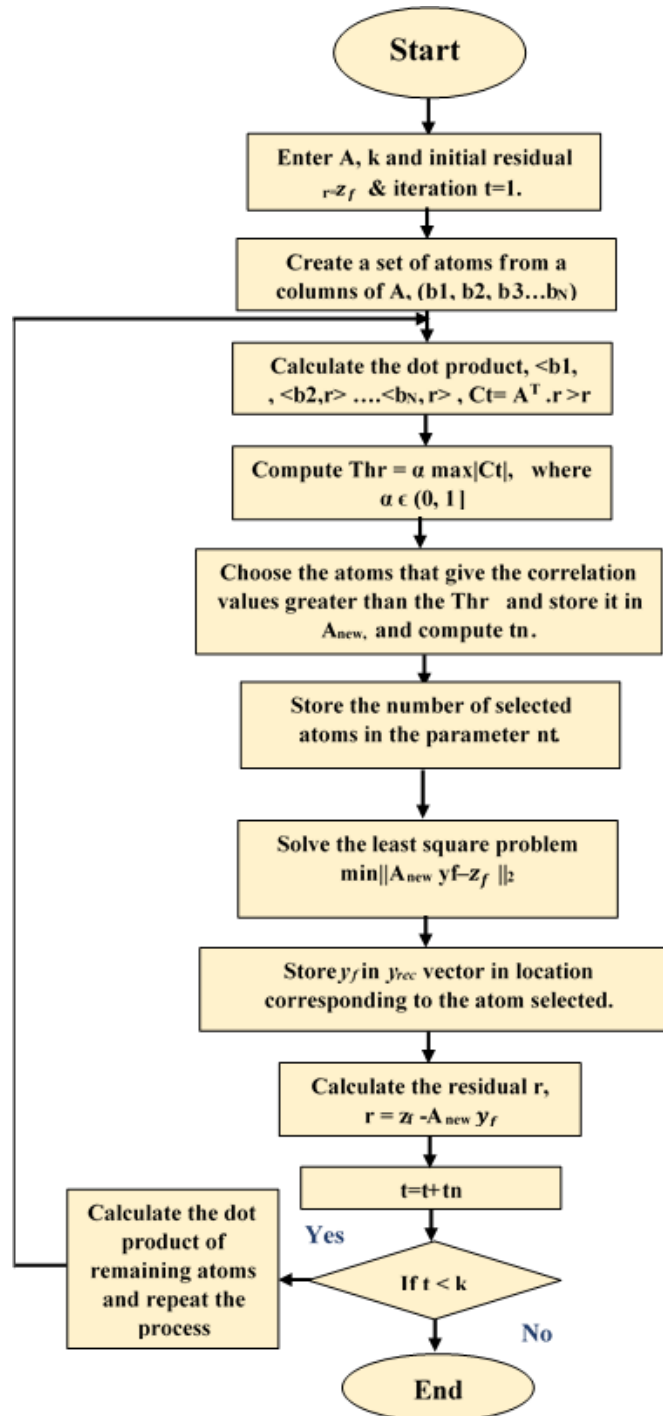
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Figure 1: SWOMP algorithm

2) The first 5000 samples of the sequence are skipped to ensure the randomness of the chaotic factors and the $M \times N$ matrix is created from the remaining sequence.

3) The created matrix is entered into a modified Gram Schmidt algorithm (MGS) to create an orthogonal matrix. The process of MGS is presented by the following Equation 7 [27]- [29]

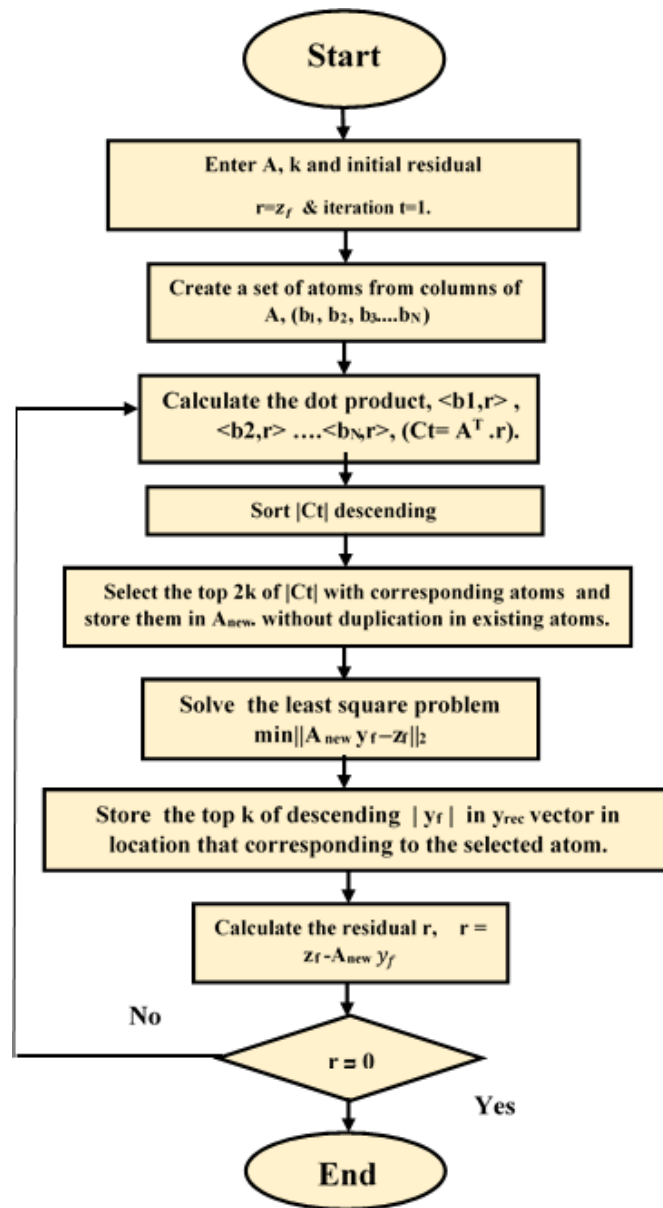$$u_k = v_k - proj_{u_1}(v_k) - proj_{u_2}(v_k) - .... - proj_{u_{k-1}}(v_k) \tag{7}$$

Figure 2: CoSaMP algorithm

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Where the sequences v_1 to v_k , and u_1 to u_k are the column vectors in the matrix entered to MGS, and the orthogonal column vectors of the matrix at the output of MGS respectively while the projection operator is given by

$$proj_u(v) = \frac{(v,u)}{(u,u)}u \tag{8}$$

The orthogonal column vectors of the matrix at the output of MGS are:

$$u_k^{(1)} = v_k - proj_{u_1}(v_k)$$
$$u_k^{(2)} = u_k^{(1)} - proj_{u_2}(u_k^{(1)})$$
$$\vdots$$
$$u_k^{(k-2)} = u_k^{(k-3)} - proj_{u_{k-2}}(u_k^{(k-3)})$$
$$u_k^{(k-1)} = u_k^{(k-2)} - proj_{u_{k-1}}(u_k^{(k-2)})$$

4) The orthogonal matrix created from step 3 is shown in Equation 9 which is used for compressive sensing.

$$A = \sqrt{\frac{8}{M}} \begin{bmatrix} u_0 & u_M & \cdots & u_{M(N-1)} \\ u & u_{M+1} & \cdots & u_{M(N-1)+1} \\ \vdots & \vdots & \vdots & \vdots \\ u_{M-1} & u_{2M-1} & \cdots & u_{MN-1} \end{bmatrix} \tag{9}$$

Where, $\sqrt{(8/M)}$ is a normalization factor.

This proposed chaotic matrix provides low reconstruction error with high compression and high security where the attackers from outside the network cannot reconstruct the sensing information when the control parameters and initial values of the chaotic matrix are slightly different. This proposed chaotic matrix provides low mutual coherence and high security where the attackers from outside the network cannot reconstruct the sensing information when the control parameters and initial values of the chaotic matrix are slightly different.

VI. SYSTEM MODEL

In this section, the design of compressive sensing based proposed chaotic matrix is presented as shown in Fig. 3. The proposed system is implemented on real TV signals in Iraq as PUs. The UHF TV band is captured using a spectrum analyzer and at time t. This band is then saved and entered into MATLAB to implement the proposed system on it. The sensing channel is the channel between PU broadcasting and spectrum analyzer which is a real channel and the received PUs signal severed from noise, multipath fading and so on. The reporting channel is the channel between SU and Fusion Center (FC) which is simulated in MATLAB and assume to be an Additive White Gaussian noise (AWGN). At the FC the original sparse signal is recovered using either SWOMP or CoSaMP recovery algorithms. Fig. 4 shows the screenshot of the received TV signal using a spectrum analyzer. The performance of the proposed system was measured using recovery error (RE), MSE and probability of detection as shown in the following equations respectively [9] [10].

$$RE = \frac{||y_{frec} - y_f||}{||y_f||} \tag{10}$$

$$MSE = ||y_f - y_{frec}||_{12} \tag{11}$$

The performance detection of the reconstructed sparse signal is tested by applying the energy detection technique of each sub band [12], as shown in Equation 12

$$T_{EDb} = \sum_{n=0}^{Nb} y_{fb}(n)^2 \, n = 1....Nb \tag{12}$$

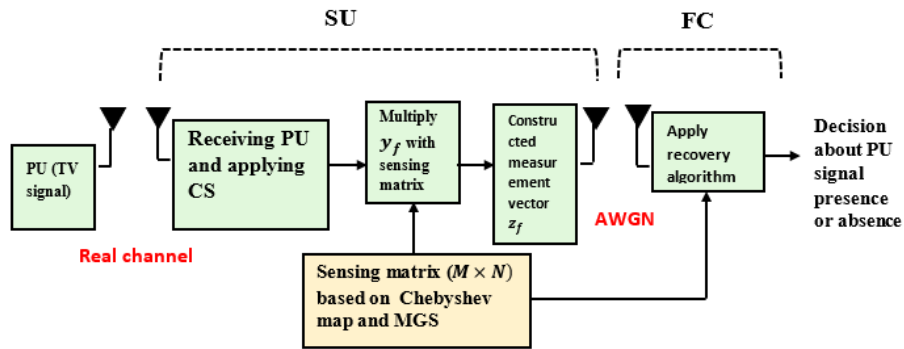Where Nb is the number of samples in i^{th} sub band. So, the sensing decision can be presented as follows [12] :
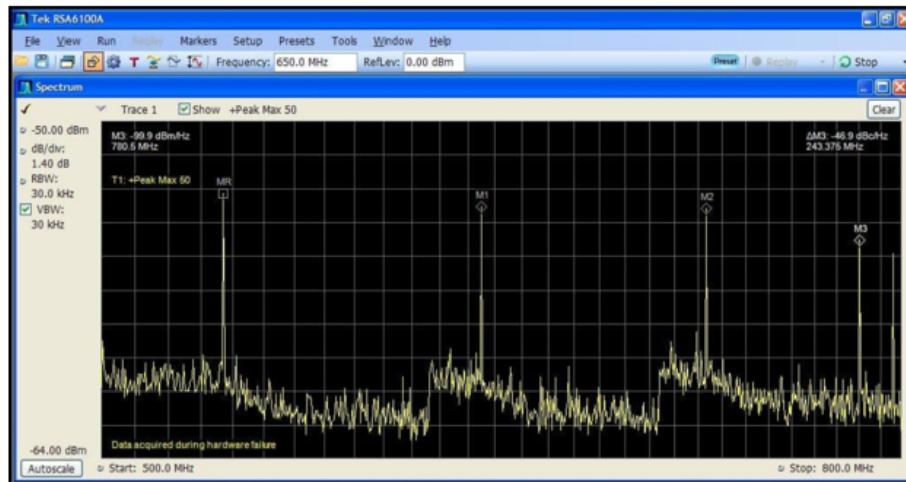

Figure 3: System model

Figure 4: Received PUs (real TV signal) using spectrum analyzer

$$if \, T_{EDb} \geq \Lambda, H_1 : PU \, is \, present$$
$$if \, T_{EDb} < \Lambda, H_0 : PU \, is \, absent \tag{13}$$

Where Λ is the sensing threshold given by the following equation

$$\Lambda = (Q^{-1}(Pf)/\sqrt{Nb} + 1) \tag{14}$$

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Where Pf is the probability of false alarm.

VII. SIMULATION RESULTS AND DISCUSSION

This section presents the performance analysis of compressive sensing in cognitive radio based on the proposed chaotic matrix and real TV-UHF signal as a PU. The spectrum band used for TV signals is 500 MHz - 800MHz. The initial value and parameter of the Chebyshev map used in the proposed matrix are $x(0) = 0.2, r = 10.5$. The original sparse signal is reconstructed using CoSaMP and SWOMP greedy algorithms. The performance of the proposed system evaluated by comparing it with Gaussian matrix, Bernoulli matrix, and chaotic matrix in [4] which is based on the modified chaotic map given as:

$$v_m = 1 - 2x_{1000+md} \tag{15}$$

Where m = 0, 1, 2 ...., d is the down sampling factor, and x is the sequence created from the logistic map. The sensing matrix created from Equation 15 is defined as :

$$A = \sqrt{\frac{2}{M}} \begin{bmatrix} v_0 & v_M & ... & v_{M(N-1)} \\ v_1 & v_{M+1} & ... & v_{M(N-1)+1} \\ : & : & : & : \\ v_{M-1} & v_{2M-1} & ... & v_{MN-1} \end{bmatrix} \tag{16}$$

Where, $\sqrt{(2/M)}$ is a normalization factor.

Fig. 5 shows the performance curve of recovery error versus SNR in CS based proposed matrix at M/N =0.56 using CoSaMP and SWOMP recovery algorithms. It can be seen that for matrices the recovery error decreases as SNR increases. Also, it shows that the results based CoSaMP algorithms have low recovery error as compared to the SWOMP algorithm since in CoSaMP the incorrectly selected atom can be rectified in the next iteration, unlike the SWOMP algorithm where it may stay in the next iteration. The CS based proposed matrix has the same performance as Gaussian and Bernoulli matrix and better performance than the chaotic matrix based logistic map in [4]. For example at SNR equals -20 dB with the case of CoSaMP, the CS based proposed matrix is reduced by 78 %. Fig. 6 presents the performance curve of MSE versus M/N in CS based proposed matrix at SNR=-20 dB using CoSaMP and SWOMP. It can be noted that in all types of matrices the MSE decreases as M/N increases and also the performance based on CoSaMP is better than SWOMP. The CS based proposed matrix has very low MSE as compared to chaotic matrix based logistic map, d=15 in [4]. For example at M/N= 0.3 when CoSaMP is used, the MSE in the proposed matrix is reduced by 99.6 % as compared to chaotic matrix based logistic map, d=15 in [4]. Fig. 7 displays the performance curve of the overall probability of detection hole in all band versus SNR based proposed matrix using CoSaMP and SWOMP, at pf (probability of false alarm)=10-5 it can be seen that the probability of detection increased as SNR increased, and the probability of detection based proposed matrix outperform Gaussian matrix, Bernoulli matrix and chaotic matrix in [4] in both recovery algorithm (CoSaMP and SWOMP) especially in low SNR values. For example at SNR = -11 dB when using CoSaMP, pd under the proposed matrix increases by 44%, and 59% as compared with Gaussian and chaotic matrices based logistic map, d=15 in [4], respectively. Also, it has the same performance as the Bernoulli matrix. Also at SNR=-6 dB when using SWOMP the CS based on the proposed

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matrix increases by 21%, and 40% as compared with Bernolli matrix and chaotic matrix based logistic map, d=15 in [4], respectively, and has the same performance of Gaussian matrix.
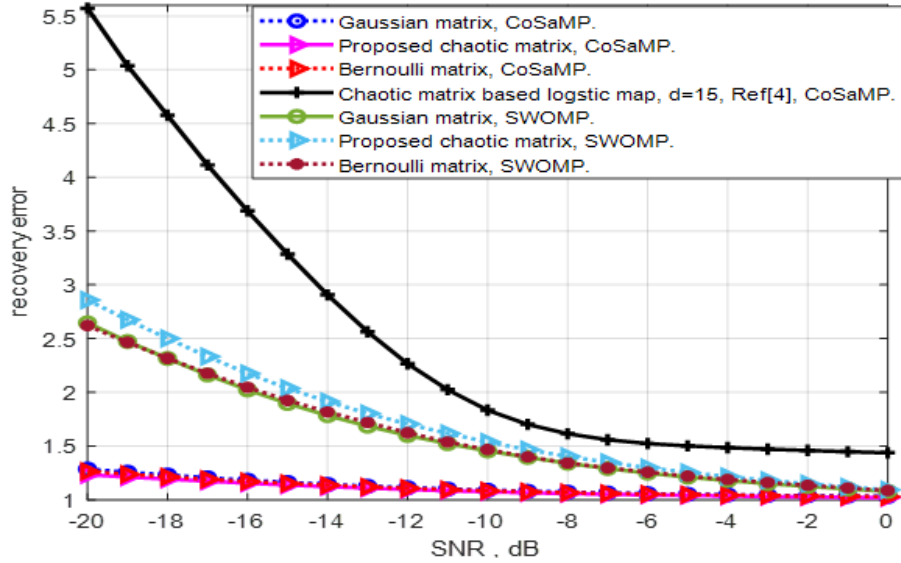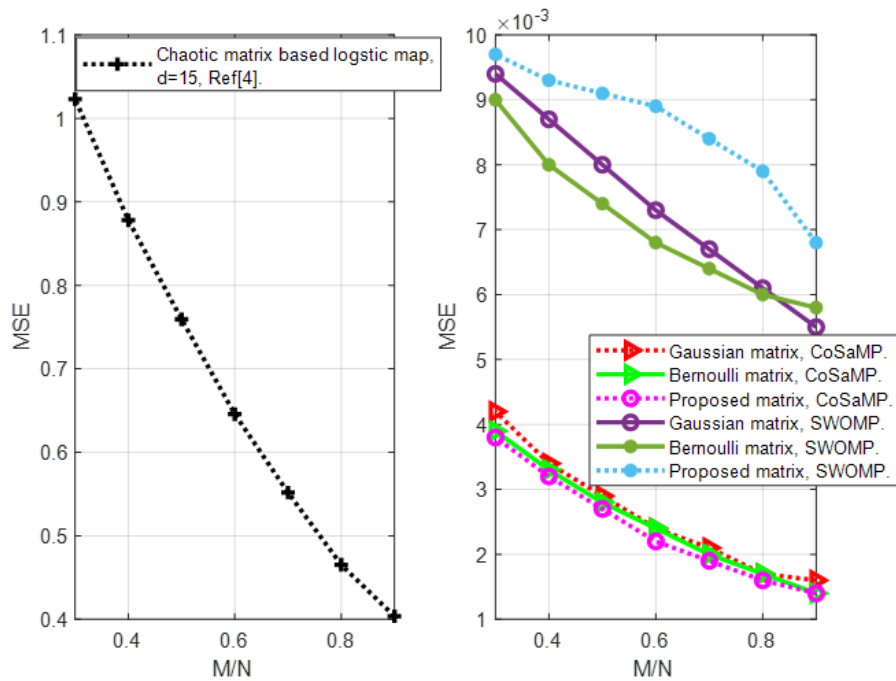

Figure 5: Recovery error versus SNR, at M/N = 0.56
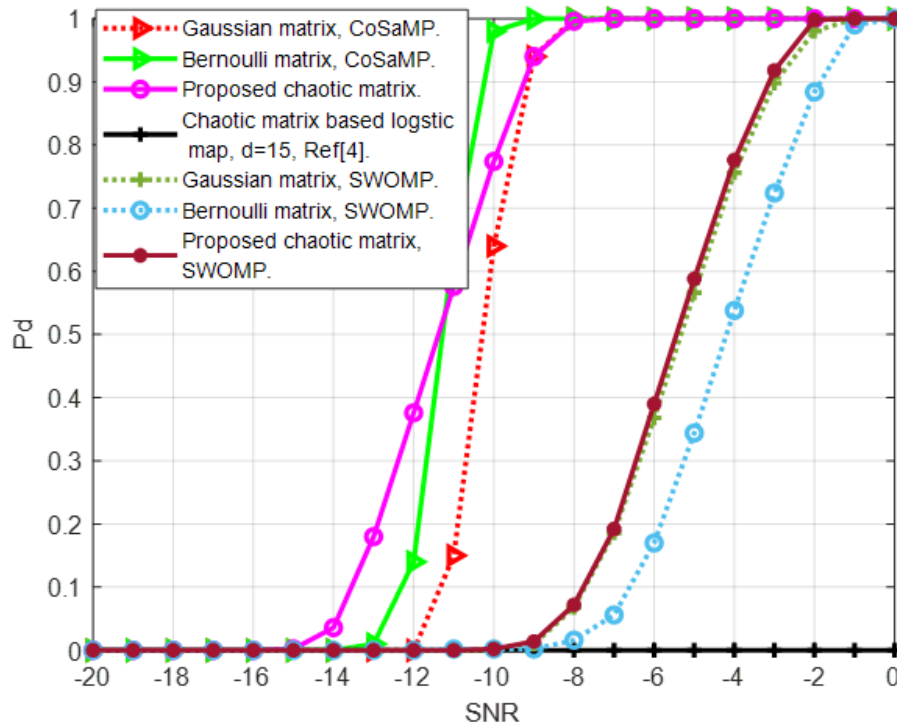

Figure 6: MSE versus M/N, at SNR = -20 dB

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Figure 7: Pd versus SNR at M/N=0.56

VIII. Conclusion

This chaotic matrix using Chebyshev and MGS is proposed as a sensing matrix of CS in CR. The CS based proposed matrix is applied in a real TV-signal in Iraq. The proposed system is tested using two types of greedy algorithms: CoSaMP and SWOMP. The performance of CS under CoSaMP is better than SWOMP, and the proposed system has low recovery error, high probability of detection under low SNR values and also has low MSE with high compression in both types of greedy algorithms used. Future works can be implementing the proposed system in the cooperative scenario and evaluating its performance in various types of fading channels.

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