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Simulation for natural convection of fluid flow numerically

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Abstract: The nonlinear behavior of fluid flow and effected of varying temperature at heated plate in an enclosure rectangular box are emphasized in this work. This problem is simulated by using a new version of differential quadrature method that is called "upwind differential quadrature(UDQM)" to demonstrated the effects of a heated through natural convection motion equation. By using UDQM, this problem is handled and good numerical results were obtained that are in agreement with that in literature.

Key words: Natural convection motion, Incompressible flow, UDQM.

1-Introduction

This subject has been studied extensively by many investigators, in differs situations. The underlying techniques [2,5,7,9] for solve numerically this problem can be complex and require a large amount of computational time to obtain accurate and reasonable stable solutions. This is certainly due to the difficulties to model such flows: the heat transfer equation can be quite complex, as well as its coupling with momentum equations. According to these reasons, and the lack information on the solution of natural convection heat transfer fluid flow by differential quadrature motivates to the present work. The differential quadrature method which is introduced by Bellman et al.[3], is able to overcome these difficulties with few grid points and less computational workloads. This fact is mentioned in many articles [1-3,8].

The upwind mechanism is important for the computation of fluid flow, and that mechanism is absence in DQM, so we introduce the DQM with upwind mechanism in this article. In addition, the effects of a heated on natural convection of fluid flow in rectangular box is examine. The results of a new version of differential quadrature technique are comparing with upwind finite difference technique and that in [2] and [9]. Good numerical results were obtained that are in agreement with existing results.

Governing equations

Natural convection of fluid flow problem can be described by the entire system of non-dimensional governing equations in conservation form as[4]

$$\frac{\partial U}{\partial t} = \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} \qquad \text{…………….(1)}$$

Where U, F, G are interpreted as column vectors, given by

$$U=\begin{pmatrix}\omega\\ -\omega\,dt\\ T\\ u\,dt\\ v\,dt\end{pmatrix}\quad F=\begin{pmatrix}-u\omega+GrT+\omega_x\\ \Psi_x\\ -uT+\Pr^{-1}T_x\\ 0\\ -\Psi\end{pmatrix}\quad G=\begin{pmatrix}-v\omega+\omega_y\\ \Psi_y\\ -vT+\Pr^{-1}T_y\\ \Psi\\ 0\end{pmatrix}\qquad \ldots\ldots\ldots\ldots\ldots(2)$$

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Where ω is the vorticity, u,v are the horizontal and vertical velocities, Ψ is the stream function, T is the temperature, x,y are the non-dimensional coordinates, $Gr,\Pr$ are the Grashof and prandtal numbers.

We shall made testing for this problem by consider liquid enclosed by rectangular box, bounded by top cold ($T_c=0$) and side walls and bottom hot ($T_h=\alpha>0$) plate (see Fgure.1)

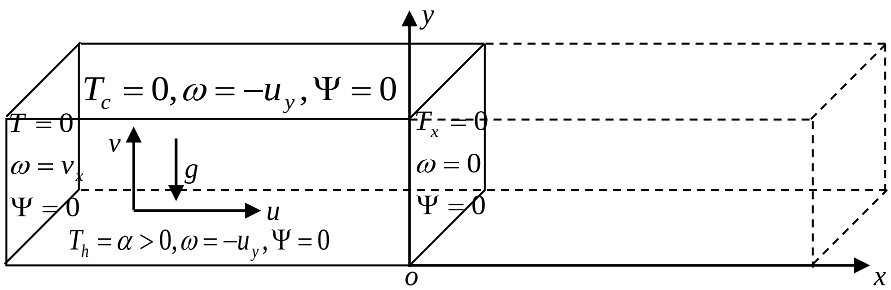


Figure.1 Schematic diagram of the test region with boundary conditions

Because the symmetry of the temperature field , the boundary condition along the $y-$axis is $\frac{\partial T}{\partial x}=0,\Psi=0$ on the three solid walls comes from the fact that there is no net flow across those boundaries. Also, because the symmetry about the y-axis, it is necessary to seek a solution for the left of the flow.

3-Numerical method

The DQM expresses a linear operator of a function with respect to a coordinate direction is expressed as a weighted linear sum of all the function values at all grid points along that direction. Thus, for a smooth function $f(x,y)$ on a domain $0\le x\le a, 0\le y\le b$, with a and b fixed, local DQM discretizes its $rth-$order x-partial derivative, and sth $-$order y-partial derivative, at the grid point (x_i,y_j) may be written as

$$\frac{\partial^r f(x_i, y_j)}{\partial x^r} = \sum_{k=1}^{N} A_{ik}^{(r)} f(x_k, y_j) \quad , \quad r = 1,2,....,N-1 \qquad \text{...............(3a)}$$

$$\frac{\partial^s f(x_i, y_j)}{\partial y^s} = \sum_{l=1}^{M} B_{jl}^{(s)} f(x_i, y_l) \quad , \quad s = 1,2,....,M-1 \qquad \text{...............(3b)}$$

For $i = 1,2,....,N$: $j = 1,2,....M$

Where N , M are the numbers of grid points in the x and y direction respectively, $A_{ik}^{(r)}$ and $B_{jl}^{(s)}$ are the respective weighting coefficients to be determined as [2].

The upwind mechanism is important for the computation of fluid flow, and that mechanism is absence in DQM, so we introduce the locally DQM with upwind mechanism in this article. The construction of the upwind mechanism goes through a partition of the grid points system in the network of the whole domain [1,8]. For convenience, we introduce the upwind cell coordinates with DQM given in more details in the next step.

Let us consider a uniform grid ($\Delta x = \Delta y = \Delta$) inside the region. The spacing between any two neighboring internal grid points is equal to Δ. The derivative values of $f(x, y)$ at an internal, grid point (i, j) can be expressed as a weighted linear sum of the function values at the p grid points near the point (i, j), along x direction or along y direction rather than at all grid points, that is, equations (3a) and (3b) can be written as

$$\left.\frac{\partial^r f}{\partial x^r}\right|_{ij} = \sum_{k=1}^{p} A_{\sigma_1 k}^{(r)} f(i - \sigma_1 + k, j) \quad , \quad r = 1,2,....,N-1 \qquad \text{...............(4a)}$$

$$\left.\frac{\partial^s f}{\partial y^s}\right|_{ij} = \sum_{l=1}^{p} B_{\sigma_2 l}^{(s)} f(i, j - \sigma_2 + l) \quad , \quad s = 1,2,....,M-1 \qquad \text{...............(4b)}$$

For $i = 1,2,....,N$: $j = 1,2,....M$

Where σ_n is an integer belonging the interval $[1, p]$, $n = 1,2$. The p grid points are internal point and for some locations include one or two boundary points. In order to, introduce upwind mechanism into (4), the value of σ must be determined in terms of horizontal (u_{ij}) and vertical (v_{ij}) velocity components subject to the direction of the flow as follow:

$$\sigma_n = \begin{cases} \dfrac{P+(n-1)}{2} + 1 \\ \dfrac{P+(n-1)}{2} - (n-1) \end{cases} \quad \text{if} \quad p \begin{array}{l} \textit{even and} \\ \textit{odd and} \end{array} \begin{array}{l} n = 1 \\ n = 2 \end{array} \quad \textit{for} \begin{array}{lll} u \geq 0 & \textit{or} & v \geq 0 \\ u < 0 & \textit{or} & v < 0 \end{array}$$

This idea is illustrate in Figure (2).

The weighting coefficients $A^{(1)}_{\sigma_1 k}, A^{(r)}_{\sigma_1 k}, B^{(1)}_{\sigma_2 l}$ and $B^{(s)}_{\sigma_2 l}$ should be determined by values of x and y coordinates of these p grid points as follows,

$$A^{(1)}_{\sigma k} = \frac{C^{(1)}(x_\sigma)}{(x_\sigma - x_k).C^{(1)}(x_k)} \quad , \quad k = 1,2,....p \qquad \text{................(5a)}$$

$$B^{(1)}_{\sigma l} = \frac{C^{(1)}(y_\sigma)}{(y_\sigma - y_l).C^{(1)}(y_l)} \quad , \quad l = 1,2,....p \qquad \text{................(5b)}$$

$$A^{(r)}_{\sigma k} = r[A^{(r-1)}_{\sigma\sigma} A^{(r)}_{\sigma k} - \frac{A^{(r-1)}_{\sigma k}}{x_\sigma - x_k}] \quad , \quad k = 1,2,....p \qquad r \geq 2 \qquad \text{................(5c)}$$

$$B^{(s)}_{\sigma l} = s[A^{(s-1)}_{\sigma\sigma} A^{(s)}_{\sigma l} - \frac{A^{(s-1)}_{\sigma l}}{y_\sigma - y_l}] \quad , l = 1,2,....p \qquad s \geq 2 \qquad \text{................(5d)}$$

For $r = 1,2,....,N-1$: $s = 1,2,....,M-1$.

Where $A^{(r)}_{\sigma\sigma} = -\sum_{\mu=1,\mu\neq\sigma}^{p} A^{(r)}_{\sigma\mu}$ $,1 \leq r \leq (N-1)$, $B^{(s)}_{\sigma\sigma} = -\sum_{\mu=1,\mu\neq\sigma}^{p} A^{(s)}_{\sigma\mu}$ $,1 \leq s \leq (M-1)$,

$C^{(1)}(x_\sigma) = \prod_{\mu=1,\mu\neq\sigma}^{p}(x_\sigma - x_\mu)$, and $C^{(1)}(y_\sigma) = \prod_{\mu=1,\mu\neq\sigma}^{p}(y_\sigma - y_\mu)$,and $\sigma = 1,......, p$

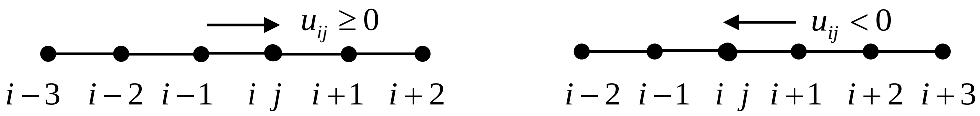


Figure 2. Upwind local DQ grid point's model in the $x-deriction$ for $p = 6$.

According to UDQM described above with the time derivatives approximate by forward difference scheme, the governing equation (1), take the following discrete forms

$$\left.\frac{\partial U}{\partial t}\right|_{ij}^{n+1} = \sum_{k=1}^{\sigma_1 - 1} A^{(1)}_{\sigma_1 k} F^{n+1}(i - \sigma_1 + k, j) + \sum_{k=\sigma_1}^{p} A^{(1)}_{\sigma_1 k} F^{n}(i - \sigma_1 + k, j) + \sum_{l=1}^{\sigma_2 - 1} B^{(1)}_{\sigma_2 l} G^{n+1}(i, j - \sigma_2 + l) + \sum_{l=\sigma_2}^{p} B^{(1)}_{\sigma_2 l} G^{n}(i, j - \sigma_2 + l) \qquad \text{................(6)}$$

Where (i, j)'s should be all internal grid points $(2 \leq i \leq (N-1), 2 \leq j \leq (M-1))$.

All the boundary conditions and the implicit quantities ω, Ψ, T, u, v that include derivative (equ.(2)) can approximate by DQM depend upon the p grid point, using equation (4).

4-<u>Error Analysis</u>

Analyzing the errors resulting from approximation of a function and derivatives is useful work. Depending on the DQ is identical to Lagrange polynomial interpolation of order $N-1$, author [8] have given a thorough error analysis for the first-order derivative (E^1) and the second-order derivative (E^2). These errors written as;

$$E^1(x_i) = \frac{f^{(N)}(\xi)C^{(1)}(x_i)}{N!}, \quad i = 1,\cdots\cdots\cdots, N$$
$$E^2(x_i) = \frac{f^{(N)}(\xi)C^{(2)}(x_i)}{N!}, \quad i = 1,\cdots\cdots\cdots, N$$

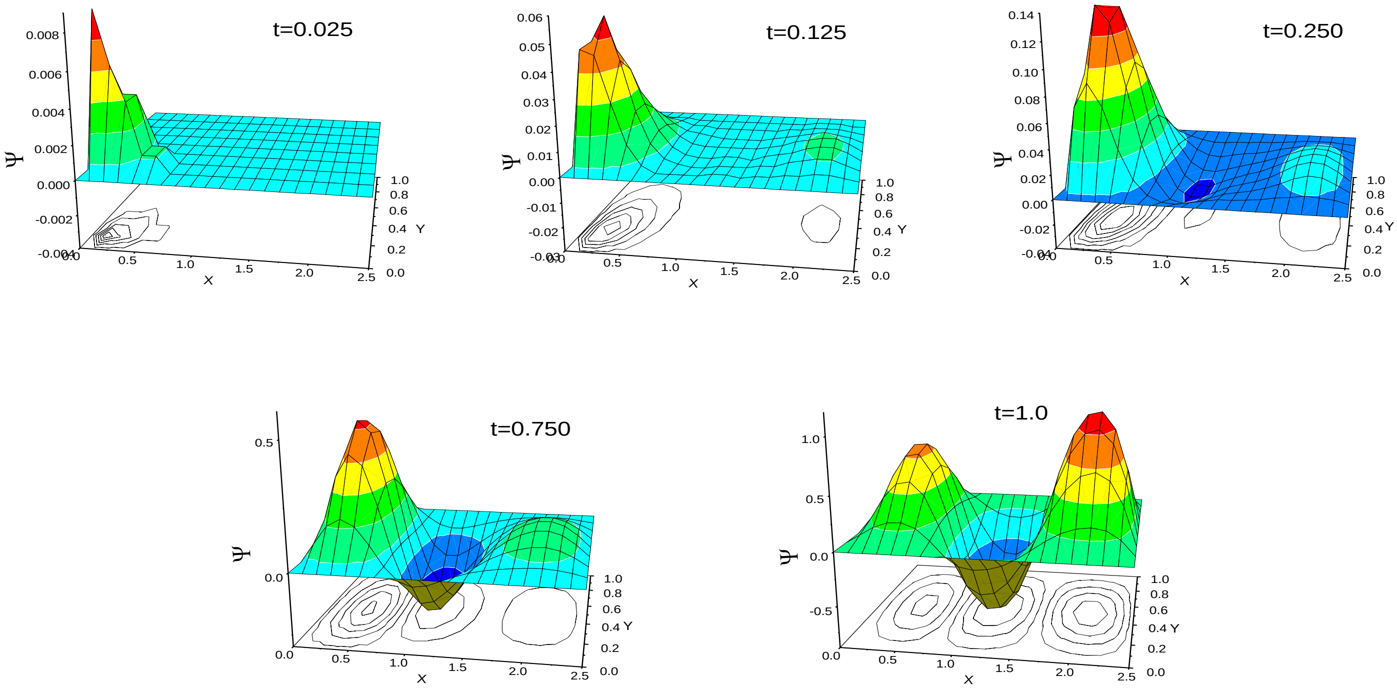
These residual estimates show that very high accuracy can be obtained if the number of grid points N is large. Accuracy is proportional to N or its powers. By "its powers", we mean here that accuracy may also be proportional to squared or cubic N, or even higher order term of N.

However, too large N may lead to instability this shown in [6].for dynamical problems small error may accumulate with each time step. A simple estimate for the first-order derivative is then that the accumulated error at each time is proportional to $N\Delta t$, where Δt is time step. Similar arguments lead us to the conclusions that the accumulated error at each time step for the second order derivative is proportional to $(N\Delta t)^2$. If large N is used, time step must be small to keep the errors within controllable range. High order DQ discretization becomes unstable faster than low-order DQ discretization. This simple analysis and the numerical results lead us to the conclusion that DQ is of high accuracy, but of poor stability. The more grid points are used, the high accuracy we obtained, but the poorer the stability is. Stability of the function values and its derivative Lagrange polynomial interpolation is a complicated problem. From this rude estimate, however, we conclude that accuracy and stability are conflicting. Accuracy requires large number of grid points, but stability requires the opposite.

5-Numerical Results and Discussion

The computationally efficiency of UDQM with Chebyshev-Gauss-Lobatto points [2] on the numerical accurate results have been well demonstrated here. In this study, the two-dimensional incompressible Navier-Stocks equation, continuity equation, an energy equation are which to describe convection motions of fluid flow, in rectangular channel flow region includes cold top and sidewalls and a hot bottom plate, Figure(1). In the present computations, we adopted Gauss siedel method to solve vorticity and energy equations (equ.(2)), and SOR with damping factor $0 < \theta < 1$, to solve stream function(equ.(2)). And the sufficient condition Max$\left|\Phi^{(n)} - \Phi^{(n)}\right| \le e$,(where $e = 10^{-5}$, $\Phi = (\omega, \Psi, \mathrm{T}, u, v)^{Tr}$, $Tr \equiv transort$) for convergence of numerical solution of UDQM ,if it is not satisfied go to repeat the calculation steps again, otherwise stop the iterative procedure and the steady state solutions of the incompressible flow can be obtained.

The streamlines in three dimensions are shown in Figure(3), at prime time the secondary flow-vortex presented at a corner where the cold and hot walls intersect. In the remaining region flow, the fluid is practically motionless. In this moment, the stream function value is positive $(+ve)$ everywhere, that is; the fluid motion is in the counterclockwise direction. Conversely, the motion is clockwise along closed streamlines of negative $(-ve)$ values for the stream function. Thus, the fluid descends along the cold vertical wall and then rises after flowing over the hot surface. At this time, the convective motion appears weakly. This motion becomes stronger gradually with the increase in time,



Figuer.3 Surface with contour of streamlines for $Gr = 10^3$, $\Pr = 6.75$, $\Delta t = 0.0025$

Also, the isotherm profiles have a slow variation with respect to time. Although a steady state has not been reached yet, the flow does not seem to have any further significant changes. Thus, additional computations with maximum step greater than 2000 have not been attempted. From Figure(4), we see that the thermo-gravitational convection is weak due to the low Grashof number. As a result, the values of Ψ at $0 < Gr < 200$ are positive $(+ve)$ and at $Gr \geq 200$ are **$(\pm ve)$.** The isotherms gather densely near the horizontal walls and the fluid flows mainly in the clockwise direction like a rotating flow around the core region. The regions of the isotherms gathering densely are in a long region of the horizontal walls. It is observed that the fluid motion at $Gr > 6500$ is not steady but oscillating in nature, this is the same phenomena with [2].

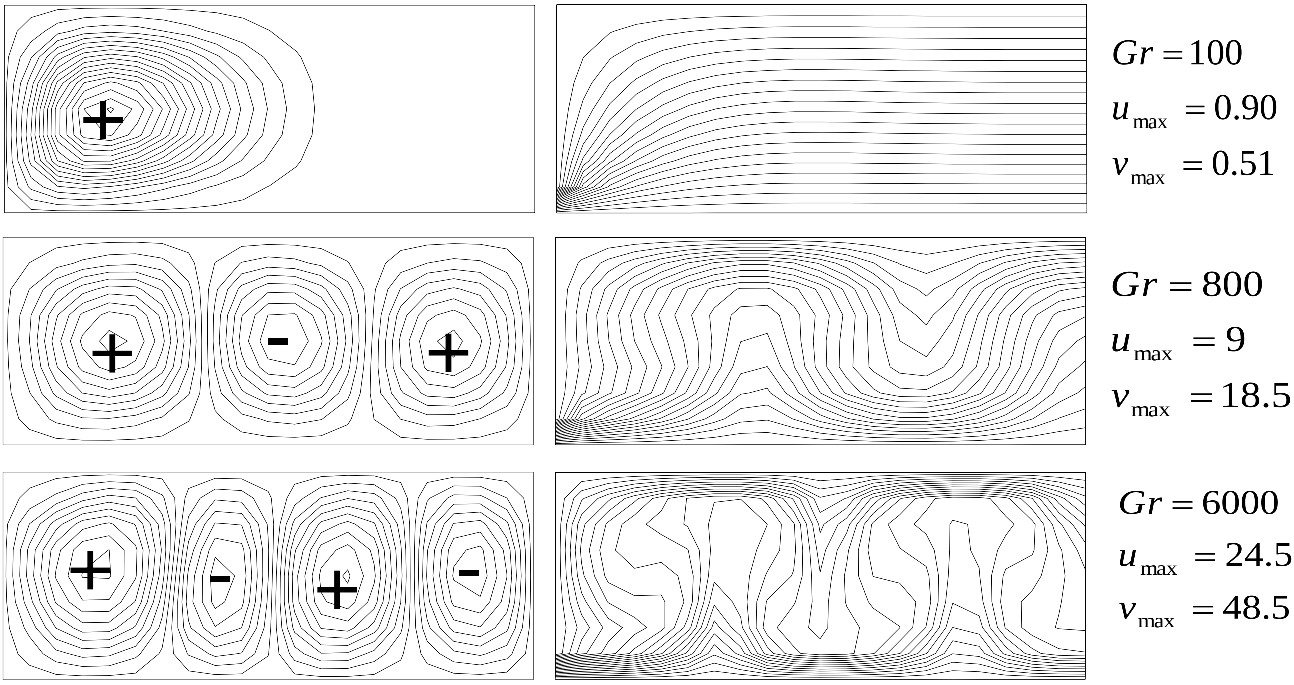
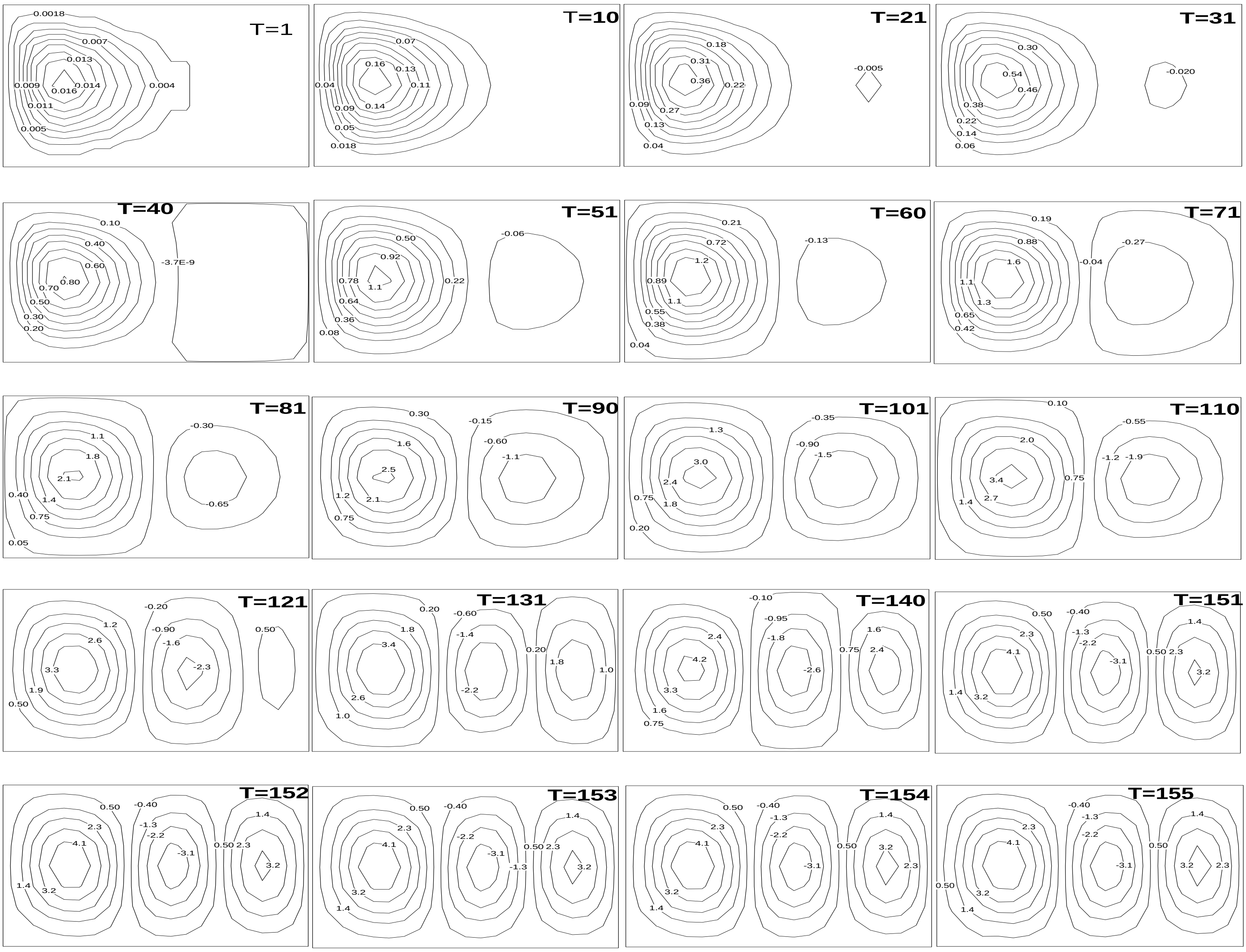


Figure.4 Stremlines and isotherm for different values of Grashof number with $\Pr = 0.75$

We shall discuss the effect of temperature on the behaviors of fluid motion in aspect of secondary flow-vortex (which is representing by sequential plots of the streamlines). Here, an important matter is to specify the critical values, which are indicated to the changing of the streamlines or the total number of the convection cells that appear in the channel. Figures (5), shows the temperature effect on the behaviors of motion of the fluid for $Gr = 50, \Pr = 0.75$, $t = 1.0$ in the channel. From these figures, we notice that, There is one cell at $0 < T < 21$, the stream function value is positive (+ve) everywhere, that is; the fluid motion is in the counterclockwise direction. The numbers of cells are changing into two cells on $20 < T \le 21$. Moreover, when $T \ge 21$ the two cells are different in size and motion direction (the left cell is counterclockwise direction and the right cells is clockwise direction). The size of the right cell is changing on $21 \le T < 63$, and the size of the left cell become small slowly on $62 < T \le 63$. This case go on for $T \ge 85$. In the remaining region flow, the fluid is practically motionless. Conversely, the motion is clockwise along closed streamlines of negative $(-ve)$ values for the stream function. Thus, the fluid descends along the cold vertical wall and then rises after flowing over the hot surface. In this case, the convective motion appears weakly. This motion becomes stronger gradually with the increase in temperature; also, the isotherm profiles have a slow variation (identical with those in Figure (4)). The change in the number of cells of the secondary flow is continue with increases of temperature values, Another change is occurs in the number of cells at $117 < T < 118$,where the total number of cells become three in the channel. These cells are different in size and direction (the left +ve , the middle is –ve , the right is

+ve). The size of the right cell starting to change until becomes identically with other cells approximately.

From the results are shown in Figure(6), we see that the results are obtained by five-point UDQM(5pt.UDQM) agreement with existing results, three-point UFDM with coarse grid mesh. It pointed out the secondary –flow vortex appears starting the wall of channel toward the whole domain consequently.



Figuer.5 Effect of the temperature of the shape on the streamlines.

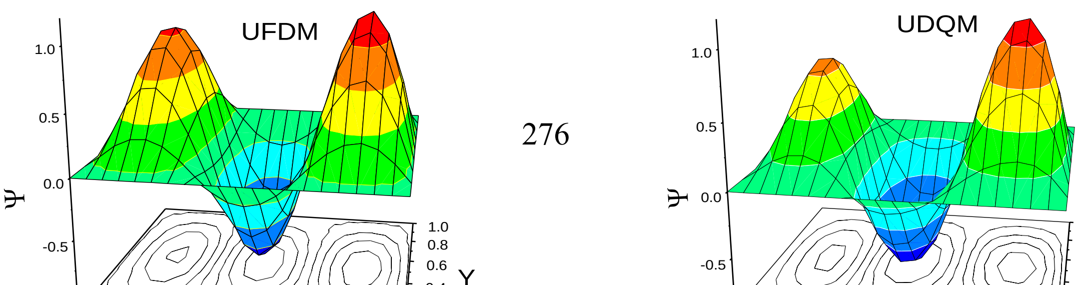


Figure.6 Ψ in 3D with contour lines for UFDM & UDQM with $Gr = 1000, \Pr = 6.75, \Delta t = 0.0025$

Table-1 shows that comparison of the solver method and numerical results of the present work with these are presented by [2] and [9]. It is most noteworthy that the present computation is very close to that in [2] and [9]. The present results are in agreement with those given by [5] in the qualitative analysis for the distribution of streamlines and isotherms (see pages 2152-2157).

Table 1.Comparison of the present study with [2,9], for $Gr = 3\times 10^4$, $\Pr = 0.015$

Authors	*Method*	*Grids*	$\Psi_{\max}$	$u_{\max}$	$v_{\max}$
Ref. [8]	QUICK	81x21	0.4579	0.6882	0 .7951
Ref. [2]	DQM	21×9	0.4552	0.5917	0.6904
Present	5pt.UDQM	21x9	0.4566	0.6760	0.7893

6-Conclusions

We conclude that the total number of the cells (secondary flow vortex) is changing with respect to temperature. That is, total number of cells is increases with temperature increase. The same phenomena are happens for the streamlines of cells. We see that the thermo-gravitational convection is weak due to the low Grashof number and low temperature, and become stronger gradually with increases in temperature. A simple study of estimation errors lead us to conclude that DQM is of high accuracy requires large number of grid points, but stability requires the opposite. This motivates us to introduce analyzing for this problem in the next future work.

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محاكاة الانتقال الطبيعي لتدفق مائع عدديا

عبدالستار جابر علي السيف/كلية التربية/قسم الرياضيات/جامعة البصرة

المستخلص:

السلوك اللاخطي لتدفق المائع وتأثير التغير في درجة الحرارة على حركة المائع في قناة مستطيلة فد حدد في هذا البحث. تم الحصول على نتائج عددية جيدة باستعمال طريقة التفاضل التربيعي الجديدة (UDQM) للمحاكاة،وكانت متماثلة والنتائج السابقة.