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The Energy Release Rate at Knik and Peak Points for Polyester-kevlar fiber and polyester-Polyethylene fiber

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Abstract:
A single fiber drag-out test was used in this study to measure the critical energy release rate of a crack at interface of composite ,using Nairn model at the initiation of crack (knik point) or the beginning of the difference in elastic behavior between fiber and matrix and the critical energy release rate when the crack length equal embedded length (peak point) or full debond , the difference between these points effected by the strength of adhesion between the fiber and the matrix and also by the wettability or diffusion of polyester monomer into interfibrillar space where polyethylene is solid fiber while kevlar is rope fiber .

ملخص البحث:
في هذا البحث تم حساب معدل الطاقه المتحرره عند شروع الانسلاخ وعند الانسلاخ الكامل في المنطقه البينيه في المواد التركيبيه باستخدام فحص سحب الليف كطريقه عمليه و استخدام موديل نيرن النظري كما تم دراسة تأثر هاتين النقطتين في عملية انتشار البوليمر بين الالياف تم الفحص على بولي استر مدعم بليف نوع كيفلر و عينه اخرى هي بولي استر مدعم بليف نوع بولي اثلين .

Introduction:

The measuring of fiber-polymer matrix adhesion has been in used for a number of years by using pull-out, microbond tests and recently using drag-out test .The analysis of test data (force-displacement curve) fall into two broad categories [1,2] namely the stress-based or energy based approaches .In stress-based analysis the interfacial shear stress (τ_{ifss}) calculated by :

$$\tau_{ifss} = \frac{F}{2\pi r_f l_e} \quad \text{-------------------- } 1$$

Where (F) is the force from force-displacement curve and if (F) is at the peak then τ_{ifss} is the interfacial shear strength , (r_f) is the fiber radius and (l_e) is the embedded length .A large differences for τ_{ifss} are obtained among various experiments test results [3] this because the tests are different in nature , the loading configuration and specimen geometry vary from test to test and therefore the stress fields induce locally are different . In this paper we matched between the theoretical energy-based approach which derived by Nairn and others [1,2,3] for axisymmetric stress analysis of two concentric cylinders which used for pull-out or microbond test and the drag-out test [4] for U-shaped specimen . Debonding is predicted based on the energy release rate for initiation of an interfacial crack at the point where the fiber enter the matrix (the knik point in force–displacement curve) and then the crack propagated until the crack length equal the embedded length of fiber (peak point in the force-displacement curve),the calculation done for polyester-kevlar fiber as kevlar is rope and polyester-polyethylene fiber as polyethylene is a solid fiber using drag-out test.

Theory :

The energy release rate for propagation of debond in single fiber pull-out test or microbond test including the effect of thermal stress and friction [5,6,7] was derived by Nairn and other fig(1) which depend on two concentric cylinders represented the fiber and matrix phases ,due to shear-lag analysis the maximum shear stress (crack initiation) where the fiber enter the matrix and the critical energy release rate for crack initiation G_{ic} and according to Sheer and Nairn model in reference [7] , solving the equation of energy release rate at the initiation of crack [equation (64) in reference (7)] :

$$G_{ic} = \frac{r_f C_{33s}}{2}\left[\frac{F_d}{\pi r^2_f} + \frac{(\alpha_A - \alpha_m)}{2C_{33s}}\right]^2 \qquad \text{----------- } 2$$

Where r_f is fiber radius and F_d is the force at the point of deflection between the theoretical and experimental line in fig(2) (kink point) , the other constants defined in the appendix (A) .

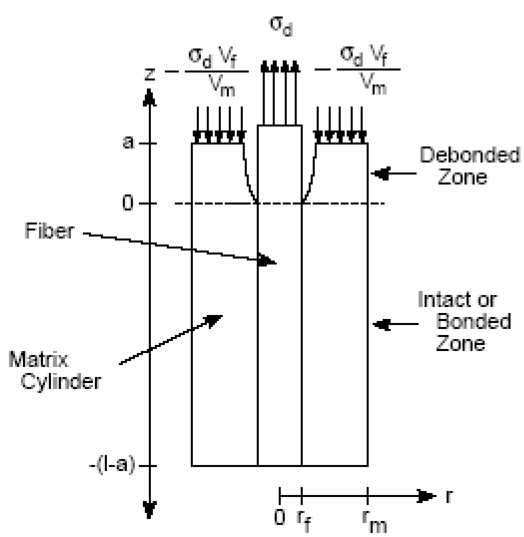

Fig-1. The coordinate system used for the axisymmetric stress analysis of two concentric cylinders.
The origin of the z axis is placed at the debond tip. The zone $0 < z < a$ is the debonded zone.
The zone $-(l - a) < z < 0$ is the intact or bonded zone.

The energy release rate model for Nairn solved for a crack of length approximately equal to the embedded length (l_e) ,equation (16) in reference [1] .

$$G(a) = \frac{r_f}{2} C_{33s}(\sigma_d - ka)^2 + \frac{r_f}{2} D_{3s}(2 + C'_T(a))(\sigma_d - ka)\Delta T + \frac{r_f}{2}\left[\left(\frac{D^2{}_3}{C_{33}} + \frac{v_m(\alpha_T - \alpha_m)^2}{v_f A_o}\right) + \right.$$

$$\frac{2D_3D_{3s}}{C_{33}}C'_T(a))\Delta T^2 - kD_{3s}C'_T(a)\Delta T] \qquad \text{----------------- 3}$$

Where:

β is the shear-lag parameter and defined in references [9,10]:

$$\beta^2 = \frac{2}{r_f{}^2E_AE_m}\left[\frac{E_Av_f + E_mv_m}{\frac{v_m}{4G_A} + \frac{1}{2G_m}\left(\frac{1}{v_m}\ln\frac{1}{v_f} - 1 - \frac{v_f}{2}\right)}\right] \qquad \text{---------- 4}$$

And :

σ_d is the external stress $\sigma_d = \frac{F_p}{\pi r_f{}^2}$,F_p is the force applied to fiber at peak point fig(2)

k is the frictional stress transfer rate $k = \frac{2T_f}{r_f}$

T_f is the absolute value of interfacial frictional force and (a) is the crack length.
The other constants defined in the appendix [A].

In figure (2) F_d is corresponding to zero initial crack length [1,4] or the deflection between the theoretical and experimental line means the difference in elastic behavior between the fiber and the matrix while F_p (the force at the peak) is corresponding to the point at which the crack length is nearly equal to the length of droplet or l_e in equivalent cylindrical model .T_f is the absolute value of the constant interfacial friction stress which can determined form force-displacement curve after complete debonding (F_p) [1,9].

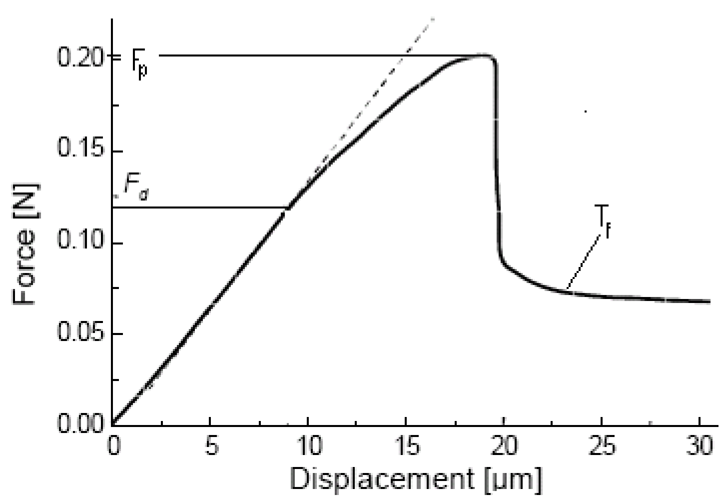

Fig-2: A typical force-displacement curve, theoretically (dash line) and experimentally (solid line), (ref.5).

In equation (2) and (3), ΔT is the difference between the specimen temperature and the stress free temperature and with the thermal expansion coefficient equation (2) would include thermal stress addition to friction stress (k) and the external stress (σ_d).
For a specimen without observed debond growth [1,5] and from force-displacement curve the peak debond force is corresponding to the point at which the debond reached the end of droplet then the critical energy release rate:

$$G_{ic} = \lim_{a \to l_e} G(a)$$

To calculate G(a) in long –droplet limit or the limit as $l_e \to \infty$ the cumulative stress transfer function would be:

$$\lim_{l_e \to \infty} C_T(a) = \frac{1}{\beta} \qquad \text{and} \qquad \lim_{l_e \to \infty} C'_T(a) = 0$$

The limiting energy release rate $G_\infty(a) = \lim_{l_e \to \infty} G(a)$

$$G_\infty(a) = \frac{r_f}{2}\left[C_{33s}(\sigma_d - ka)^2 + D_{3s}(2\sigma_d - k(2a + \frac{1}{\beta}))\Delta T + (\frac{D^2{}_3}{C_{33}} + \frac{v_m(\alpha_T - \alpha_m)^2}{v_f A_o})\Delta T^2 \right]$$

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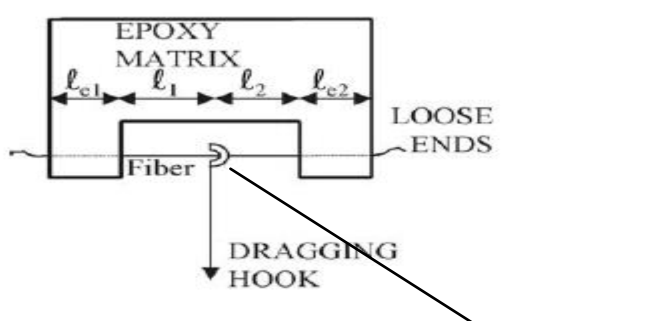
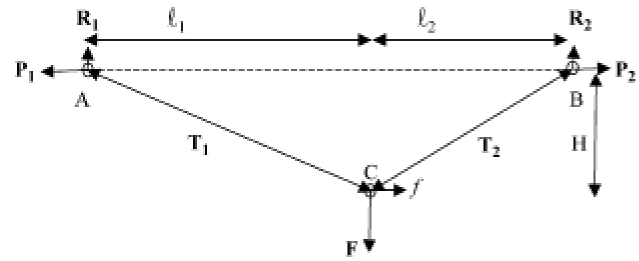
The previous equations (Nairn model) derived for cylindrical geometry for microbond and pull-out tests in this paper we used a drag-out test using a U-shipped specimen .In drag-out test the knik point (F_d) represent the difference in elastic behavior between the fiber and the matrix induce a shear stress at interface [4], form the force equilibrium for drag-out configuration fig(3-b) and for the case $l_1=l_2=l_{1/2}$ (see appendix [B]) .



(A) Schematic diagram of drag out specimen



(B) Force equilibrium for the drag-out configuration
Fig (3): Drag-out representation test [ref 4]

The pull-out component :

$$P_1 = P_2 = P = \frac{F}{2}\frac{l_{1/2}}{H} \qquad \text{---------- } 6$$

Where :

$l_{1/2}=l_1=l_2$,half of free fiber (when the balance force f= 0)
H= cross head displacement
F= drag out force
P is the pull out force for one side of specimen

$$F = \frac{2AE_fH}{l_{1/2}}\left(1-\frac{l_{1/2}}{\sqrt{H^2+l^2_{1/2}}}\right) \qquad \text{----------------------}7$$

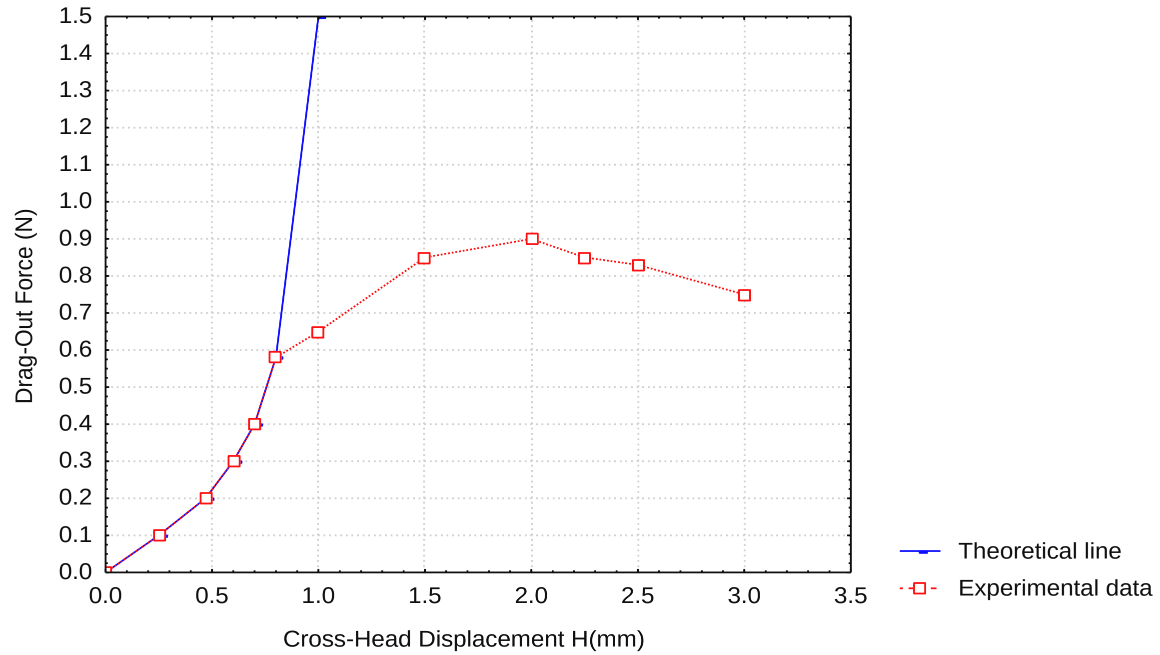
$$P = AE_f\left(1-\frac{l_{1/2}}{\sqrt{H^2+l^2_{1/2}}}\right) \qquad \text{----------------------}8$$

(F) Drag-out force (from force-displacement curve)
(P) The component pull-out force
(E_f) the Modulus of the fiber
(A) fiber cross section

The equations of energy release rate in Nairn model derived for pull-out test ,from equation (7) and (8) the pull-out component .

$$P = \frac{F}{2}\frac{l_{1/2}}{H} \qquad \text{------------------- } 9$$

Equations (7),(8) derived in elastic region and governed by Hook's law before debonding between fiber and matrix occurs. The deflection between the theoretical line fig(4) predict by equation (7) or (8) and the experimental data (drag-out force and cross head displacement) indicate a kink point (F_{dd}) and the drag-out force at peak (F_{dp}) point in which the crack equal approximately the length of embedded length (l_e) [1,6] ,these points can convert to pull-out component using equation (6)



$$F_d = \frac{F_{dd}}{2}\frac{l_{1/2}}{H} \qquad \ldots\ldots\ldots\ldots\ldots..\ 10$$

$$F_p = \frac{F_{dp}}{2}\frac{l_{1/2}}{H} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 11$$

F_d,F_p used in calculating the energy release rate at knik point and at peak point using equations (2) and (5) respectively .

Fig(4):typical graph of deflection between the theoretical line and experimental data in drag-out test

Sample an F_{dp} **ılation Preparation**

A U-shapeu pu1ysiloxane mold of free length fiber (10 mm) and different embedded lengths of fiber as shown in figure (5-A) and (5-B).A kevlar-49(tow) and polyethylene fiber of diameter (250 µm) usec F_{dd} ›olyester matrix ,cured at room temperature for five days before tensile test by Instron(1122) , ΔT in equation (5) equal to (-12C^{o}) and the volume fraction of fiber V_f and matrix V_m in the previous equations (which calculated as in appendix A) taken as the actual volume fraction (fig 6).the drag-out rate used was 0.5 mm/min . Fiber and matrix properties listed in table (1) and table (2).

Table (1) ; The properties of fibers :

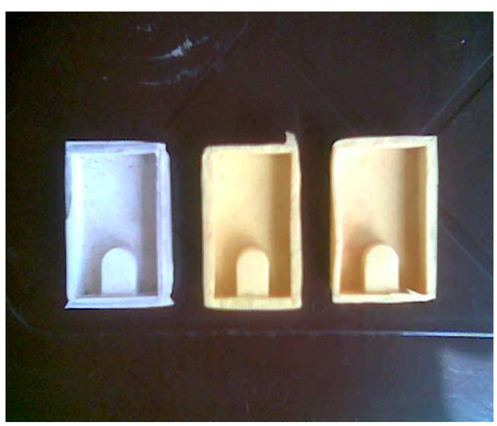
Properties	Kevlar fiber	Polyethylene fiber
Tensile modulus(Gpa)	130	170
Transverse modulus (Gpa)	10	117
Axial shear modulus(Gpa)	15	5.61
Axial poission ratio	0.2	0.32

Transverse poission ratio	0.35	0.61
Axial coefficient of Thermal expansion ppm/c^0	-2	120
Transverse coefficient of Thermal expansion ppm/c^0	-60	48

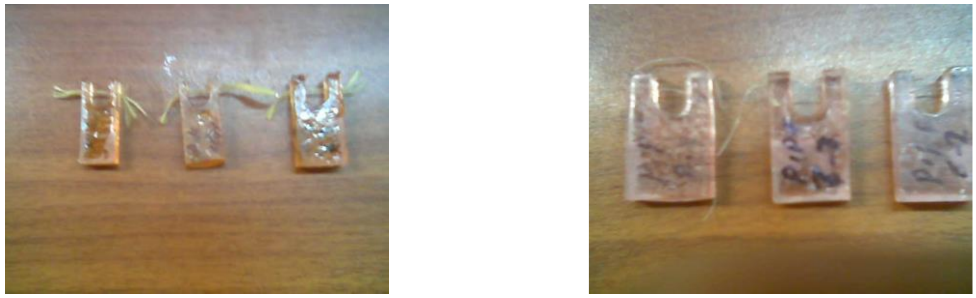
Table (2):The properties of matrix :

Properties	Polyester
Tensile modulus (Gpa)	4
Axial shear modulus (Gpa)	1.16
Axail poission's ratio	0.5
Axial coefficient of Thermal expansion ppm/c^0	70

From tables (1) and (2) the constants in equation (2) and equation (5) calculated as defined in appendix [A] ,using equations (10) and (11) the knik force and the peak force also calculated for graphs in figure (7) and (8) the from equation (2) and (5) the critical energy release rate for initiation of crack and full debonding estimated .



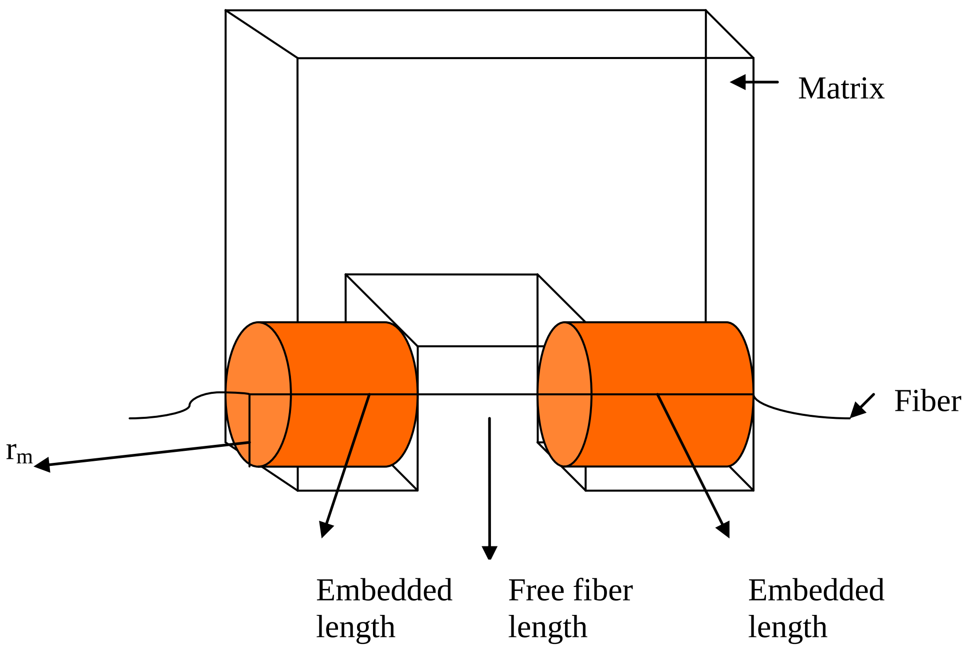
(A)



(B)

Fig(5) A: Polysiloxane U-shaped mold

B: U-shaped drag-out specimen



Fig(6): Matrix radius representation for U-shaped specimens

Results and Discussion:

Figures (7,8) shows the deflection between the theoretical data (equation 7) and the experimental drag-out data from which the knik point (F_d) calculated and the peak point (maximum point in drag-out force-displacement curve) and by using equation (6) the pull-out component calculated .The energy release rate (J/m^2) calculated at these points all the results summarized in table (3) for polyester-kevlar fiber and table (4) for polyester-polyethylene fiber .

For polyester-kevlar fiber the energy release rate initiation of crack (66.17 J/m^2) for all embedded length while for polyester-polyethylene fiber is (approximately 22.4 J/m^2) this indicted that the initiation of crack did not depend on the embedded length of the fiber but depend on the bonding between the fiber and the matrix which is high in polyester-kevlar fiber than in polyester-polyethylene fiber in addition to that the interfibrillar effect due to diffusion of polyester monomer into interfibrillar spaces of kevlar fiber

The energy release rate at peak point for polyester-kevlar (245.7 J/m^2, 267.6J/m^2, 283.7J/m^2) increased as the embedded length increased in the range of test (5.9mm , 6.7mm , 8mm) the increasing is due to increasing in bonding area , while for polyester- polyethylene there is no high difference in the values of energy release rate at peak point because the adhesion depend primely on the shrinkage of polyester matrix.

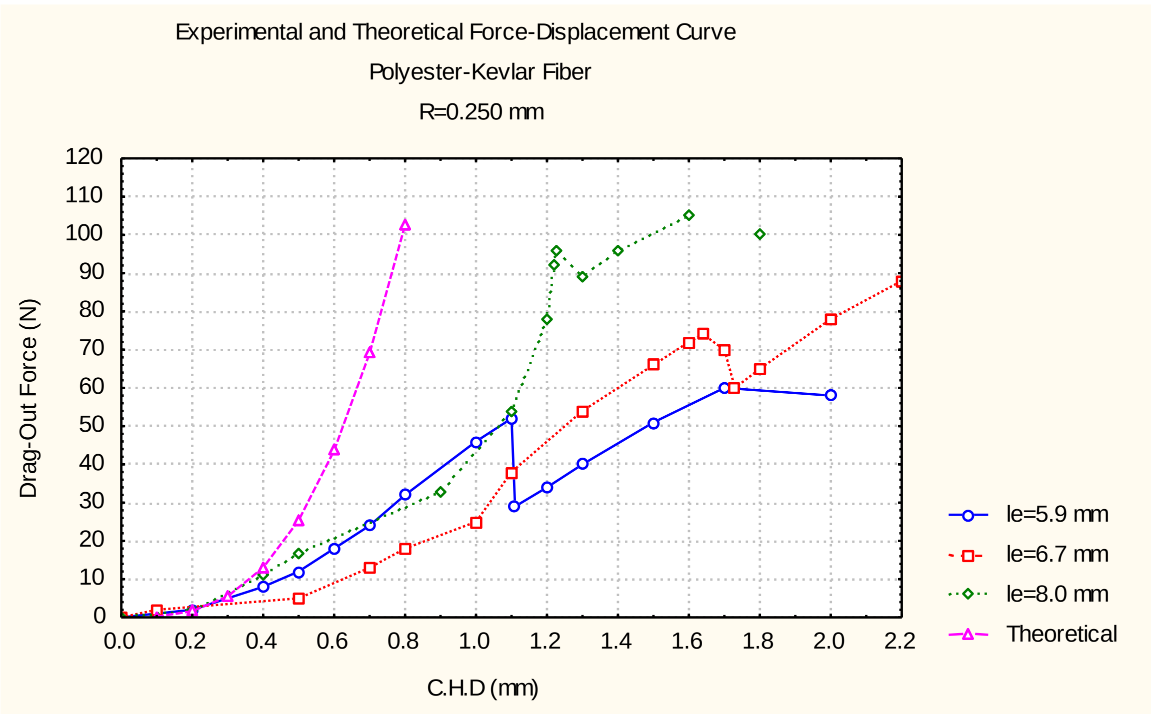
The difference between the energy release rate for knik point (initiation of crack) and peak point (crack of complete debond) approximately four times in polyester-kevlar fiber and (1.7) times in polyester-polyethylene fiber. This can be attributed to the strong adhesive interaction between polyester and kevlar fiber and the diffusion of polyester into interfibrillar spaces of kevlar filaments (wettability effect) while didn't appear in the single polyethylene fiber additional to the poor adhesion between the polyester matrix and polyethylene fiber.

Table-3
Matrix-Fiber: Polyester-Kevlar Fiber
Radius=0.250 mm

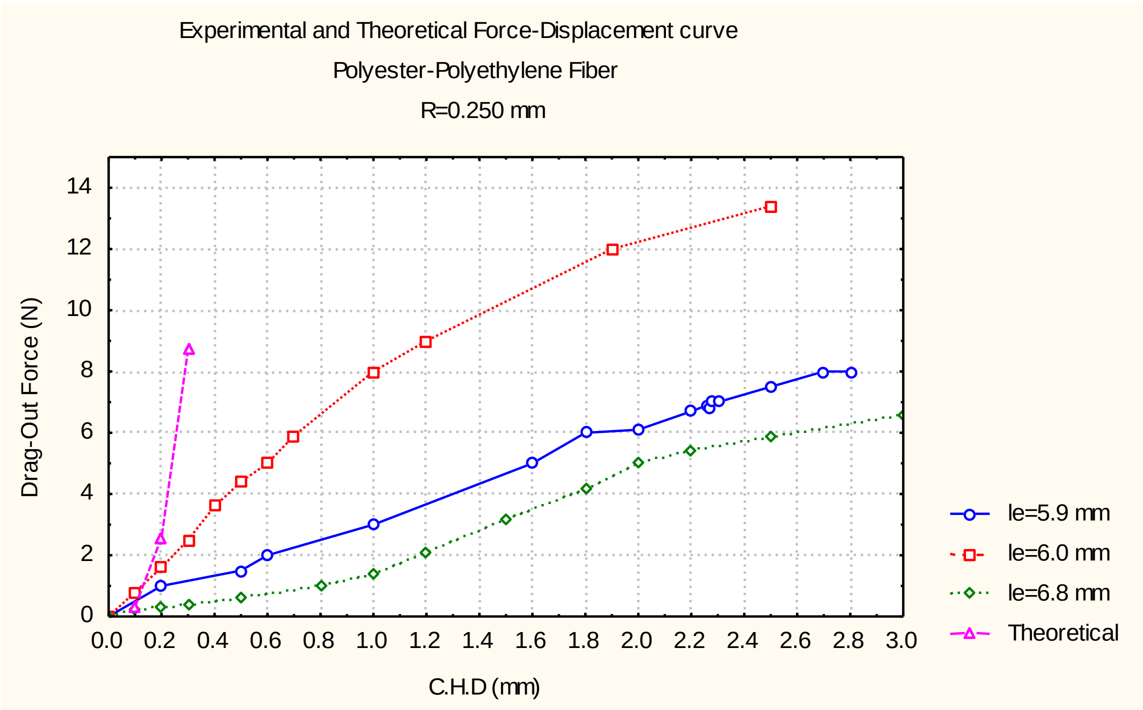
Embedded length(mm)	5.9	6.7	8.0
Drag-out at knik point (N)	7	7	7
Drag-out at peak point(N)	52	74	96
Drag-out friction (N)	58	50	100
Pull-out at knik point(N)	58.3	58.3	58.3
Pull-out at peak point (N)	118.18	112.8	195.12
Pull-out at friction (N)	58	50	138
IFSS at peak point (MPa)	21.2	17.8	25.86
Shear-Lag parameter $\beta(\mu m)^{-1}$	0.00579	0.00579	0.00579
Energy release rate at peak Point (J/m^2)	245.77	267.6	283.7
Energy release rate at knik Point (J/m^2)	66.17	66.17	66.17

Table-4
Matrix-Fiber: Polyester-Polyethylene fiber
Radius: 0.250 mm

Embedded length(mm)	5.9	6.0	6.8
Drag-out at knik point (N)	0.54	0.54	0.46
Drag-out at peak point(N)	6.9	6.2	9.0
Drag-out friction (N)	6.2	5.4	7.0
Pull-out at knik point(N)	13.7	13.7	7.9
Pull-out at peak point (N)	7.632	7.75	9.615
Pull-out at friction (N)	5.78	5	6.04
IFSS at peak point (MPa)	0.747	0.747	0.818
Shear-Lag parameter $\beta(\mu m)^{-1}$	0.000346	0.000346	0.000346
Energy release rate at peak Point (J/m^2)	39.42	38.14	37.26
Energy release rate at knik Point (J/m^2)	22.4	22.8	22.4



Fig(7): Experimental and theoretical drag-out force-displacement curve for polyester matrix and kevlar fiber.



Fig(8): Experimental and theoretical drag-out force-displacement curve for polyester matrix and polyethylene fiber.

Appendix A

The defined *Ai*, *Cij* , and *Di* required for the calculations described in this paper are listed below:

$$\begin{aligned}
v_f A_0 &= \frac{v_m(1-\nu_T)}{E_T} + \frac{v_f(1-\nu_m)}{E_m} + \frac{1+\nu_m}{E_m} \\
A_3 &= -\left(\frac{\nu_A}{E_A} + \frac{v_f \nu_m}{v_m E_m}\right) \\
C_{33} &= \frac{1}{2}\left(\frac{1}{E_A} + \frac{v_f}{v_m E_m}\right) - \frac{v_m A_3^2}{v_f A_0} \\
C_{33_s} &= \frac{1}{2}\left(\frac{1}{E_A} + \frac{v_f}{v_m E_m}\right) \\
D_3 &= -\frac{v_m A_3}{v_f A_0}\left[\alpha_T - \alpha_m\right] + \frac{1}{2}\left[\alpha_A - \alpha_m\right] \\
D_{3_s} &= \frac{1}{2}\left(\alpha_A - \alpha_m\right)
\end{aligned}$$

Here v_f and v_m are the volume fractions of fiber and matrix within the droplet. For the concentric cylinders in macroscopic specimens

$$v_f = \frac{r_f^2}{r_m^2} \qquad \text{and} \qquad v_m = \frac{r_m^2 - r_f^2}{r_m^2}$$

E_A and E_T are the axial and transverse moduli of the fiber,V_A and V_T are the axial and transverse Poisson's ratios of the fiber, E_m is the modulus of the matrix, V_m in the Poisson's ratio of the matrix, α_A and α_T are the axial and transverse thermal expansion coefficients of the fiber, and αm is the thermal expansion coefficient of the matrix. The fiber is treated as transversely isotropic with the axial direction along the axis of the fiber. The results for isotropic fibers are easily generated be setting $E_A = E_T = E_f$, $V_A = V_T = V_f$,and $\alpha_A = \alpha_T = \alpha_f$ where subscript *f* indicates thermomechanical properties of an isotropic fiber. The matrix is here always considered to be isotropic.

Appendix B

The configuration of U-shaped specimen used in drag-out test [4] as in fig (2-6-a) which has a free length and two embedded fiber areas . A force is applied at a point on the free length in direction perpendicular to the fiber , the balance of forces for this configuration is shown in figure (2-6-b) .A hook applies a tensile force (F) at a distance (l_1) and (l_2) from the left (A) and right(B) edges respectively ,(H) is the distance perpendicular to the base line AB the force cause a tension (T) in the fiber and the subscripts 1,2 refer to the left and the right sides respectively .The tension vector with components (P_1) or (P_2) parallel to the base line AB and the component (R_1),(R_2) perpendicular to AB , the parallel components is equal to the pull-out force that act to debond the fiber from the matrix

If $l_1 \neq l_2$ then $p_1 \neq p_2$ and there is horizontal balancing force f

$$P_1=P_1+f \qquad \text{………… (B-1)}$$

$$F=R_1+R_2 \qquad \text{………… (B-2)}$$

And the torque balance

$$R_1 l_1=R_2 l_2+fH \qquad \text{……….. (B-3)}$$

Inserting equation (B-2) in (B-3)

$$R_1 l_1=(F-R_1)l_2+H \qquad \text{………. (B-4)}$$

$$R_1 = \frac{Fl_2 + fH}{l_1 + l_2} \qquad \text{…………… (B-5)}$$

$$R_2 = \frac{Fl1 - fH}{l_1 + l_2} \qquad \text{…………… (B-6)}$$

The ratio between P and R equal to the ratio between l and H

$$\frac{P_1}{R_1} = \frac{l_1}{H} \qquad \text{…………… (B-7)}$$

And

$$\frac{P_2}{R_2} = \frac{l_2}{H} \qquad \text{………….. (B-8)}$$

Isolating P in equations (B-7),(B-8) and inserting the expressions for R from equations (B-5),(B-6) gives:

$$P_1 = \frac{Fl_2 + fH}{l_1 + l_2} \times \frac{l_1}{H} \qquad \text{……… (B-9)}$$

And

$$P_2 = \frac{Fl_1 - fH}{l_1 + l_2} \times \frac{l_2}{H} \qquad \text{……….. (B-10)}$$

If $l_1 = l_2 = l_{1/2}$ then f=0 and equations B-1,B-7, B-8 become

$$R_1 = R_2 = \frac{F}{2} \qquad \text{……. (B-11)}$$

$$P_1 = P_2 = \frac{F}{2} \times \frac{l_{1/2}}{H} \qquad \text{………… (B-12)}$$

The tension T in the fiber is :

$$T_1 = \sqrt{p_1^2 + R_1^2} \qquad \text{………. (B-13)}$$

$$T_2 = \sqrt{p_2^2 + R_2^2} \qquad \text{……… (B-14)}$$

In the elastic region Hook's law

$\sigma = E\varepsilon$ and $\sigma = \frac{F}{A}$ where A is area of cross section of fiber

$$\frac{F}{A} = E_f \varepsilon$$

If $F = T_1$

Then:

$$\frac{T_1}{A} = E_f \varepsilon$$

$$\frac{T_1}{AE_f} = \varepsilon = \frac{\sqrt{H^2 + l_1^2} - l_1}{l_1}$$

Right-hand side from the geometry of fig (3-b)

$$\sqrt{H^2 + l_1^2} = l_1\left(1 + \frac{T_1}{AE_f}\right) \qquad \text{....... (B-15)}$$

In the same way

$$\sqrt{H^2 + l_2^2} = l_2\left(1 + \frac{T_2}{AE_f}\right) \qquad \text{.. ..... (B-16)}$$

If $l_1 = l_2 = l_{1/2}$ then $P_1 = P_2 = P$ and $T_1 = T_2 = T$

$$\sqrt{H^2 + l^2_{1/2}} = l_{1/2}\left(1 + \frac{\sqrt{P^2 + R^2}}{AE_f}\right)$$

From equation (2- 28)

$$\sqrt{H^2 + l^2_{1/2}} = l_{1/2}\left(1 + \frac{\sqrt{R^2\left(\frac{l_{1/2}}{H}\right)^2 + R^2}}{AE_f}\right)$$

In equation (B-12) , $R_1 = R_2$ if $l_1 = l_2 = l_{1/2}$ then:

$$R = \frac{F}{2}$$

So

$$\sqrt{H^2 + l^2_{1/2}} = l_{1/2}\left(1 + \frac{F}{AE_f}\sqrt{\left(\frac{l_{1/2}}{H}\right)^2 + 1}\right)$$

$$F = \frac{2AE_fH}{l_{1/2}}\left(1 - \frac{l_{1/2}}{\sqrt{H^2 + l^2_{1/2}}}\right) \qquad \text{.......... (B-17)}$$

From equation (B-17) if $l_1 = l_2 = l_{1/2}$

$$P = R\frac{l_{1/2}}{H} = \frac{l_{1/2}}{2}\times\frac{F}{H}$$ ………. (B-18)

And

$$P = AE_f\left(1-\frac{l_{1/2}}{\sqrt{H^2+l^2_{1/2}}}\right)$$ ………….. (B-19)

Where F is the applied force in drag-out test while P if the pull-out force

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