Al-Qadisiyah Journal of Pure Science Vol.(27) Issue (1)(2022) math pp. x-x

Al-Qadisiyah Journal of Pure Science

ISSN(Printed): 1997-2490 ISSN(Online): 2411-3514

DOI: /10.29350/jops.

http://qu.edu.iq/journalsc/i

Soft i-Open Sets in Soft Bi-Topological Spaces

Authors Names

a. ***Sabih W. Askandar***

b. Beyda S. Abdullah

c. Luma Ahmed Khaleel

Article History

Received on: 4 / 12/2022
Revised on: 15 / 12 /2022
Accepted on: 22 /12 /2022

Keywords s: Soft i-open sets, soft bi-topological spaces, i-open sets, soft topological space.

DOI: https://doi.org/10.29350/jops. 2022.27. 1.1592

ABSTRACT

In our study we introduced soft i-open sets and soft i-star-generalized-w-closed sets in soft bi-topological spaces, (X,τ_1,τ_2,E), using the notion of soft i-open sets in soft-topological-space, (X,τ,E). We besides that give examples to clarify these relationships while presenting some essential characteristics and relationships between various groups of sets. Besides that we studied the property of soft bi-topologically extended and non-soft bi-topologically extended of soft i-open sets in soft bi-topological spaces by proofs and examples.

1. Introduction

g*-closed sets were first introduced to bi-topological spaces in 2004 ([13]) by Sheik and Sundaram. Al- Zoubi in 2005, studied the concept of generalized w-closed sets([1]). Introducing regular star generalized closed sets in bitopological spaces was done by Kannan in 2006 ([5]). Mahdi first discussed semi-open and semi-closed sets in bitopological spaces in 2007 ([8]). R. K., Chandrasekhara and D., Narasemhan, in **2009**, defined semi Star Generalized w-closed sets in Bitopological spaces([4]). w-locally closed sets in bitopological spaces were first introduced by Benchalli, Patil, and Rayanagoudar in 2010 ([3]). In 2010 ([12]), Sheik and Maragathavalli first discussed the idea of strongly $\alpha g*-$ closed sets in bitopological spaces. The concepts of GRW-closed sets and GRW-continuity in bitopological spaces were presented by Nagaveni and Rajarubi in 2012 ([11]). Askandar and Mohammed, A subset W of a topological space (X,τ) is referred to as an i-open set ([9]) if there is an open set $O\neq\phi$, X s.t. $W\subseteq Cl(W\cap O)$. They could be combined with many other ideas of generalized open sets. The i-closed set is the complement of an i-open set, also they introduced the i-open sets in bi-topological spaces. Molodtsov established the idea of soft sets and their characteristics in 1999([10]). Askandar Mohammed In 2020 defined a soft i-open set (in short $s\tau$-$open\ set$)([2]) in a single soft topological space as follows: a soft set (W,E) is a soft i-open($SIOS$) in (X,τ,E), whether a soft open set $(O,E)\neq\phi,X$ is existent where $(W,E)\tilde{\subseteq} Cl((W,E)\tilde{\cap}(O,E))$. In 1963([7]),Kelly, J.C., defined the concept of bi-topological Spaces. In 2014, Ittanagi, B. M., defined the concept of soft bi-topological spaces([6]) (in short $S.BI.T.S.$)as follows: if (X,τ_1,E) and (X,τ_2,E) are two different soft topologies on X, then (X,τ_1,τ_2,E) is called a soft bi-topological space($S.BI.T.S$). In this study, the idea of soft i-open sets in soft bi-topological spaces (X,τ_1,τ_2,E) is introduced. This set class might be introduced together with other soft set classes, objects have been listed above for comparison in order to identify common characteristics and qualities. $s\tau^i$ is a family of all soft i-open sets of X. There are two components to this work. In the first, soft i-open sets in soft bi-topological spaces are defined, and numerous instances are provided. We create soft i-star generalized w-closed sets and soft i-star generalized w-open sets in the second part and look at their fundamental features in soft bi-topological spaces. (X,τ_1,τ_2,E) denotes a soft bi-topological space, where (X,τ_1,E) and (X,τ_2,E) are soft-topological-spaces. For $(W,E)\subseteq X$, $s\tau_i$-$Int(M,E)$ and $s\tau_i$-$Cl(M,E)$ denote the soft interior, soft closure of a soft set (M,E) with respect to the soft topology τ_i. A point $x\in X$ is called a condensation point of (M,E) if for each $(U,E)\in\tau$ with $x\in(U,E)$, the set $(U,E)\cap(M,E)$ is uncountable. (M,E) is called soft $w-closed$ if it contains all its condensation points. soft $w-open$ set is the complement of soft $w-closed$ set. The soft $w-closure$ and soft $w-\text{int}erior$ of (M,E) designated by $s\tau_i$-$Cl_W(M,E)$ and $s\tau_i$-$int_W(M,E)$, respectively. $(M,E)^C$ Denote the soft complement of (M,E) in X. $S.BI.T.S$ denotes soft bi-topological space, sTs denotes soft topological space.

1. Soft i-Open Sets in Soft Bi-Topological Spaces.

With using numerous related instances, we define soft i-open sets and many other concepts of soft generalized open sets in soft bi-topological spaces in this part. We also examine the sets' characteristics.

Definition1.1. Let (X,τ_1,τ_2,E) be *S.BI.T.S*, a subset *(M,E)* of X is called "$(s\tau_1\tau_2\text{-}i\text{-}open\ set)$" assuming there is $s\tau_1\text{-}openset$ $(U,E)\neq\varphi,X$ s.t. $(W,E)\subseteq s\tau_2\text{-}Cl((W,E)\cap(U,E))$. The complement of $(s\tau_1\tau_2\text{-}i\text{-}open\ set)$ is called $s\tau_1\tau_2\text{-}i\text{-}closed\ set)$.

Definition1.2. A *S.BI.T.S* (X,τ_1,τ_2,E) is called "soft Bi-Topologically Extended" for *SIOS* in short (*S.Bi.T.E.S.I.*) if $(X,s\tau_1\tau_2\text{-}i\text{-}opensets)$ is *sTs*. On the other hand, if $(X,s\tau_1\tau_2\text{-}i\text{-}opensets)$ is not *sTs*, then, (X,τ_1,τ_2,E) is called "not soft Bi-Topologically Extended" for *SIOS* (not *S.Bi.T.E.S.I.*). Where, $(X,s\tau_1\tau_2\text{-}i\text{-}opensets)$ denote the family of all *SIOS* in the *S.BI.T.S* (X,τ_1,τ_2,E).

Example1.3. Let $X=\{r,z,w\}$, $\tau_1=\{\varphi,(M_1,E),X\}$, $\tau_2=\{\varphi,(M_1,E),(M_2,E),X\}$, $E=\{e_1,e_2\}$.
Where, $(M_1,E)=\{(e_1,\{r\}),(e_2,\{r\})\}$, $(M_2,E)=\{(e_1,\{r,z\}),(e_2,\{r,z\})\}$.

$s\tau_1\text{-}open\ sets\ are:\ \varphi,(M_1,E),X$. $s\tau_2\text{-}closed\ sets\ are:$
$\phi,(M_1,E)^C=\{(e_1,\{z,w\}),(e_2,\{z,w\})\},(M_2,E)^C=\{(e_1,\{w\}),(e_2,\{w\})\},X$.

$(M_1,E)\subset(s\tau_2-Cl((M_1,E)\cap(M_1,E))=X)$,
$(M_2,E)\subset(s\tau_2-Cl((M_2,E)\cap(M_1,E))=X)$
$\{(e_1,\{r,w\}),(e_2,\{r,w\})\}\subset(s\tau_2-Cl(\{(e_1,\{r,w\}),(e_2,\{r,w\})\}\cap(M_1,E))=X)$.

Then, $(M_1,E),(M_2,E)\ \{(e_1,\{a,c\}),(e_2,\{a,c\})\}$, are $s\tau_1\tau_2-i-opensets$ .But, $\{(e_1,\{z\}),(e_2,\{z\})\},\{(e_1,\{w\}),(e_2,\{w\})\},\{(e_1,\{z,w\}),(e_2,\{z,w\})\}$ are not $s\tau_1\tau_2\text{-}i-opensets$, because there is no exist $s\tau_1\text{-}openset\ U$ s.t., $\{(e_1,\{z\}),(e_2,\{z\})\}\subset(s\tau_2-Cl(\{(e_1,\{z\}),(e_2,\{z\})\}\cap U))$.

$\{(e_1,\{z,w\}),(e_2,\{z,w\})\}\subset(s\tau_2-Cl(\{(e_1,\{z,w\}),(e_2,\{z,w\})\}\cap U))$.

$\{(e_1,\{w\}),(e_2,\{w\})\}\subset(s\tau_2-Cl(\{(e_1,\{w\}),(e_2,\{w\})\}\cap U))$.

Therefore, $s\tau_1\tau_2\text{-}i\text{-}open\ sets=\{\phi,(M_1,E),(M_2,E),\{(e_1,\{r,w\}),(e_2,\{r,w\})\},X\}$.

$s\tau_1\tau_2\text{-}i\text{-}closed\ sets=\phi,\ (M_1,E)^C,(M_2,E)^C$, $\{(e_1,\{z\}),(e_2,\{z\})\}$,

Where, $(X,s\tau_1\tau_2\text{-}i\text{-}opensets)$ is *sTs*. Then, (X,τ_1,τ_2,E) is a *S.Bi.T.E.S.I.*

Example1.4. Let $X=\{r,u,z,w\}$, $\tau_1=\{\phi,(M_1,E),(M_2,E),X\}$,

$\tau_2=\{\phi,(M_1,E),(M_3,E),(M_4,E),X\}$, $E=\{e_1,e_2\}$.Where, $(M_1,E)=\{(e_1,\{r\}),(e_2,\{r\})\}$,
$(M_2,E)=\{(e_1,\{u,z,w\}),(e_2,\{u,z,w\})\}$. $(M_3,E)=\{(e_1,\{z\}),(e_2,\{z\})\}$, $(M_4,E)=\{(e_1,\{r,z\}),(e_2,\{r,z\})\}$.

$s\tau_1\text{-}open\ sets\ are:\ \phi,(M_1,E),(M_2,E),X$. $s\tau_2\text{-}closed\ sets\ are:$
$\phi,(M_1,E)^C=\{(e_1,\{u,z,w\}),(e_2,\{u,z,w\})\},(M_3,E)^C=\{(e_1,\{r,u,w\}),(e_2,\{r,u,w\})\}$,
$(M_4,E)^C=\{(e_1,\{u,w\}),(e_2,\{u,w\})\}X$

By the same way, in Example 1.3, we have: , $s\tau_1\tau_2 - i - opensets$ are ϕ, (M_1, E), $\{(e_1,\{u\}),(e_2,\{u\})\}$, $\{(e_1,\{z\}),(e_2,\{z\})\}$, $\{(e_1,\{w\}),(e_2,\{w\})\}$, $\{(e_1,\{u,z\}),(e_2,\{u,z\})\}$, $\{(e_1,\{u,w\}),(e_2,\{u,w\})\}$, $\{(e_1,\{z,w\}),(e_2,\{z,w\})\}$, (M_2,E), $\{(e_1,\{r,u\}),(e_2,\{r,u\})\}$, $\{(e_1,\{r,w\}),(e_2,\{r,w\})\}$, $\{(e_1,\{r,u,w\}),(e_2,\{r,u,w\})\}$, X .

$s\tau_1\tau_2$ - i - *closed sets* $= \phi$, $\{(e_1,\{u,z,w\}),(e_2,\{u,z,w\})\}$, $\{(e_1,\{r,z,w\}),(e_2,\{r,z,w\})\}$, $\{(e_1,\{r,u,w\}),(e_2,\{r,u,w\})\}$, $\{(e_1,\{r,u,z\}),(e_2,\{r,u,z\})\}$, $\{(e_1,\{r,w\}),(e_2,\{r,w\})\}$, $\{(e_1,,\{r,z\}),(e_2,\{r,z\})\}$, $\{(e_1,\{r,u\}),(e_2,\{r,u\})\}$, $\{(e_1,\{r\}),(e_2,\{r\})\}$, $\{(e_1,\{z,w\}),(e_2,\{z,w\})\}$, $\{(e_1,\{u,z\}),(e_2,\{u,z\})\}$, $\{(e_1,\{z\}),(e_2,\{z\})\}$, X .

Where, $(X, s\tau_1\tau_2\text{-}i\text{-}opensets)$ is not *sTs*, Then, (X,τ_1,τ_2,E) is not a *S.Bi.T.E.S.I*.

Definition1.5. Let (X, τ^i, E) be *sTs* and let *(W,E)* be a soft subset of X . Recall that the term "soft i-closure of*(W,E)*" is the intersection of all soft i-closed sets(*SICS*) that contain *(W,E)*, designated by $sCl_i(W,E)$: $sCl_i(W,E) = \bigcap_{i\in\Lambda}(F_i,E).(W,E)\subseteq(F_i,E)\ \forall i$ where, (F_i,E) is *SICS* $\forall i$ in (X, τ^i, E). $sCl_i(W,E)$ is the smallest *SICS* containing *(W,E)*.

Definition1.6. Let (X, τ^i, E)be *sTs* and let *(W,E)* be a soft subset of X. Recall that the union of all *SIOS* contained in *(W,E)* is named soft i-interior of *(W,E)*, denoted by $sInt_i(W,E)$. $sInt_i(W,E) = \bigcup_{i\in\Lambda}(I_i,E)$ $(I_i,E)\subseteq(W,E)$ $\forall i$. Where, (I_i,E) is *SIOS* $\forall i$ in (X, τ^i, E). $sInt_i(W,E)$ is the largest *SIOS* contained in *(W,E)*.

Theorem1.7. Each $s\tau_1$ - *open set* is *si - openset* in (X,τ_1,τ_2,E) or ($s\tau_1 \subset (s\tau_1\tau_2$ - i – open sets)).

Proof: Assume that *X* is a finite nonempty set.

Let $\tau_1 = \{\phi,(W_1,E),(W_2,E),.......(W_n,E),X\}$, $\tau_2 = \{\phi,(Z_1,E),(Z_2,E),.......,(Z_n,E),X\}$.

Where, $(W_i,E)\subset X, (Z_i,E)\subset X\ \forall i$. $s\tau_1$ - *open sets are :* $\phi,(W_1,E),(W_2,E),.......(W_n,E),X$.

$s\tau_2$ - *closed sets are :* $\phi,(Z_1,E)^C,(Z_2,E)^C,.......,(Z_n,E)^C,X$.

$s\tau_2\text{-}Cl((W_i,E)\cap(W_i,E)) = \bigcap_{(W_i,E)\cap(W_i,E)\subset(F,E)}(F,E)$, where (F,E) is $s\tau_2$ - *closedset* .

At least, X is a $s\tau_2$ - *closed set* contains $(W_i,E)\cap(W_i,E)$ $\forall i$. Hence, $s\tau_2\text{-}Cl((W_i,E)\cap(W_i,E)) = \bigcap_{(W_i,E)\cap(W_i,E)\subset(F,E)}(F,E) = X$ .Therefore, $(W_i,E)\subset(s\tau_2\text{-}Cl((W_i,E)\cap(W_i,E)) = \bigcap_{(W_i,E)\cap(W_i,E)\subset(F,E)}(F,E) = X$, $\forall i$ .Then, ($s\tau_1 \subset (s\tau_1\tau_2$ - i – open sets)).

Theorem 1.7's converse is untrue. In fact, Example 1.4, $\{(e_1,\{u,z\}),(e_2,\{u,z\})\}$ is $s\tau_1\tau_2$ - i - *openset* , but it is not $s\tau_1$ - open set.■

Definition1.8. Recall that extension $s\tau^i$ is the family of all *SIOS* subsets of space X.

Remark 1.9. ***(X, τ^i,E)*** need not to be *sTs***.**

Definition1.10. A *sTs* (X,τ,E) is called soft topologically extended for *SIOS* (shortly *S.T.E.S.I.)* if and only if (X, τ^i,E) is *sTs*. If not, it's referred to as not *S.T.E.S.I.*

Theorem1.11. Let $X \neq \phi$ be a finite set, with $\tau=\{\varphi,(W,E),X\}$ where, *(W,E)* is a soft single subset of *X* (containing only one element). Then, (X, τ,E) is *S.T.E.S.I.* (i.e. (X, τ^i,E) is *sTs*).

Corollary1.12. Let (X,τ_1,τ_2,E) be a *S.BI.T.S* and let (X,τ_1,E) be a (*S.T.E.S.I.*). Similar to Theorem 1.11, let $s\tau_2 = s\tau_1^i$ where, $s\tau_1^i$ the largest family of all *SIOS* in (X,τ_1,E), then, $s\tau_1\tau_2$ - $i-opensets = s\tau_2$.

Proof: Assume that $X=\{j_1,j_2,\dots\dots\dots,j_n\}$ and $\tau_1 = \{\phi,\{(e_1,\{j_1\}),(e_2,\{j_1\})\},X\}$.

$s\tau_1$ - *open sets are:* $\phi,\{(e_1,\{j_1\}),(e_2,\{j_1\})\},X$. By definition of soft i-open sets, we have:

$s\tau_1^i=\{\phi,\{(e_1,\{j_1\}),(e_2,\{j_1\})\},\{(e_1,\{j_1,j_2\}),(e_2,\{j_1,j_2\})\},\{(e_1,\{j_1,j_3\}),(e_2,\{j_1,j_3\})\},\dots\dots\dots,$
$\{(e_1,\{j_1,j_n\}),(e_2,\{j_1,j_n\})\},\{(e_1,\{j_1,j_2,j_3\}),(e_2,\{j_1,j_2,j_3\})\},\{(e_1,\{j_1,j_2,j_4\}),(e_2,\{j_1,j_2,j_4\})\},\dots\dots,$
$\{(e_1,\{j_1,j_2,j_n\}),(e_2,\{j_1,j_2,j_n\})\},\dots,\{(e_1,\{j_1,j_3,j_4,j_n\}),(e_2,\{j_1,j_3,j_4,j_n\})\},$
$\{(e_1,\{j_1,j_2,j_3,..,j_n\}),(e_2,\{j_1,j_2,j_3,..,j_n\})\}=X\}$. Since, $s\tau_2 = s\tau_1^i$, Then, $s\tau_2-closed\ sets are$:
$\{(e_1,\{j_1,j_3,j_4,j_n\}),(e_2,\{j_1,j_3,j_4,j_n\})\}=X\{(e_1,\{j_3,j_4,\dots\dots,j_n\}),(e_2,\{j_3,j_4,\dots\dots,j_n\})\},$
$\{(e_1,\{j_2,j_4,\dots\dots,j_n\}),(e_2,\{j_2,j_4,\dots\dots,j_n\})\},\dots\dots,\{(e_1,\{j_2,\dots\dots,j_{n-1}\}),(e_2,\{j_2,,\dots\dots,j_{n-1}\})\},$
$\{(e_1,\{j_4,\dots\dots,j_n\}),(e_2,\{j_4,\dots\dots,j_n\})\},\{(e_1,\{j_3,j_5,\dots\dots,j_n\}),(e_2,\{j_3,j_5,\dots\dots,j_n\})\},..,$
$\{(e_1,\{j_3,\dots\dots,j_{n-1}\}),(e_2,\{j_3,\dots\dots,j_{n-1}\})\},\dots,\{(e_1,\{j_2\}),(e_2,\{j_2\})\},\phi$.

Since, $\{(e_1,\{j_1\}),(e_2,\{j_1\})\}$ is the alone s $s\tau_1-openset\neq\phi,X$ and the intersection between $\{(e_1,\{j_1\}),(e_2,\{j_1\})\}$ and the soft sets $\{(e_1,\{j_2\}),(e_2,\{j_2\})\},\{(e_1,\{j_3\}),(e_2,\{j_3\})\},\dots,$
$\{(e_1,\{j_n\}),(e_2,\{j_n\})\},\dots,\{(e_1,\{j_2,j_3\}),(e_2,\{j_2,j_3\})\},..,\{(e_1,\{j_2,j_n\}),(e_2,\{j_2,j_n\})\},$
$\{(e_1,\{j_2,j_3,j_4\}),(e_2,\{j_2,j_3,j_4\})\},\dots.,\{(e_1,\{j_2,j_3,j_n\}),(e_2,\{j_2,j_3,j_n\})\},..,$
$\{(e_1,\{j_3,j_4,j_n\}),(e_2,\{j_3,j_4,j_n\})\},\dots,\{(e_1,\{j_3,j_4,j_n\}),(e_2,\{j_{n-2},j_{n-1},j_n\})\}$ which does not contain $\{(e_1,\{j_1\}),(e_2,\{j_1\})\}$, equal to ϕ, similarly, in Theorem 1.11, we have:

$s\tau_1\tau_2 - i-opensets = s\tau_2$ where, $s\tau_2 = s\tau_1^i$. ■

Example1.13. Let $X=\{r,z,w\}$, $E=\{e_1,e_2\}$ $s\tau_1=\{\phi,\{(e_1,\{r\}),(e_2,\{r\})\},X\}$,

$s\tau_2 = s\tau_1^i = \{\phi,\{(e_1,\{r\}),(e_2,\{r\})\},\{(e_1,\{r,z\}),(e_2,\{r,z\})\},\{(e_1,\{r,w\}),(e_2,\{r,w\})\},X\}$
$s\tau_2 - closed\ sets are\ \phi,\{(e_1,\{z,w\}),(e_2,\{z,w\})\},\{(e_1,\{w\}),(e_2,\{w\})\},\{(e_1,\{z\}),(e_2,\{z\})\},X$

$s\tau_1\tau_2 - i-opensets = s\tau_2$.

Definition 1.14. A subset (M,E) of *S.BI.T.S* (X,τ_1,τ_2,E) is called:

1. $s\tau_1\tau_2-generalized\ closed\ set\ (s\tau_1\tau_2-g-closed\,set)$ if $s\tau_2-Cl(M,E)\subseteq(U,E)$ where $(M,E)\subseteq(U,E)$ and $(U,E)\subseteq X$ is $s\tau_1-openset.$

2. $s\tau_1\tau_2-g-openset$ if $(M,E)^C$ is $s\tau_1\tau_2-g-closed.$

3. $s\tau_1\tau_2-g\,i-openset$ if $(F,E)\subseteq s\tau_2-Int_i(M,E)$ where $(F,E)\subseteq(M,E)\subseteq X$ is $s\tau_1-closed\,set.$

4. $s\tau_1\tau_2-g\,i-closed\,set$ if $(M,E)^C$ is $s\tau_1\tau_2-g\,i-open.$.

5. $s\tau_1\tau_2-i-star\,genralzedclosed\,set\ (s\tau_1\tau_2-i*g-closed\,set)$ if $s\tau_2-Cl(M,E)\subseteq(U,E)$ where, $(M,E)\subseteq(U,E)$ and $(U,E)\subseteq X$ is $s\tau_1-\ i-openset.$

6. $s\tau_1\tau_2-i-star\,genralzedopenset\ (s\tau_1\tau_2-i*g-openset)$ if $(M,E)^C$ is $s\tau_1\tau_2-i*g-closed.$

7. $s\tau_1\tau_2-genralzedw-closed\,set\,(s\tau_1\tau_2-gw-closed\,set)$ if $s\tau_2-Cl_w(A)\subseteq(U,E)$ where $(M,E)\subseteq(U,E)$ and $(U,E)\subseteq X$ is $s\tau_1-openset.$

8. $s\tau_1\tau_2-genralzedw-openset\,(s\tau_1\tau_2-gw-openset)$ if $(M,E)^C$ is $s\tau_1\tau_2-gw-closed.$

Example 1.15. Let $X=\{q,r,z\}$ (Finite), $\tau_1=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},X\}$ $\tau_2=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},X\}$. $E=\{e_1,e_2\}$

$s\tau_1-opensets=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},X\}$, $s\tau_1-closed\,sets=\{\phi,\{(e_1,\{r,z\}),(e_2,\{r,z\})\},X\}$

$s\tau_1$ - w - $closed\ sets=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},\{(e_1,\{r\}),(e_2,\{r\})\},\{(e_1,\{z\}),(e_2,\{z\})\},$ $\{(e_1,\{q,r\}),(e_2,\{q,r\})\},\{(e_1,\{q,z\}),(e_2,\{q,z\})\},\{(e_1,\{r,z\}),(e_2,\{r,z\})\},X\}=s\tau_1$ - w - $opensets$

$s\tau_1$ - i - $opensets=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},\{(e_1,\{q,r\}),(e_2,\{q,r\})\},\{(e_1,\{q,z\}),(e_2,\{q,z\})\},X\}$

$s\tau_1$ - i - $closed\,sets=\{\phi,\{(e_1,\{r,z\}),(e_2,\{r,z\})\},\{(e_1,\{z\}),(e_2,\{z\})\},\{(e_1,\{r\}),(e_2,\{r\})\},,X\}$

$s\tau_2-opensets=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},X$, $s\tau_2-closed\,sets:\phi,\{(e_1,\{r,z\}),(e_2,\{r,z\})\},X$

$s\tau_2$ - w - $closed\ sets=\phi,\{(e_1,\{q\}),(e_2,\{q\})\},\{(e_1,\{r\}),(e_2,\{r\})\},\{(e_1,\{z\}),(e_2,\{z\})\},$ $\{(e_1,\{q,r\}),(e_2,\{q,r\})\},\{(e_1,\{q,z\}),(e_2,\{q,z\})\},\{(e_1,\{r,z\}),(e_2,\{r,z\})\},X\}$

$=s\tau_2$ - w - $opensets$

$s\tau_2$ - i - $opensets=\{\phi,\{(e_1,\{q\}),(e_2,\{q\})\},\{(e_1,\{q,r\}),(e_2,\{q,r\})\},\{(e_1,\{q,z\}),(e_2,\{q,z\})\},X\}$

$s\tau_2$ - i - $closed\,sets=\{\phi,\{(e_1,\{r,z\}),(e_2,\{r,z\})\},\{(e_1,\{z\}),(e_2,\{z\})\},\{(e_1,\{r\}),(e_2,\{r\})\},,X\}$

$s\tau_1\tau_2$ - g - $closed\,sets=\{\phi,\{(e_1,\{r\}),(e_2,\{r\})\},,\{(e_1,\{z\}),(e_2,\{z\})\},\{(e_1,\{q,r\}),(e_2,\{q,r\})\},$ $\{(e_1,\{q,z\}),(e_2,\{q,z\})\}\{(e_1,\{r,z\}),(e_2,\{r,z\})\},,X\},$

$\{(e_1,\{a\}),(e_2,\{a\})\}$, is not $s\tau_1\tau_2-g-closed\,set\ \ because$

$s\tau_2-Cl(\{(e_1,\{q\}),(e_2,\{q\})\})=X\subseteq X$
$but\ s\tau_2-Cl(\{(e_1,\{q\}),(e_2,\{q\})\})=X\not\subset\{(e_1,\{q\}),(e_2,\{q\})\}(definition(1.14)(1))$.

$s\tau_1\tau_2\text{-}g\text{-}open\,sets=\{\phi,\ \{(e_1,\{q,z\}),(e_2,\{q,z\})\},\{(e_1,\{q,r\}),(e_2,\{q,r\})\},\ \{(e_1,\{z\}),(e_2,\{z\})\},$
$\{(e_1,\{r\}),(e_2,\{r\})\},,\{(e_1,\{q\}),(e_2,\{q\})\},,X\}$ But, $\{(e_1,\{r,z\}),(e_2,\{r,z\})\}$, is not $s\tau_1\tau_2-g-open\,set$ because, $\{(e_1,\{q\}),(e_2,\{q\})\}^C=\{(e_1,\{r,z\}),(e_2,\{r,z\})\}$, and $\{(e_1,\{q\}),(e_2,\{q\})\}$,
$is\ not\ s\tau_1\tau_2-g-closed\,set(difinition1.14(2))$

$s\tau_1\tau_2-gi-open\,sets=\{\phi,X\},\ s\tau_1\tau_2-gi-closed\,sets=\{\phi,X\}$

$s\tau_1\tau_2\text{-}\mathrm{i}*g\text{-}closed\,sets=\{\phi,\ \{(e_1,\{r,z\}),(e_2,\{r,z\})\},X\}$

$s\tau_1\tau_2\text{-}\mathrm{i}*g\text{-}open\,sets=\{\phi,\ \{(e_1,\{q\}),(e_2,\{q\})\},X\}$

$s\tau_1\tau_2\text{-}\mathrm{g}w\text{-}closed\ sets=\{\phi,\ \{(e_1,\{q\}),(e_2,\{q\})\},\ \{(e_1,\{r\}),(e_2,\{r\})\},\ \{(e_1,\{z\}),(e_2,\{z\})\},$
$\{(e_1,\{q,r\}),(e_2,\{q,r\})\},\ \{(e_1,\{q,z\}),(e_2,\{q,z\})\},\{(e_1,\{r,z\}),(e_2,\{r,z\})\},X\}=s\tau_1\tau_2\text{-}\mathrm{g}w\text{-}open\,sets$

Example 1.16. Let $X=R$(" infinite"), $s\tau_1=\{\phi,R-Q,R\}$ $s\tau_2=\{\phi,Q,R\}$. Where, R is the set of real numbers, Q is the set of rational numbers and $R-Q$ is the set of irrational numbers.

From definitions mentioned above, we have:

$s\tau_1-open\,sets:\phi,R-Q,R\ .\ s\tau_1-w-closed\,sets=\{\phi,R-Q,Q,R\}=s\tau_1-w-open\,sets$,
$s\tau_1-i-open\,sets:\phi,R-Q,R,\ s\tau_2-open\,sets:\phi,Q,R,\ s\tau_2-w-closed\,sets\{\phi,R-Q,Q,R\}$

$s\tau_2-i-open\,sets:\phi,Q,R,\ s\tau_1\tau_2-g-closed\,sets=\{\phi,R-Q,Q,R\}\ s\tau_1\tau_2-gi-open\,sets:\phi,Q,R,$
$s\tau_1\tau_2-i*g-closed\,sets=\{\phi,R-Q,Q,R\},\ s\tau_1\tau_2-gw-closed\,sets=\{\phi,R-Q,Q,R\}$

2. Soft i-Star Generalized w-Closed and Soft i-Star Generalized w-Open Sets in Soft Bi-Topological Spaces.

Definition 2.1. A set (M,E) of *S.BI.T.S* (X,τ_1,τ_2,E) is said to be
$soft\,\tau_1\tau_2-i-star\,genralized\,w-closed\,set\ (s\tau_1\tau_2-i*g\,w-closed\ set\ )$, if
$s\tau_2-Cl_w(M,E)\subseteq(U,E)$, $(M,E)\subseteq(U,E)$ and $(U,E)\subseteq X$ is a $s\tau_1-i-open\,set.$

Example 1.15 shows us: $s\tau_1\tau_2\text{-}\mathrm{i}*\mathrm{g}w\text{-}closed\ sets=\phi,\ \{(e_1,\{q\}),(e_2,\{q\})\},\ \{(e_1,\{r\}),(e_2,\{r\})\},$
$\{(e_1,\{z\}),(e_2,\{z\})\},\ \{(e_1,\{q,r\}),(e_2,\{q,r\})\},\ \{(e_1,\{q,z\}),(e_2,\{q,z\})\},\{(e_1,\{r,z\}),(e_2,\{r,z\})\},X.$

Example 1.16, shows us: $s\tau_1\tau_2-i*gw-closed\,sets:\phi,R-Q,Q,R,$

Remark 2.2. [2] Each soft open set in (X,τ,E) is *SIOS* .

Theorem 2.3. Let (X,τ_1,τ_2,E) be *S.BI.T.S* and $(M,E)\subseteq X$. The following statements are correct:

1. If (M,E) is $s\tau_2 - w - closed$ then, (M,E) is $s\tau_1\tau_2 - i * gw - closed$.
2. If (M,E) is $s\tau_1 - i - open$ and $s\tau_1\tau_2 - i * gw - closed$ then, (M,E) is $s\tau_2 - w - closed$.
3. If (M,E) is $s\tau_1\tau_2 - i * gw - closed$ then, (M,E) is $s\tau_1\tau_2 - gw - closed$.

Proof:

1. Let (M,E) be $s\tau_2 - w - closed$, $(M,E) \subseteq (U,E)$ and $(U,E) \subseteq X$ are $s\tau_1 - i - open$ then $s\tau_2 - Cl_w(M,E) = (M,E) \subseteq (U,E)$. Therefore, (M,E) is $s\tau_1\tau_2 - i * gw - closed$.

2. Assume that (M,E) is $s\tau_1 - i - open$ and $s\tau_1\tau_2 - i * gw - closed$. Let $(M,E) \subseteq (M,E)$ and (M,E) is $s\tau_1 - i - open$. Then, $s\tau_2 - Cl_w(M,E) \subseteq (M,E)$. Therefore, $s\tau_2 - Cl_w(M,E) = (M,E)$. Then, (M,E) is $s\tau_2 - w - closed$.

3. Suppose that (M,E) is $s\tau_1\tau_2 - i * gw - closed$. Let $(M,E) \subseteq (U,E)$ and $(U,E) \subseteq X$ is $s\tau_1 - open$. Since, (U,E) is $s\tau_1 - i - open$ in X "Remark 2.2", we get, $s\tau_2 - Cl_w(M,E) \subseteq (U,E)$. Then, (M,E) is $s\tau_1\tau_2 - gw - closed$. ■

Theorem 2.4. Let (X,τ_1,τ_2,E) be *S.BI.T.S*, then every $s\tau_1\tau_2 - i * g - closed\, set$ in X is $s\tau_1\tau_2 - i * gw - closed$.

Proof: Let (M,E) be $s\tau_1\tau_2 - i * g - closed\, set$, we have, "$s\tau_2 - Cl(M,E) \subseteq (U,E)$", where "$(M,E) \subseteq (U,E)$" and $(U,E) \subseteq X$ are $s\tau_1 - i - openset$.

Since, $s\tau_2 - Cl_w(M,E) \subseteq s\tau_2 - Cl(M,E)$, we get $s\tau_2 - Cl_w(M,E) \subseteq s\tau_2 - Cl(M,E) \subseteq (U,E)$. Therefore, (M,E) is $s\tau_1\tau_2 - i * gw - closed$. ■

Remark 2.5. The inverse of Theorem 2.4 is untrue. In fact, "Example 1.15", *(M,E)= {(e₁,{q,r}),(e₂,{q,r})}* is $s\tau_1\tau_2 - i * gw - closed$ set, but is not $s\tau_1\tau_2 - i * g - closed$.

$$s\tau_1\tau_2 - i^* g - closed\, set \;\; \begin{matrix} \Longrightarrow \\ \not\Longleftarrow \end{matrix} \;\; s\tau_1\tau_2 - i^* gw - closed$$

Theorem 2.6. If (M,E) is $s\tau_1\tau_2 - i * gw - closed\, set$ in X and $(M,E) \subseteq (B,E) \subseteq s\tau_2 - Cl_w(M,E)$, then *(B,E)* is $s\tau_1\tau_2 - i * gw - closed\, set$.

Proof: Suppose that (M,E) is $s\tau_1\tau_2 - i * gw - closed\, set$ in X and $(M,E) \subseteq (B,E) \subseteq s\tau_2 - Cl_w(M,E)$. Let $(B,E) \subseteq (U,E)$ and (U,E) is $s\tau_1 - i - openset$. Then, $(M,E) \subseteq (U,E)$. Since, (M,E) is $s\tau_1\tau_2 - i * gw - closed\, set$, we have $s\tau_2 - Cl_w(M,E) \subseteq (U,E)$. Since, $(B,E) \subseteq s\tau_2 - Cl_w(M,E)$, $s\tau_2 - Cl_w(B,E) \subseteq s\tau_2 - Cl_w(M,E) \subseteq (U,E)$. Hence, (B,E) is $s\tau_1\tau_2 - i * gw - closed$. ■

Theorem 2.7. If (M,E) and (B,E) are $s\tau_1\tau_2 - i*gw-closed\,sets$ then, so is $(M,E)\bigcup(B,E)$.

Proof: Suppose that (M,E) and (B,E) are $s\tau_1\tau_2 - i*gw-closed\,sets$. Let $(U,E)\subseteq X$ be $s\tau_1 - i - openset$ and $(M,E)\subseteq(U,E)$. Then, $(M,E)\bigcup(B,E)\subseteq(U,E)$ and $(B,E)\subseteq(U,E)$. Since, (M,E) and (B,E) are $s\tau_1\tau_2 - i*gw-closed\,sets$, we have, $s\tau_2 - Cl_w(M,E)\subseteq(U,E)$ and $s\tau_2 - Cl_w(B,E)\subseteq(U,E)$. Then, $s\tau_2 - Cl_w((M,E)\bigcup(B,E))\subseteq(U,E)$. Therefore, $(M,E)\bigcup(B,E)$ is $s\tau_1\tau_2 - i*gw-closed\,set.$ ■

Theorem 2.8. Let $(M,E)\subseteq X$, then:

1. If (M,E) is $s\tau_2 - closed$ then, (M,E) is $s\tau_2 - w - closed$.
2. If (M,E) is " $s\tau_1\tau_2 - i*g - closed$" then, (M,E) is $s\tau_1\tau_2 - g - closed$.
3. If (M,E) is "$s\tau_1\tau_2 - g - closed$"then, (M,E) is $s\tau_1\tau_2 - gw - closed$.

Proof:

1.Let (M,E) be $s\tau_2 - closed$ .Then $s\tau_2 - Cl\ (M,E) = (M,E)$.

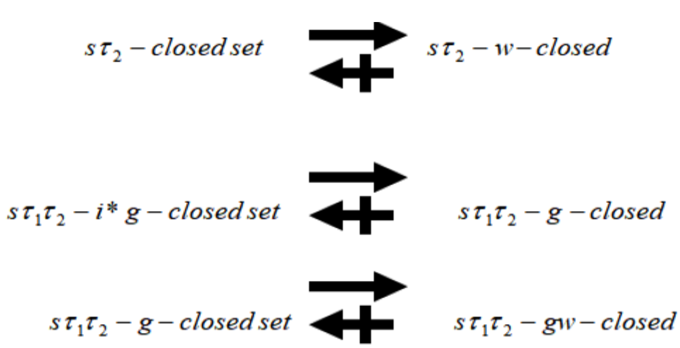
Since, $s\tau_2 - Cl_w(M,E)\subseteq s\tau_2 - Cl\ (M,E) = (M,E)$, we have $s\tau_2 - Cl_w(M,E) = (M,E)$. Therefore, (M,E) is $s\tau_2 - w - closed$.

2. Let (M,E) be $s\tau_1\tau_2 - i*g - closed$. Let "$(M,E)\subseteq(U,E)$" and $(U,E)\subseteq X$ is $s\tau_1 - open$. Therefore, $s\tau_2 - Cl\ (M,E)\subseteq(U,E)$. Then, (M,E) is $s\tau_1\tau_2 - g - closed$.

3. Let (M,E) be $s\tau_1\tau_2 - g - closed$. Let "$(M,E)\subseteq(U,E)$" and $(U,E)\subseteq X$ are $s\tau_1 - open$. Therefore, $s\tau_2 - Cl\ (M,E)\subseteq(U,E)$.

Since $s\tau_2 - Cl_w(M,E)\subseteq s\tau_2 - Cl\ (M,E)\subseteq(U,E)$, we have, $s\tau_2 - Cl_w(M,E)\subseteq(U,E)$. Then, (M,E) is $s\tau_1\tau_2 - gw - closed$. ■

Remark 2.9. Theorem 2.8's converse is untrue. In fact, Example 1.15, *(M,E) = {(e*$_1$*,{q, z}), (e*$_2$*,{q,z})}* is $s\tau_2 - w - closed\,set$, but it is not $s\tau_2 - closed$, *(M,E)* is $s\tau_1\tau_2 - g - closed\,set$, but it is not $s\tau_1\tau_2 - i*g - closed\,set$. Also *(M*$_2$*,E) = {(e*$_1$*,{q}), (e*$_2$*,{q})}* is $s\tau_1\tau_2 - gw - closed$ set but, it is not $s\tau_1\tau_2 - g - closed$.



Definition 2.10. A soft set (M,E) of $S.BI.T.S$ (X,τ_1,τ_2,E) is said to be $s\tau_1\tau_2-i-star\,genralzed\,w-open\,set$ $(shortly(s\tau_1\tau_2-i*gw-open\,set))$, if $(M,E)^C$ is $s\tau_1\tau_2-i*gw-closed\,set.$

Example 1.15, shows us: $s\tau_1\tau_2$ -i*gw-open sets$=\phi$, $\{(e_1,\{q\}),(e_2,\{q\})\}$, $\{(e_1,\{r\}),(e_2,\{r\})\}$, $\{(e_1,\{z\}),(e_2,\{z\})\}$, $\{(e_1,\{q,r\}),(e_2,\{q,r\})\}$, $\{(e_1,\{q,z\}),(e_2,\{q,z\})\}$, $\{(e_1,\{r,z\}),(e_2,\{r,z\})\}$, $X.$

Example 1.16, shows us: $s\tau_1\tau_2-i*gw-open\,sets:\phi,R-Q,Q,R,$

Theorem 2.11. (M,E) is $s\tau_1\tau_2-i*gw-open\,set$ if and only if $(F,E)\subseteq s\tau_2-Int_w(A)$, where $(F,E)\subseteq(M,E)$ and $(F,E)\subseteq X$ is $s\tau_1-i-closed\,set$.

Proof: Assume that (M,E) is $s\tau_1\tau_2-i*gw-open\,set$, $(F,E)\subseteq X$ is $s\tau_1-i-closed\,set$ and $(F,E)\subseteq(M,E)$. Then $(F,E)^C$ is $s\tau_1-i-open$ and $(M,E)^C\subseteq(F,E)^C$. Since, $(M,E)^C$ is $s\tau_1\tau_2-i*gw-closed\,set$, we have $s\tau_2-Cl_w((M,E)^C)\subseteq(M,F)^C$. Since, $s\tau_2-Cl_w((M,E)^C)=[s\tau_2-Int_w(M,E)]^C$, we have, $(F,E)\subseteq s\tau_2-Int_w(M,E)$.

Conversely, suppose that $(F,E)\subseteq s\tau_2-Int_w(M,E)$ where $(F,E)\subseteq(M,E)$ and $(F,E)\subseteq X$ is $s\tau_1-i-closed\,set$. Then, $(M,E)^C\subseteq(F,E)^C$ and $(F,E)^C$ is $s\tau_1-i-open$. Since, $(F,E)\subseteq s\tau_2-Int_w(M,E)$ and $s\tau_2-Cl_w((M,E)^C)=[s\tau_2-Int_w(M,E)]^C$, we have, $s\tau_2-Cl_w((M,E)^C)\subseteq(F,E)^C$. Then, $(M,E)^C$ is $s\tau_1\tau_2-i*gw-closed\,set$. Therefore, (M,E) is $s\tau_1\tau_2-i*gw-open\,set$.■

Theorem 2.12. If (S_1,E) and (S_2,E) are separated $s\tau_1\tau_2-i*gw-open\,sets$, then so is $(S_1,E)\bigcup(S_2,E)$.

Proof: Suppose that (S_1,E) and (S_2,E) are $s\tau_1\tau_2-i*gw-open\,sets$. Let $(F,E)\subseteq X$ be $s\tau_1-i-closed\,set$ and $(F,E)\subseteq(S_1,E)\bigcup(S_2,E)$. Since (S_1,E) and (S_2,E) are separated soft sets, we have, $s\tau_1-Cl(S_1,E)\bigcap(S_2,E)=(S_1,E)\bigcap s\tau_1-Cl(S_2,E)=\phi$.

Also, $s\tau_2-Cl(S_1,E)\bigcap(S_2,E)=(S_1,E)\bigcap s\tau_2-Cl(S_2,E)=\phi$.

Then, $(F,E)\bigcap s\tau_2-Cl(S_1,E)\subseteq((S_1,E)\bigcup(S_2,E))\bigcap s\tau_2-Cl(S_1,E)=(S_1,E)$. By the same way, we have, $(F,E)\bigcap s\tau_2-Cl(S_2,E)\subseteq(S_2,E)$. Since, $(F,E)\subseteq X$ is $s\tau_1-i-closed\,set$, we have, $(F,E)\bigcap s\tau_1-Cl(S_1,E)$ and $(F,E)\bigcap s\tau_1-Cl(S_2,E)$ are $s\tau_1-i-closed\,sets$. Since, (S_1,E) and (S_2,E) are $s\tau_1\tau_2-i*gw-open\,sets$, we have, $(F,E)\bigcap s\tau_2-Cl(S_1,E)\subseteq s\tau_2-Int_w(S_1,E)$ and $(F,E)\bigcap s\tau_2-Cl(S_2,E)\subseteq s\tau_2-Int_w(S_2,E)$.

Now $(F,E)=(F,E)\bigcap((S_1,E)\bigcup(S_2,E))\subseteq((F,E)\bigcap s\tau_2-Cl(S_1,E))\cup((F,E)\bigcap s\tau_2-Cl(S_2,E))$ $\subseteq s\tau_2-Int_w((S_1,E)\bigcup(S_2,E))$. Therefore, $(S_1,E)\bigcup(S_2,E)$ is $s\tau_1\tau_2-i*gw-open\,set.$■

Theorem 2.13. If (S_1,E) and (S_2,E) are $s\tau_1\tau_2-i*gw-opensets$ then so is $(S_1,E)\bigcap(S_2,E)$

Proof: Suppose that (S_1,E) and (S_2,E) are $s\tau_1\tau_2-i*gw-opensets$. Let $(F,E)\subseteq X$ be $s\tau_1-i-closed\,set$ and $(F,E)\subseteq(S_1,E)\bigcap(S_2,E)$, we have $(F,E)\subseteq(S_1,E)$ and $(F,E)\subseteq(S_2,E)$. Since, (S_1,E) and (S_2,E) are $s\tau_1\tau_2-i*gw-opensets$, we have $(F,E)\subseteq s\tau_2-Int_w(S_1,E)$ and $(F,E)\subseteq s\tau_2-Int_w(S_2,E)$.Then $(F,E)\subseteq s\tau_2-Int_w((S_1,E)\bigcap(S_2,E))$. Therefore, $(S_1,E)\bigcap(S_2,E)$ is $s\tau_1\tau_2-i*gw-openset$. ■

Theorem 2.14. If (M_1,E) is $s\tau_1\tau_2-i*gw-openset$ in X and $s\tau_2-Int_w(M_1,E)\subseteq(M_2,E)\subseteq(M_1,E)$, then (M_2,E) is $s\tau_1\tau_2-i*gw-openset$.

Proof: Suppose that (M_1,E) is $s\tau_1\tau_2-i*gw-openset$ in X and $s\tau_2-Int_w(M_1,E)\subseteq(M_2,E)\subseteq(M_1,E)$. Let $(F,E)\subseteq X$ be $s\tau_1-i-closed\,set$ and $(F,E)\subseteq(M_2,E)$. Since, $(F,E)\subseteq(M_2,E)$ and $(M_2,E)\subseteq(M_1,E)$, we have $(F,E)\subseteq(M_1,E)$. Since, (M_1,E) is $s\tau_1\tau_2-i*gw-openset$, we have, $(F,E)\subseteq s\tau_2-Int_w(M_1,E)$ and Since, $s\tau_2-Int_w(M_1,E)\subseteq(M_2,E)$, we have, $s\tau_2-Int_w(M_1,E)\subseteq s\tau_2-Int_w(M_2,E)$. Then, $(F,E)\subseteq s\tau_2-Int_w(M_2,E)$. Therefore, (M_2,E) is $s\tau_1\tau_2-i*gw-openset$. ■

Theorem 2.15. Let (X,τ_1,τ_2,E) be $S.BI.T.S$ and $(M\ ,E)\subseteq X$ then the followings are true:

1. If $(M\ ,E)$ is $s\tau_2-w-open$, then it is $s\tau_1\tau_2-i*gw-open$.
2. If $(M\ ,E)$ is $s\tau_1-i-closed$ and $s\tau_1\tau_2-i*gw-open$, then it is $s\tau_2-w-open$.
3. If $(M\ ,E)$ is $s\tau_1\tau_2-i*gw-open$, then it is $s\tau_1\tau_2-gw-open$.
4. If $(M\ ,E)$ is $s\tau_1\tau_2-i*g-open$ then it is $s\tau_1\tau_2-i*gw-open$.
5. If $(M\ ,E)$ is $s\tau_1\tau_2-i*g-open$ then it is $s\tau_1\tau_2-g-open$.
6. If $(M\ ,E)$ is $s\tau_1\tau_2-g-open$ then it is $s\tau_1\tau_2-gw-open$.

Proof:

1. Suppose that $(M\ ,E)$ is $s\tau_2-w-open$. We have $(M\ ,E)^C$ is $s\tau_2-w-closed$. Then, $(M\ ,E)^C$ is $s\tau_1\tau_2-i*gw-closed$ (Theorem2.3(1)) .Therefore, $(M\ ,E)$ is $s\tau_1\tau_2-i*gw-open$.

2. Suppose that $(M\ ,E)$ is $s\tau_1-i-closed$ and $s\tau_1\tau_2-i*gw-open$. Then, $(M\ ,E)^C$ is $s\tau_1-i-open$ and $s\tau_1\tau_2-i*gw-closed$.

Then, $(M\ ,E)^C$ is $s\tau_2-w\ closed$ (Theorem2.3(2)). Therefore , $(M\ ,E)$ is $s\tau_2-w-open$.

3. Suppose that $(M\ ,E)$ is $s\tau_1\tau_2-i*gw-open$. Then, $(M\ ,E)^C$ is $s\tau_1\tau_2-i*gw-closed$, hence $(M\ ,E)^C$ is $s\tau_1\tau_2-gw-closed$ (Theorem2.3(3)). Therefore, $(M\ ,E)$ is $s\tau_1\tau_2-gw-open$.

4. Suppose that (M,E) is $s\tau_1\tau_2 - i*g - open$. Then, $(M,E)^C$ is $s\tau_1\tau_2 - i*g - closed$, hence $(M,E)^C$ is $s\tau_1\tau_2 - i*gw - closed$ (Theorem 2.4). Therefore, (M,E) is $s\tau_1\tau_2 - i*gw - open$.

5. Suppose that (M,E) is $s\tau_1\tau_2 - i*g - open$. Then, $(M,E)^C$ is $s\tau_1\tau_2 - i*g - closed$, hence $(M,E)^C$ is $s\tau_1\tau_2 - g - closed$ (Theorem 2.8(2))Therefore, (M,E) is $s\tau_1\tau_2 - g - open$.

6. Suppose that (M,E) is $s\tau_1\tau_2 - g - open$. Then, $(M,E)^C$ is $s\tau_1\tau_2 - g - closed$, hence $(M,E)^C$ is $s\tau_1\tau_2 - gw - closed$ (Theorem 2.8(3)).Therefore, (M,E) is $s\tau_1\tau_2 - gw - open$. ■

Remark 2.16. The converses of Theorem 2.15(4)(5)(6) are not true. Indeed, In Example 1.15 , $(M,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$ is $s\tau_1\tau_2 - i*gw - open$, but it is not $s\tau_1\tau_2 - i*g - open$ and $(M,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$ is $s\tau_2 - w - openset$, but it is not $s\tau_2 - open$. Also, $(M,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$ **is** $s\tau_1\tau_2 - g - openset$, but it is not $s\tau_1\tau_2 - i*g - openset$.

$(M,E)=\{(e_1,\{b,c\}),(e_2,\{b,c\})\}$ is $s\tau_1\tau_2 - gw - open$ set but it is not $s\tau_1\tau_2 - g - open$.

$$s\tau_1\tau_2 - i^*g - open\,set \;\; \underset{\not\Leftarrow}{\Rightarrow} \;\; s\tau_1\tau_2 - i^*gw - open$$

$$s\tau_2 - open\,set \;\; \underset{\not\Leftarrow}{\Rightarrow} \;\; s\tau_2 - w - open$$

$$s\tau_1\tau_2 - i^*g - open\,set \;\; \underset{\not\Leftarrow}{\Rightarrow} \;\; s\tau_1\tau_2 - g - open$$

$$s\tau_1\tau_2 - g - open\,set \;\; \underset{\not\Leftarrow}{\Rightarrow} \;\; s\tau_1\tau_2 - gw - open$$

Conclusions: From above we concluded that *(X, τ^i,E)* is not necessary to be *sTs* and *(X, τ,E)* is *S.T.E.S.I.* (i.e. *(X, τ^i,E)* is *sTs* if $\tau=\{\varphi,(W,E),X\}$ where, *(W,E)* is a soft single subset of *X* (containing only one element). Each $s\tau_1\tau_2 - i*g - openset$ is $s\tau_1\tau_2 - i*gw - open$, each $s\tau_2 - openset$ is $s\tau_2 - w - open$, each $s\tau_1\tau_2 - i*g - openset$ is $s\tau_1\tau_2 - g - open$ and each $s\tau_1\tau_2 - g - openset$ is $s\tau_1\tau_2 - gw - open$ set, but the converses are not true.

Acknowledgment : "The Authors are very grateful to the University of Mosul/ College of Education for Pure Sciences for their provided facilities, which helped to improve the quality of this work"

Author Contributions: All authors contributed equally in writing this article. All authors read and approved the final manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] K. Y., Al- Zoubi. On generalized w-closed sets, *Int. J. of Mathematics and Math. Sciences*, 13(2005), pp. 2011-2021.

[2] S.W., Askandar and A.A. Mohammed. Soft ii-open sets in soft topological spaces, *Open Access Library Journal*,7(5)(2020), pp. 1-18.

[3] S.S., Benchalli, P.G., Patil and T.D., Rayanagoudar. On w-Locally Closed Sets in Bitopological Spaces, *Int. Journal of Math. Analysis*, 4(39)(2010), pp. 1937-1944.

[4] R. K., Chandrasekhara and D., Narasemhan. Semi Star Generalized w-closed sets in Bitopological spaces, *Int. J. Contemp. Math. Sciences*, 4(12)(2009), pp. 587-595.

[5] R. K., Chandrasekhara and K., Kannan. Regular Star Generalized Closed Sets in Bitopological Spaces, *Thai J. of Math*., 4(2)(2006), pp. 341-349.

[6] B. M., Ittanagi. Soft Bitopological Spaces. *International Journal of Computer Applications*, 107(7)(2014), pp. 1-4.

[7] J.C., Kelly. Bitopological Spaces. *Proc. London Math. Soc*., 13(1963), pp. 71-81.

[8] Y.K., Mahdi. Semi-open and Semi-closed sets in Bitopological spaces, accepted in the first science Conference of education college , Babylon Univ., (2007), February 18-19.

[9] A.A., Mohammed, and S.W., Askandar. i-open sets in bi-topological spaces, *Al-Rafidain Journal of Computer Sciences and Mathematics*, 12(1)(2018), p. 13-23.

[10] D.A., Molodtsov. Soft set theory-first results, *Comp. Math. App.*, 37(4)(1999), pp. 19-31.

[11] N., Nagaveni, and P., Rajarubi. GRW-Closed Sets and GRW-Continuity in Bitopological spaces, *European Journal of Scientific Research*,78(1)(2012), pp. 5-14.

[12] M., Sheik John and S., Maragathavalli. Strongly $\alpha g*-$closed sets in Bitopological spaces, *Int. J. Contemp. Math. Sciences*, 5(17)(2010), pp. 805-813.

[13] M., Sheik John and P., Sundaram. g*-Closed Sets in Bitopological Spaces, *Indian J. Pure appl. Math.*, 35(1)(2004), pp. 71-80.