مجلة القادسية للعلوم الصرفة المجلد 13 العدد 1 لسنة 2008
المؤتمر العلمي الاول لكلية العلوم المنعقد في 26-27 اذار لسنة 2008

ALMOST TC – CONTINUOUS FUNCTIONS

By Prof. Dr. Hadi Jaber Mustafa , Mohammad S. Mechi

Dept of math.
College of Science
Kufa university

ABSTRACT

In this work a new concept namely Almost TC - continuous functions is introduced , which is stronger than the concept of almost of C-continuous functions that has been introduced by S. G. Hwang and studied by T. Noiri. ,Several properties of Almost TC-continuous functions have been stated and proved .

1. INTRODUCTION

Quite recently , S.G. Hwang [7] has introduced anew class of functions ,called almost C-continuous functions ,which contains the class of C-continuous functions , and that of almost continuous functions .The purpose of the present paper is to introduce anew concept which is stronger than the concept of almost C-continuous functions namely almost TC-continuous functions .Some of properties of almost TC-continuous functions has been proved.

2. PRELIMINARIES

Through bout the present paper spaces mean always topological spaces

,let A be a subset of a space X , The closure of A and the interior of A are denoted by cl(A) and int(A) respectively .

Def 2.1

A function $f : X \to Y$ is said to be C-continuous functions[2] if for each $x \in X$ and each open set v of f(x) in Y such that v^c is compact there exists an open set u of x in X such that $f(u) \subseteq v$

Before we introduce the next definition , we recall the followings definition :

Def 2.2

(a) Let (X,τ) be a Topological space , let $T : \tau \to p(X)$ be a function such that $w \subseteq T(w)$ where $w \in \tau$ then we say that T is an operator associated with τ and the triple (X,τ,T) is called an operator Topological space (O.T.S) [4] .

(b) Let (X,τ,T) be an (O.T.S) and $w \subseteq X$,we say that w is T-open in X if $\forall\ x \in w, \exists G\ \ open\ \ in\ \ X\ \ni \mathrm{x} \in \mathrm{G} \subseteq \mathrm{T(G)} \subseteq \mathrm{W}$ [4] it is clear that every T-open is open .

(c) Let (X,τ,T) be an O.T.S and $k \subseteq X$,we say that K is T- compact if : for each open cover of $\mathrm{A} = \{G_\alpha : \alpha \in \Lambda\}$ of K $\exists \alpha_1, \alpha_2, \alpha_3, ..., \alpha_n \ni K \subseteq \bigcup_{i=1}^{n} T(G_{\alpha_i})$

It is clear that every compact is T-compact [5]

Now we are ready to introduce the concept of TC-continuous functions as follows:

Def 2.3

Let $f:(X,\tau)\rightarrow(Y,\sigma,T))$ be a function from a T.s. (X,τ) to an o. T.s. (Y,σ,T) we say that f is TC-continuous if : for each $x\in X$ and each open set v of f(x) in Y such that V^{c} is T-compact , $\exists$ an open set u of x in X such that $f(u)\subseteq v$

It is clear that every Tc- continuous function is C – continuous.

A subset S of a space X is said to be H-closed [6] if for every cover $\{v_{\alpha}:\alpha\in\Omega\}$ of S by open sets of X, there exists $\alpha_1,\alpha_2,\alpha_3,...,\alpha_n$ such that : $S\subseteq\bigcup_{i=1}^{n}cl(v_{\alpha_i})$

Now we introduced the concept of TH-closed as set as follows:

Def 2.4

Let S be a a subset of an o.T.s. (X,τ,T) we say that S is TH-closed in X if for every cover $\{v_{\alpha}:\alpha\in\Omega\}$ of S by open sets of X there exists $\alpha_1,\alpha_2,\alpha_3,...,\alpha_n$ such that $S\subseteq\bigcup_{i=1}^{n}T(v_{\alpha_i})$

Def 2.5

A function $f:X\rightarrow Y$ is said to be H-continuous [1] if for each $x\in X$ and each open set V of f(x) such that V^{c} is H-closed ,there exists an open set U of x such that $f(u)\subseteq v$

Now we introduce the following definitions :

Def 2.6

A function $f:(X,\tau)\rightarrow(Y,\sigma,T))$ is said to be TH-continuous if for each $x\in X$ and each open set V of f(x) such that V^c is TH-closed , there exists an open set U of x such that

$f(u)\subseteq v$ when T is the closure operator we get the concept of H-continuous function

Now we recall definition of almost –continuous function :

Def 2.7:

A function $f:X\rightarrow Y$ is said to be almost-continuous [7]

If for each $x\in X$ and each open set V of f(x) , there exists an open set U of x such that $f(u)\subseteq Int(cl(v))$

Now we introduce the following definition:

Def 2.8

A function $f:(X,\tau)\rightarrow(Y,\sigma,T))$ is said to be almost T-continuous if for each $x\in X$ and each open set V of f(x) , there exists an open set U of x such that $f(u)\subseteq Int(T(v))$

When T is the closure operator almost T- continuous becomes Almost Continuous

Def 2.9

A function $f:X\rightarrow Y$ is said to be almost C – continuous [7] if for each $x\in X$ and each

open set v of f(x) such that v^c is compact , there exists an open set U of x such that

$f(u) \subseteq Int(cl(v))$ **almost continuous functions are almost c-continuous**

functions but the converse not true in general [7] the following theorem shows the relationships between the functions defined above.

Theorem 2-10

The followings implications hold and none of these implications can in general be reversed :

$$\text{Continuity} \Rightarrow H-\text{Continuity} \Rightarrow C-\text{Continuity} \Rightarrow \text{almost C-Continuity}$$

Proof

See[2] and [7]

Now we are ready to introduce the main concept of this work :

Def 2-11

A function $f:(X,\tau) \longrightarrow (Y,\sigma,T)$ **is said to be almost TC-continuous if for each** $x \in X$ **and each open set V of f(x) such that** V^c **is T-compact , there exists an open set U of x such that**

$$f(u) \subseteq Int(T(v))$$

Remark 2-12

We have the followings implications :

$$\begin{array}{ccc} \text{T-Continuity} & \Longrightarrow & almost-TC-\text{Continuity} \\ \Downarrow & & \Downarrow T = cl \\ C\text{-Continuity} & \Longrightarrow & almostC-continuity \end{array}$$

3- T-STRONGLY – CLOSED GRAPH

let $f : X \to Y$ be a function of a space X into Y . the subset $\{(x, f(x)) : x \in X\}$ of the product space X x Y is

called the graph of f and usually denoted by G(f)

we recall the followings definition :

Def 3.1

The graph G(f) is said to be strongly –closed [3] if for each

$(x, y) \not\in G(f)$ there exists

open sets $u \subset X$ and $v \subset Y$ and containing x and y respectively , such

that:

$$[u \ x \ \ cl(v)] \cap G(f) = \Phi$$

Now we Introduce the definition of T- strongly –closed graph as

Follows :

Def 3.2

Let $f : (X, \tau) \to (Y, \sigma, T))$ be a function , we say that the graph G(f) is T-strongly –closed if for each

$(x, y) \not\in G(f)$ there exist open sets $u \subset X, v \subset Y$ and containing x and y

respectively, such that : $[uxT(v)] \cap G(f) = \Phi$

The following lemma is a useful characterization of functions with

strongly – closed graphs , we give the proof for completeness.

Lemma 3-3 ([3])

The graph G(f) is strongly –closed if and only if for each

$(x, y) \notin G(f)$ **there exists**

$u \subset X, v \subset Y$ **and containing x and y , respectively such that**

$f(u) \cap cl(v) = \Phi$

Proof:

$\Longrightarrow$

Suppose G(f) is strongly –closed then for each $(x, y) \notin G(f)$

there exist open

sets $u \subset X, v \subset Y$ **and containing x and y , respectively such that**

$[u\ x\ cl(v)] \cap G(f) = \Phi$

We want to prove that $f(u) \cap cl(v) = \varphi$

Suppose $f(u) \cap cl(v) \neq \varphi$

Then $\exists y \in f(u) \cap cl(v)$

So $y \in f(u)$ **and** $y \in cl(v)$

$\exists x \in u \ni y = f(x)$

$(x, y) \in G(f)$ **And** $(x, y) \in uxcl(v)$

So that $[u\ x\ c\,l(v)] \cap G(f) \neq \Phi$ **which is a contradiction**

$\Longleftarrow$ **Suppose** $f(u) \cap c\,l(v) = \varphi$

Now we want to prove that $[u \, x \, c\,l(v)] \bigcap G(f) = \varphi$

Suppose $[u \, x \, c\,l(v)] \bigcap G(f) \neq \varphi$

So

$$\exists (x, y) \in [u \, x \, c\,l(v)] \bigcap G(f)$$

It mean that $(x, y) \in u \, x \, c\,l(v)$ **and** $(x, y) \in G(f)$

$$y = f(x), x \in U, y \in f(U)$$

So that $f(U) \bigcap cl(v) \neq \varphi$

Which is a contradiction

The proof of the following lemma is similar

Lemma 3-4

Let $f : (X, \tau) \to (Y, \sigma, T))$

Be a function . The graph G(f) is T- strongly-closed if and only if for each $(x, y) \notin G(f)$**,there exist open sets** $U \subset X$ **and** $V \subset Y$ **containing x and y respectively , such that**

$$f(U) \bigcap T(v) = \Phi$$

Now we are ready to prove the following important theorem

Theorem 3-5

If a function $f : (X, \tau) \to (Y, \sigma, T))$ **has a T- strongly –closed graph , then it is TH-Continuous.**

Proof

Suppose that G(f) is T-strongly –closed . let K be any TH-closed set of Y

and $x \notin f^{-1}(k)$ **for each** $y \in k$ **,** $(x, y) \notin G(f)$ **hence by lemma 3-4 ,**

there exists open sets

$U_y(x) \subset X$ **and** $v(y) \subset Y$ **containing x and y , respectively ,**

such that $f(U_y(x)) \bigcap T(v(y)) = \varphi$

Now , The family $\{v(y) : y \in k\}$ **is a cover of K by open sets of y**

Hence , there exists a finite subset k_0 **o f k such that**

$k \subset \bigcup \{T(v(y)) : y \in k_0\}$

Put $U = \cap \{U_y(x)) : y \in k_0\}$

Then U is an open set of X containing x and $U \bigcap f^{-1}(k) = \varphi$

This show that $f^{-1}(k)$ **is a closed set of X**

Now we claim that f is TH-continuous

Let $x \in X$ **, let v be an open set of f(x) such that** v^c **is TH-closed**

Let $k = v^c$

The above proof shows that $f^{-1}(k)$ **is closed in X**

Let $U = X - f^{-1}(k)$

Then U is open in X and $x \in U$

Now

$$\begin{aligned} f(U) &= f(X - f^{-1}(K)) = f(X) - K \\ &\subseteq \mathrm{Y - k = k^c = v} \end{aligned}$$

$f(U) \subset v$

So that f is TH- continuous

Before we state the next theorem we need the following definition :

Def 3-6

Let (Y,σ,T) **be an o.T.s. we say that [5]**

- **Y is T-Hausdorf if given** $y_1, y_2 \in Y, y_1 \neq y_2$ **then there exist the open sets** v_1, v_2 **in Y**

  such that $y_1 \in v_1, y_2 \in v_2, T(v_1) \bigcap T(v_2) = \varphi$

- **Y is T-locally compact regular if given** $y \in Y$ **,** v_1 **open in Y**

  containing y , then there exists open set v in Y such that :

  $y \in v \subset T(v) \subset v_1$ **and T(v) is compact**

- **T is called regular closed operator if :**
  - **T(v) is regular closed**
  - **Int(T(v))=v ,** $v \in \tau$

Theorem 3-7

if Y is T- Hausdor and T-locally compact regular and

$f:(X,\tau) \to (Y,\sigma,T))$ **is almost TC-continuous function also T is regular closed, then G(f) is T- strongly – closed**

Proof

Let $(x,y) \notin G(f)$ **then** $y \neq f(x)$ **and**

Hence there exist disjoint open sets v_1 **and** v_2 **containing y and f(x) ,**

respectively such that

$$T(v_1)\bigcap T(v_2)=\varphi$$

Since Y is T – locally compact regular , there exists an open set V such that $y\in v\subset T(v)\subset v_1$

and T(v) is compact

Now we claim that $f^{-1}(T(v))$ **is closed in X**

Let $x^{*}\notin f^{-1}(T(v))$

Consider $y^{*}=f(x^{*})$

Now $f(x^{*})\in\big(T(v)\big)^{c}$ **which is open**

Now f is almost TC-continuous , then there exists u^{*} **open in X**

containing x^{*} **and**

$f(u^{*})\subset Int(T((T(v))^{c}))$ **Hence** $f(u^{*})\in\big(T(v)\big)^{c}$

So that $u^{*}\bigcap f^{-1}\big(T(v)\big)=\varphi$ **and Hence** $u^{*}\subset\Big(f^{-1}\big(T(v)\big)\Big)^{c}$

This means that $\Big(f^{-1}\big(T(v)\big)\Big)^{c}$ **is open**

Hence $f^{-1}\big(T(v)\big)$ **is closed**

Let $U=X-f^{-1}(T(v))$

So $x\in U$ **and U is open**

And $f(u)\bigcap T(v)=\varphi$

So G(f) is T-strongly closed .

REFERENCES

1. P.E. long and T.R. Hamlett , H-continuous functions , Boll. un. mat. tal (4) , II(1975),552-558 .
2. P.E. long and m.p. Hendrix , properties of C-continuous functions, York hama Math. J.22 (1974) ,117-123 .
3. P.E. long and L.L Herrington Functions with strongly-closed graphs,B011.Un Mat. Ital- (4,12,(1975)-384.
4. Hadi J. Mustafa and A. Adul Haisan ,T-open sets ,M.Sc. thesis , Mutta university,2004 .
5. Hadi J. Mustafa and A . Abdul Hassan , T-compact spaces , Journal of Mutta university,2005 .
6. J.porter and J . Thomas hausdurf spaces , Trans ,Amer math. Sec. 138(1969),159-170 .
7. S.G.Hwang, almost C-continuous functions ,J. kerean Math . sec. 14,(1978),229-234 .